Seznam vzorců pro zápočtovou písemku.

$$i\hbar\frac{|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle$$

Důležité komutátory

$$[\hat{x}_\alpha, \hat{p}_\beta] = i\hbar\delta_{\alpha,\beta} \qquad \left[\hat{J}_\alpha, \hat{V}_\beta\right] = i\hbar\varepsilon_{\alpha,\beta,\gamma}\hat{V}_\gamma$$

Moment hybnosti a sférické harmoniky

$$\hat{J}^2|jm\rangle = \hbar^2 j(j+1)|jm\rangle \qquad \hat{J}_z|jm\rangle = \hbar m|jm\rangle \qquad \hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$$

$$\hat{J}_\pm|jm\rangle = \hbar\sqrt{(j\mp m)(j\pm m+1)}|jm\pm 1\rangle$$

Pauliho matice

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Několik prvních sférických harmonik:

$$\begin{array}{lll}
Y_{00}=\frac{1}{\sqrt{4\pi}} & Y_{22}=\sqrt{\frac{15}{32\pi}}\frac{(x+iy)^2}{r^2} & Y_{33}=-\sqrt{\frac{35}{64\pi}}\frac{(x+iy)^3}{r^3} \\
= & Y_{21}=-\sqrt{\frac{15}{8\pi}}\frac{z(x+iy)}{r^2} & Y_{32}=\sqrt{\frac{105}{32\pi}}\frac{z(x+iy)^2}{r^3} \\
= & Y_{20}=\sqrt{\frac{5}{16\pi}}\frac{2z^2-x^2-y^2}{r^2} & Y_{31}=-\sqrt{\frac{21}{64\pi}}\frac{(4z^2-x^2-y^2)(x+iy)}{r^3} \\
Y_{11}=-\sqrt{\frac{3}{8\pi}}\frac{x+iy}{r} & Y_{2-1}=\sqrt{\frac{15}{8\pi}}\frac{z(x-iy)}{r^2} & Y_{30}=\sqrt{\frac{7}{16\pi}}\frac{z(2z^2-3x^2-3y^2)}{r^3} \\
Y_{10}=\sqrt{\frac{3}{4\pi}}\frac{z}{r} & Y_{2-2}=\sqrt{\frac{15}{32\pi}}\frac{(x-iy)^2}{r^2} & Y_{3-1}=\sqrt{\frac{21}{64\pi}}\frac{(4z^2-x^2-y^2)(x-iy)}{r^3} \\
Y_{1-1}=\sqrt{\frac{3}{8\pi}}\frac{x-iy}{r} & = & Y_{3-2}=\sqrt{\frac{105}{32\pi}}\frac{z(x-iy)^2}{r^3} \\
= & = & Y_{3-3}=\sqrt{\frac{35}{64\pi}}\frac{(x-iy)^3}{r^3}
\end{array}$$

Harmonický oscilátor

$$x_0 = \sqrt{\frac{\hbar}{m\omega}} \qquad p_0 = \frac{\hbar}{x_0} \qquad \hat{a} = \frac{\hat{x}/x_0 + i\hat{p}/p_0}{\sqrt{2}}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2) \qquad \left[\hat{a}, \hat{a}^\dagger\right] = 1$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\langle x|n\rangle = \frac{H_n(x/x_0)}{\sqrt{\sqrt{\pi}x_0 n!2^n}}\exp(-\tfrac{1}{2}(\tfrac{x}{x_0})^2) \qquad \langle p|n\rangle = (-i)^n\frac{H_n(p/p_0)}{\sqrt{\sqrt{\pi}p_0 n!2^n}}\exp(-\tfrac{1}{2}(\tfrac{p}{p_0})^2)$$

Hermitovy polynomy

$$H_0(x) = 1,\ H_1(x) = 2x,\ H_2(x) = 4x^2 - 2,\ H_3(x) = 8x^3 - 12x.$$

Coulombický potenciál

$$V(x) = \gamma/r, \qquad E_n = -\frac{m\gamma^2}{2\hbar^2 n^2}, \qquad a = \frac{\hbar^2}{m|\gamma|}$$

Radiální vlnová $R_{nl}(r)$ funkce pro prvních pár stavů:

$$\begin{aligned}
R_{10}(r) &= 2\sqrt{\frac{1}{a^3}}\exp(-r/a) \\
R_{20}(r) &= \sqrt{\frac{1}{2a^3}}(1 - r/2a)\exp(-r/2a) \\
&= \frac{1}{\sqrt{2}}\sqrt{\frac{1}{a^3}}(1 - r/2a)\exp(-r/2a)
\end{aligned}$$