

Ilustrace metody fiktivních nábojů pro dva vodiče ve tvaru koule

Příklad se počítá na cvičení z elektrodynamiky

(viz <http://utf.mff.cuni.cz/~ledvinka/?278656> a <http://utf.mff.cuni.cz/~ledvinka/PrElektrodynamika/Kapacity.pdf>)

(* funkce počítají to, co my na cvičení, jen kvůli pohodlí začínám s indexy od jedné *)

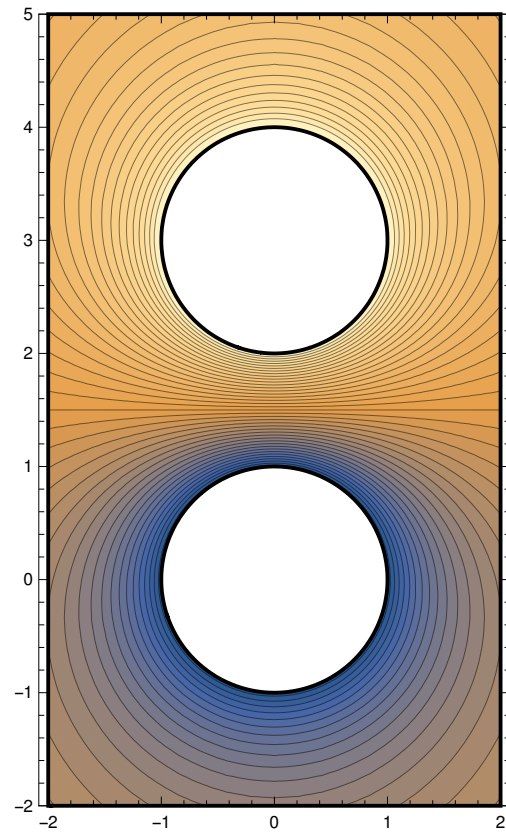
```
spoctiQS[a_, b_, d_, U1_, U2_, n_Integer] := Block[{s, S, q, Q},
  s[1] = 0;
  S[1] = 0;
  q[1] = a U1;
  Q[1] = b U2;
  Do[
    s[i] = a^2 / (d - S[i - 1]);
    S[i] = b^2 / (d - s[i - 1]);
    q[i] = -a / (d - S[i - 1]) Q[i - 1];
    Q[i] = -b / (d - s[i - 1]) q[i - 1];
    , {i, 2, n}];
  Transpose[Table[{q[i], Q[i], s[i], S[i]}, {i, 1, n}]]
]
Phi[a_, b_, d_, U1_, U2_, n_Integer] := Block[{s, S, q, Q, i, tmp},
  {q, Q, s, S} = spoctiQS[a, b, d, U1, U2, n];
  tmp = Evaluate[Sum[q[[i]] / Sqrt[x^2 + (z - s[[i]])^2] + Q[[i]] / Sqrt[x^2 + (z - d + S[[i]])^2], {i, 1, n}]];
  Function[{x, z}, Evaluate[tmp]]
]
maticeKapacit[a_, b_, d_, n_Integer] := Block[{s, S, q, Q, U1, U2, i, tmp},
  {q, Q, s, S} = spoctiQS[a, b, d, U1, U2, n];
  D[{Total[q], Total[Q]}, {{U1, U2}}]
]
maticeKapacit[a, b, d, 4] // Simplify // MatrixForm (*Bez Simplify není vidět, že Cab je symetrická*)
```

$$\begin{pmatrix} a - \frac{a^2 b}{b^2 - d^2} & \frac{a b (a^2 - a b + b^2 - d^2)}{d (-a^2 - b^2 + d^2)} \\ \frac{a b (a^2 - a b + b^2 - d^2)}{d (-a^2 - b^2 + d^2)} & b - \frac{a b^2}{a^2 - d^2} \end{pmatrix}$$

```
vzdalenost = 3;  
polomer1 = 1;  
polomer2 = 1;
```

