Homework #4

Assigned: 13.12.2019 **Deadline: 10.1.2020**

Group $GA(1, \mathbb{R})$

Group $GA(1, \mathbb{R})$ is two-dimensional Lie group ($g = g(a, b)$) of linear transformations of the form

$$x' = ax + b, \quad a, b \in \mathbb{R},\ a \neq 0.$$

1. *(5 points)* Find general form of left-invariant vector fields on this group and determine (in some suitable basis) structure constants of the corresponding Lie algebra $\mathfrak{ga}(1, \mathbb{R})$.
2. *(5 points)* Find the one-parametric subgroup corresponding to general element of the Lie algebra $\mathfrak{ga}(1, \mathbb{R})$.

Group $SL(2, \mathbb{R})$

Group $SL(2, \mathbb{R})$ is group of real matrices $A_{2\times 2}$ with $\det A = 1$.

1. *(7 points)* Find one-parametric subgroups of $SL(2, \mathbb{R})$ and determine traces of the matrices of these subgroups.

   Hint: Write down general form of a matrix $C \in \mathfrak{sl}(2, \mathbb{R})$ and compute directly $\exp(tC)$ (depending on the sign of $\det C$).
2. *(3 points)* Show that exponential mapping does not cover the whole (non-compact) group $SL(2, \mathbb{R})$, even though the group is connected.