Classical and Quantum Scattering in Impulsive Backgrounds

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Problem

We investigate the scattering of classical (i.e.trajectories) as well as quantum (i.e.wave-equations) particles by impulsive gravitational as well as Yang-Mills fields. In both cases the pulse will be modellized to be concentrated on a null-hypersurface. This immedeately entails free propagation "above" and "below" the pulse, the physical dynamics being encoded in the in the junction conditions. The physical simplicity, however, comes at a price of singular products of distributions. These will be handled within the framework of Colombeaus's algebra of new generalized functions, leaving us, mathematically, with finite ambiguities at the distributional level. However, by invoking additional *physical* conditions we show that the value of this ambiguity can be determined.

Impulsive pp-wave

Prototypic example: Particle scattering in impulsive pp-waves

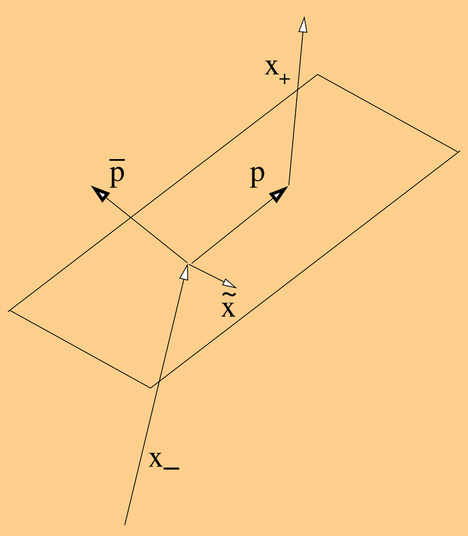
$$g_{ab} = \eta_{ab} + f p_a p_b \qquad \begin{array}{ll} f & \text{wave profile} \\ p_a & \text{cov.const null vector} \end{array}$$

Geodesic equation $(u\nabla)u^a = 0$ in impulsive, i.e. $f = \delta \tilde f$, pp-wave geometries

$$\left[\begin{array}{ll} (\bar p x)'' - \delta(\tilde x' \partial)\tilde f - \frac{1}{2}\delta' \tilde f & \approx \; 0 \\ \tilde x^{a\,\prime\prime} - \frac{1}{2}\delta \tilde\partial^a \tilde f & \approx \; 0 \end{array}\right]$$

gluing the solutions of $(u\partial)u^a = 0$ "above" and "below" the pulse

$$x^a = \theta_+ x^a_+ + \theta_- x^a_- \qquad \begin{array}{l} \theta_+ = \theta \\ \theta_- = 1 - \theta \end{array}$$



we obtain by inserting into the full equations

$$\begin{aligned} \tilde x^a_+(0) &= \tilde x^a_-(0) \\ \tilde x^a_+{}'(0) &= \tilde x^a_-{}'(0) + \frac{1}{2}\tilde\partial^a \tilde f(\tilde x(0)) \\ (\bar p x)_+(0) &= (\bar p x)_-(0) + \frac{1}{2}\tilde f(\tilde x(0)) \\ (\bar p x)'_+(0) &= (\bar p x)'_-(0) + \frac{1}{2}\tilde\partial \tilde f(\tilde x(0)) \cdot \\ & \quad (\tilde x'_-(0) + \frac{A}{2}\tilde\partial \tilde f(\tilde x(0))) \end{aligned}$$

where $\theta\delta \approx A\delta$ gives rise to the "ambiguity" A. From the physical condition of constant time-flow along the geodesic, i.e. $(u\partial)(u^a u^b g_{ab}) \approx 0$, which in the smooth context is a consequence of geodeticity of u^a A is fixed to $1/2$.

