

# Homework #2

Assigned: 1.11.2019

Deadline: 15.10.2019

## Symmetry group of benzene and character table of its subgroup $C_{6v}$ (13 points)

1. (7 points) Determine the symmetry group of benzene molecule ( $C_6H_6$ ) in its equilibrium geometry and construct character table of its 12-element subgroup  $C_{6v}$ . The solution must contain unambiguous step by step explanation of the procedure.
2. (6 points) Consider function space with a basis of  $\{x^2, y^2, z^2, xy, xz, yz\}$ . Determine irreducible invariant subspaces of this vector space under the action of  $C_{6v}$  and assign them (i.e., their bases) to the irreducible representations found in the previous sub-task.

*Note:* Systematic solution of the second sub-task will be only discussed in the 8.11. lecture. The key is the decomposition of a representation obtained as a direct product of two vector representations (with representation space defined by the basis  $\{x, y, z\}$ ). However, the problem can be solved already now with the aid of some geometrical visualization.

## Cyclic group (7 points)

1. (4 points) Construct the character table of an abstract 4-element cyclic group.
2. (3 points) Find some isomorphic point group and determine according to which irreducible representations are transformed the vectors (linear functions  $x, y, z$ ) and pseudovectors.

*Hint:* Cyclic group is Abelian and has, therefore, only one-dimensional complex irreducible representations. For the sub-task 2 it is however necessary to find real representations, which might be higher-dimensional and, strictly speaking, reducible).