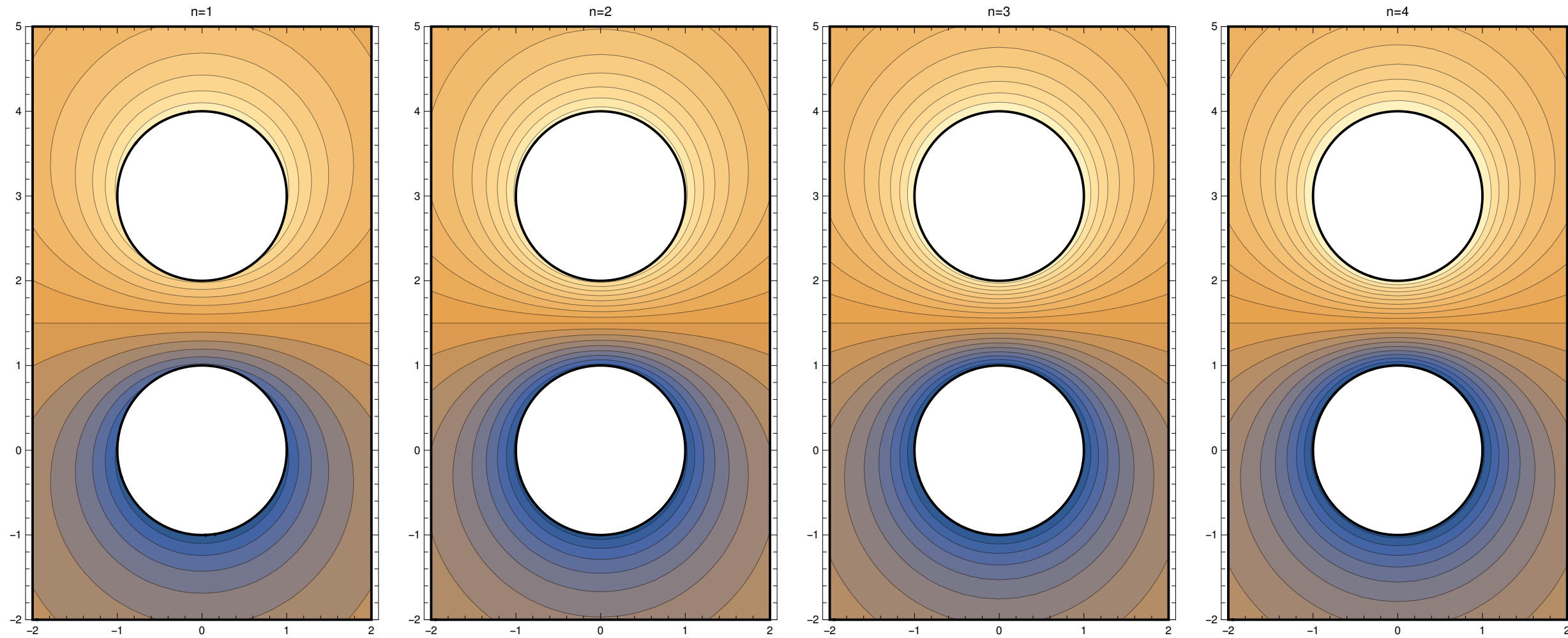
```
 $\phi = \text{Phi}[\text{polomer1}, \text{polomer2}, \text{vzdalenost}, -1, 1, 5];$ 
```

```
ContourPlot[ $\phi[x, z]$ , {x, -2, 2}, {z, -2, 2 + vzdalenost}, AspectRatio  $\rightarrow$  Automatic, PlotPoints  $\rightarrow$  20,  
Contours  $\rightarrow$  40, PlotRange  $\rightarrow$  {-1, 1},  
RegionFunction  $\rightarrow$  Function[{x, z},  $x^2 + z^2 > \text{polomer1}^2 \ \&\& \ x^2 + (z - \text{vzdalenost})^2 > \text{polomer2}^2$  ], BoundaryStyle  $\rightarrow$  {Thick, Black}]
```



(*Ukázka vlivu počtu fiktivních nábojů*)

```
Table[ContourPlot[Phi[polomer1, polomer2, vzdalenost, -1, 1, n][x, z], {x, -2, 2}, {z, -2, 2 + vzdalenost}, AspectRatio -> Automatic, PlotPoints -> 20,
  Contours -> 20, ImageSize -> 300, PlotRange -> {-1, 1}, PlotLabel -> "n=" <> ToString[n],
  RegionFunction -> Function[{x, z}, x^2 + z^2 > polomer1^2 && x^2 + (z - vzdalenost)^2 > polomer2^2], BoundaryStyle -> {Thick, Black}],
{n, 1, 4}] // GraphicsRow
```



Varianta s $U_1=0$, $U_2=1$

```
Table[ContourPlot[Phi[polomer1, polomer2, vzdalenost, 0, 1, n][x, z], {x, -2, 2}, {z, -2, 2 + vzdalenost}, AspectRatio -> Automatic, PlotPoints -> 20,
  Contours -> 20, ImageSize -> 300, PlotRange -> {0, 1}, PlotLabel -> "n=" <> ToString[n],
  RegionFunction -> Function[{x, z}, x^2 + z^2 > polomer1^2 && x^2 + (z - vzdalenost)^2 > polomer2^2], BoundaryStyle -> {Thick, Black}],
{n, 1, 4}] // GraphicsRow
```