Spinning particle in an impulsive gravitaional field

Classical spinning particle in curved spacetime (Matthisson-Papapetrou equation):

$$(u\nabla)S^{ab} = 0 \quad (u\nabla)u^a + \frac{1}{2}R^a{}_{bcd}S^{cd}u^b = 0$$

Corresponding quantum particle (Einstein-Dirac equation):

$$(\gamma^\alpha D_\alpha - m)\psi = (\gamma^a(E_\alpha - \frac{1}{2}\Sigma_{\gamma\delta}\omega^{\gamma\delta}{}_\alpha) - m)\psi = 0$$

Analogous Yang-Mills system:
Classical non-Abelian, charged particle (Yang-Mills-Lorentz equation):

$$(uD)q^i = 0 \quad (u\partial)u^a + q^i F^{ia}{}_b u^b = 0$$

Corresponding quantum particle (Yang-Mills-Dirac equation):

$$(\gamma^a D_\alpha - m)\psi = (\gamma^\alpha(\partial_\alpha - [A_\alpha, .]) - m)\psi = 0$$

pp-wave Yang-Mills field:

$$A^i_a = f^i p_a \quad F = dA + \frac{1}{2}[A, A] = \tilde d f \wedge p$$

Impulsive YM-pp-field

Particle scatering in impulsive Yang-Mills profiles $f^i = \delta \tilde f^i$
Lorentz-Yang-Mills equation becomes:

$$\left[\begin{array}{ll} q^{i\,\prime} + \delta[\tilde f, q]^i & \approx \; 0 \\ (\bar p x)'' + \delta q^i (\tilde x' \tilde\partial)\tilde f^i & \approx \; 0 \\ \tilde x^{a\,\prime\prime} + \delta q^i \tilde\partial^a \tilde f^i & \approx \; 0 \end{array}\right]$$

gluing the solutions of $(u\partial)q^i = 0$ and $(u\partial)u^a = 0$ "above" and "below" the pulse

$$\begin{array}{lll} q^i &= \theta_+ q^i_+ + \theta_- q^i_- & \theta_+ = \theta \\ x^i &= \theta_+ x^i_+ + \theta_- x^i_- & \theta_- = 1 - \theta \end{array}$$

we obtain by inserting into the full equations

$$\begin{aligned} &(id + A[\tilde f, .])q^i_+(0) = \\ &\qquad (id + (1-A)[\tilde f, .])q^i_-(0) \\ &\tilde x^a_+(0) = \tilde x^a_-(0) \\ &\tilde x^a_+{}'(0) = \tilde x^a_-{}'(0) \\ &\quad + (Aq^i_+(0) + (1-A)q^i_-(0))\tilde\partial^a \tilde f^i(\tilde x(0)) \\ &(\bar p x)_+(0) = (\bar p x)_-(0) \\ &(\bar p x)'_+(0) = (\bar p x)'_-(0) + \big[Bq^i_+(0)\tilde x^a_+{}'(0) \\ &\quad + (A-B)(q^i_+(0)\tilde x^a_-{}'(0) + q^i_-(0)\tilde x^a_+{}'(0)) \\ &\quad + (1 - 2A + B)(q^i_-(0)\tilde x^a_-{}'(0))\big] \\ &\qquad \tilde\partial^a \tilde f^i(\tilde x(0)) \end{aligned}$$

where two "ambiguities" A and B arise from $\theta\delta \approx A\delta$ and $\theta^2\delta \approx B\delta$. From the physical conditions of constant time flow $(u\partial)(u^2) = 0$ and charge conservation $(u\partial)(q^2) = 0$, which in the smooth context are follow from the Lorentz-Yang-Mills equations, we obtain $A = 1/2$ and $B = 1/4$.

Colombeau theory

Nonlinear generalized functions

(f_ϵ) family of (moderate) C^∞ functions $\hat=$ regularisation models the microscopic aspect of the object; objects form an algebra, i.e.non-linear operations are well-defined.

$(f_\epsilon) \approx (g_\epsilon)$ association modellizes macroscopic - i.e.distributional - equality

$$:\Leftrightarrow \lim_{\epsilon \to 0} \int (f_\epsilon - g_\epsilon)\varphi d^n x = 0 \;\forall \varphi$$

Special case: $(f_\epsilon) \approx T \in \mathcal{D}'$ Colombeau object associated to a distribution

$$\lim_{\epsilon \to 0}\left[\int f_\epsilon \varphi d^n x - (T, \varphi)\right] = 0 \;\forall \varphi$$

Important: Association respects linear operations like addition, scalar multiplication, multiplication by C^∞ functions and differentiation BUT in general NOT multiplication

cf. J.F.Colombeau, E.E.Rosinger