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The twin paradox in static spacetimes and Jacobi fields

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The twin paradox in special relativity has a clear geometrical meaning, yet in most textbooks its resolution is given only in the simplest case and provides no deeper understanding of the effect. Vaguely speaking, in flat spacetime the twin moving at a higher nonuniform velocity is the younger one. In curved spacetimes a multitude of possibilities occur and the simplest case in Schwarzschild spacetime, a non-geodesic twin at (absolute) rest and a geodesic twin moving on a circular orbit, is in some sense misleading. In general one can only determine which worldline connecting the separation and the reunion point makes the twin following it the oldest one. This is the timelike geodesic without points conjugate to the initial (separation) point on the segment ending at the reunion point. The conjugate points exist if any Jacobi vector field (any solution of the geodesic deviation equation) vanishes at a point provided that the field vanishes at the initial point. We therefore investigate Jacobi fields on timelike geodesics in a number of static space- times. For physical reasons we study only simplest geodesic curves: circular orbits (if they exist) and radial trajectories (flights upwards and downwards) and calculate their lengths. For comparison we compute the length of a non-geodesic worldline of a static observer between the separation and the reunion. In all the spacetimes under consideration the radial geodesics are the longest ones and except de Sitter space they contain conjugate points outside the relevant segment; there may be two points or an infinite sequence of conjugate points. All static spherically symmetric spacetimes have the same properties concerning circular geodesics (if they exist): there exist two infinite sequences of conjugate points. We also compare the radial time- like geodesics in Robertson-Walker spacetime with these in static metrics and find qualitative similarities. The Jacobi fields may be effectively studied only in spacetimes with a high symmetry since the existence of integrals of motion generated by Killing vector fields is crucial for solving the geodesic deviation equation.