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Stability of Black Holes and Black Branes

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I describe recent work with Stefan Hollands (arXiv:1201.0463) that establishes a close relationship between dynamical stability and thermodynamic stability for black holes and black branes in classical general relativity in spacetime dimensions $D \geq 4$. We show that for axisymmetric perturbations of an arbitrary stationary, axisymmetric black hole, dynamical stability is equivalent to the positivity of canonical energy of perturbations that have vanishing linearized ADM mass and angular momentum at infinity. We further show that positivity of canonical energy is equivalent to thermodynamic stability. A thermodynamically unstable black hole may be dynamically stable (as is the case for a Schwarzschild black hole) if the only perturbations with negative canonical energy have nonvanishing linearized mass and/or angular momentum. However, we show that all black branes associated with thermodynamically unstable black holes must be dynamically unstable, as conjectured by Gubser and Mitra. We also prove that positivity of canonical energy for perturbations with vanishing linearized mass and angular momentum is equivalent to the satisfaction of a "local Penrose inequality," thus showing that satisfaction of this local Penrose inequality is necessary and sufficient for dynamical stability. Although we explicitly consider vacuum general relativity, most of our results are derived using general Lagrangian and Hamiltonian methods and therefore can be generalized to allow for the presence of matter fields and/or to the case of an arbitrary diffeomorphism covariant gravitational action.