Exact Black Holes and Universality in the backreaction of scalar fields in AdS4.

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The Einstein scalar system.

Very old problem… (Posed by Wheeler. Different theorems by Bekenstein, Heusler, Sudarsky). Renewed interest from AdS/CFT and future direct observations of SgrA*.

$$S(g,\phi) = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right],$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu},$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial_\beta\phi g^{\alpha\beta} - g_{\mu\nu}V(\phi),$$

$$ds^2 = \Omega(r)(-F(r)dt^2 + \frac{dr^2}{F(r)} + d\Sigma^2),$$

The Petrov type D solutions.

$$C_{\mu\nu\rho\sigma} = \lambda^{-1} C^{(N)}_{\mu\nu\rho\sigma} + \lambda^{-2} C^{(III)}_{\mu\nu\rho\sigma} + \lambda^{-3} C^{(II)}_{\mu\nu\rho\sigma} + \lambda^{-4} C^{(I)}_{\mu\nu\rho\sigma} + \mathcal{O}(\lambda^{-5})$$

$$ds^2 = S(q,p)\left(\frac{1+p^2q^2}{Y(q)}dq^2 + \frac{1+p^2q^2}{X(p)}dp^2 - \frac{Y(q)}{1+p^2q^2}\left(p^2 d\sigma + d\tau\right)^2 + \frac{X(p)}{1+p^2q^2}\left(d\sigma - q^2 d\tau\right)^2\right)$$

$$ds^2 = \Omega(r)(-F(r)dt^2 + \frac{dr^2}{F(r)} + d\Sigma^2),$$

It is actually possible to solve a slightly more general model.

$$S(g,\phi) = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\xi}{12}\phi^2 R - V(\phi)\right],$$

$$E_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \kappa T_{\mu\nu},$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(\partial\phi\right)^2 - g_{\mu\nu}V(\phi) + \frac{\xi}{6}\left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\phi^2$$

$$\Box\phi = \frac{\xi}{6}R\phi + \frac{\partial V}{\partial\phi},$$

$$ds^2 = \frac{S(q,p)}{-6+\xi\kappa\phi^2}\left(\frac{1+p^2q^2}{Y(q)}dq^2 + \frac{1+p^2q^2}{X(p)}dp^2 - \frac{Y(q)}{1+p^2q^2}\left(p^2d\sigma + d\tau\right)^2 + \frac{X(p)}{1+p^2q^2}\left(d\sigma - q^2d\tau\right)^2\right),$$

$$\phi = \phi(q,p).$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(\partial\phi\right)^2 - g_{\mu\nu}V(\phi) + \frac{\xi}{6}\left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\phi^2$$

$$\xi = 0 \Rightarrow T^\tau_\sigma = 0 = T^\sigma_\tau$$

The system is integrable!

From two field equations, that are independent from the scalar field model, it is possible to integrate the gravitational field that this generic scalar model generates.

$$X(p) = C_0 + C_2 p^2 + C_4 p^4 + C_1 p^{-\nu+2} + C_3 B_3 p^{\nu+2},$$
$$Y(q) = C_4 - C_2 q^2 + C_0 q^4 + C_3 C_1 q^{-\nu+2} + B_3 q^{\nu+2}.$$

$$S(q,p) = C \frac{p^{\nu-1} q^{\nu-1}}{(C_3 p^\nu + q^\nu)^2}.$$

The scalar field can also be found.

$$E_\tau^\tau - E_q^q = 0 \Leftrightarrow (\partial_q \phi)^2 = \frac{\nu^2 - 1}{12 q^2 \kappa} \frac{\left(\xi \kappa \phi^2 - 6\right)^2}{\left(\kappa \xi \left(\xi - 1\right) \phi^2 + 6\right)},$$

$$E_\sigma^\sigma - E_p^p = 0 \Leftrightarrow (\partial_p \phi)^2 = \frac{\nu^2 - 1}{12 p^2 \kappa} \frac{\left(\xi \kappa \phi^2 - 6\right)^2}{\left(\kappa \xi \left(\xi - 1\right) \phi^2 + 6\right)},$$

$$E_p^q = 0 \Leftrightarrow \partial_q \phi \partial_p \phi = -\frac{\nu^2 - 1}{12 p q \kappa} \frac{\left(\xi \kappa \phi^2 - 6\right)^2}{\left(\kappa \xi \left(\xi - 1\right) \phi^2 + 6\right)}.$$

$$q \partial_q \phi + p \partial_p \phi = 0 \Leftrightarrow \phi = F\left(\frac{q}{p}\right) \equiv F(z)$$

$$\left(F'\right)^2 = \frac{\left(\nu^2 - 1\right)\left(\xi \kappa F^2 - 6\right)^2}{12 \kappa z^2 \left(\kappa \xi \left(\xi - 1\right) F^2 + 6\right)}$$

The most general potential compatible with Einstein Gravity and this Petrov type D gravitational field is (when there are no vectors):

$$V(\phi, z) = (12\kappa C)^{-1}\left[C_0(\nu-1)(\nu-2)z^{\nu+1} + C_4(\nu+1)(\nu+2)z^{\nu-1} - 4C_3(\nu^2-1)(C_0 z + C_4 z^{-1}) + C_3^2 C_0(\nu+1)(\nu+2)z^{-\nu+1} + C_3^2 C_4(\nu-1)(\nu-2)z^{-\nu-1}\right](-6+\xi\kappa\phi^2)^2$$

$$\phi = \pm\sqrt{\frac{\nu^2-1}{2\kappa}}\ln(h\frac{q}{p})$$

$$\phi = \pm\sqrt{\frac{6}{\kappa}\frac{q^\mu - p^\mu + h(q^\mu + p^\mu)}{q^\mu + p^\mu + h(q^\mu - p^\mu)}}, \qquad \mu^2 = \frac{\nu^2-1}{3}.$$

$$z = \left[\frac{h-1}{h+1}\frac{\phi\sqrt{\kappa}+6}{\phi\sqrt{\kappa}-6}\right]^{\frac{1}{\mu}}$$

The result is Universal: The non-linear sigma model.

$$S(g,\phi^C) = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} - \frac{1}{2}G_{AB}(\phi)\,\partial_\mu\phi^A\partial_\nu\phi^B g^{\mu\nu} - V(\phi)\right]$$

$$T^\tau_\sigma = 0 = T^\sigma_\tau$$

Many interesting solutions.

The first exact AF Hairy BHS in D=4.

Probably what is most remarkable is the connection with the Schwarzschild solution

$$ds^2 = S(r)(-F_\nu(r)dt^2 + \frac{dr^2}{F_\nu(r)} + d\Omega),$$

$$S(r) = \frac{\nu^2\eta^{\nu-1}r^{\nu-1}}{(r^\nu - \eta^\nu)^2},$$

$$F_\nu(r) = \nu^{-1}\left(1 + \frac{2\eta^\nu r^{-\nu}}{\nu-2}\right)r^2 - \frac{\eta^2}{\nu-2} + \left(\left(\eta^\nu - \frac{r^\nu}{\nu+2} + \frac{\eta^{2\nu}r^{-\nu}}{\nu-2}\right)r^2 - \frac{\nu^2\eta^{\nu+2}}{\nu^2-4}\right)6M$$

$\nu = 1 \quad r = \eta + \frac{1}{\rho}$

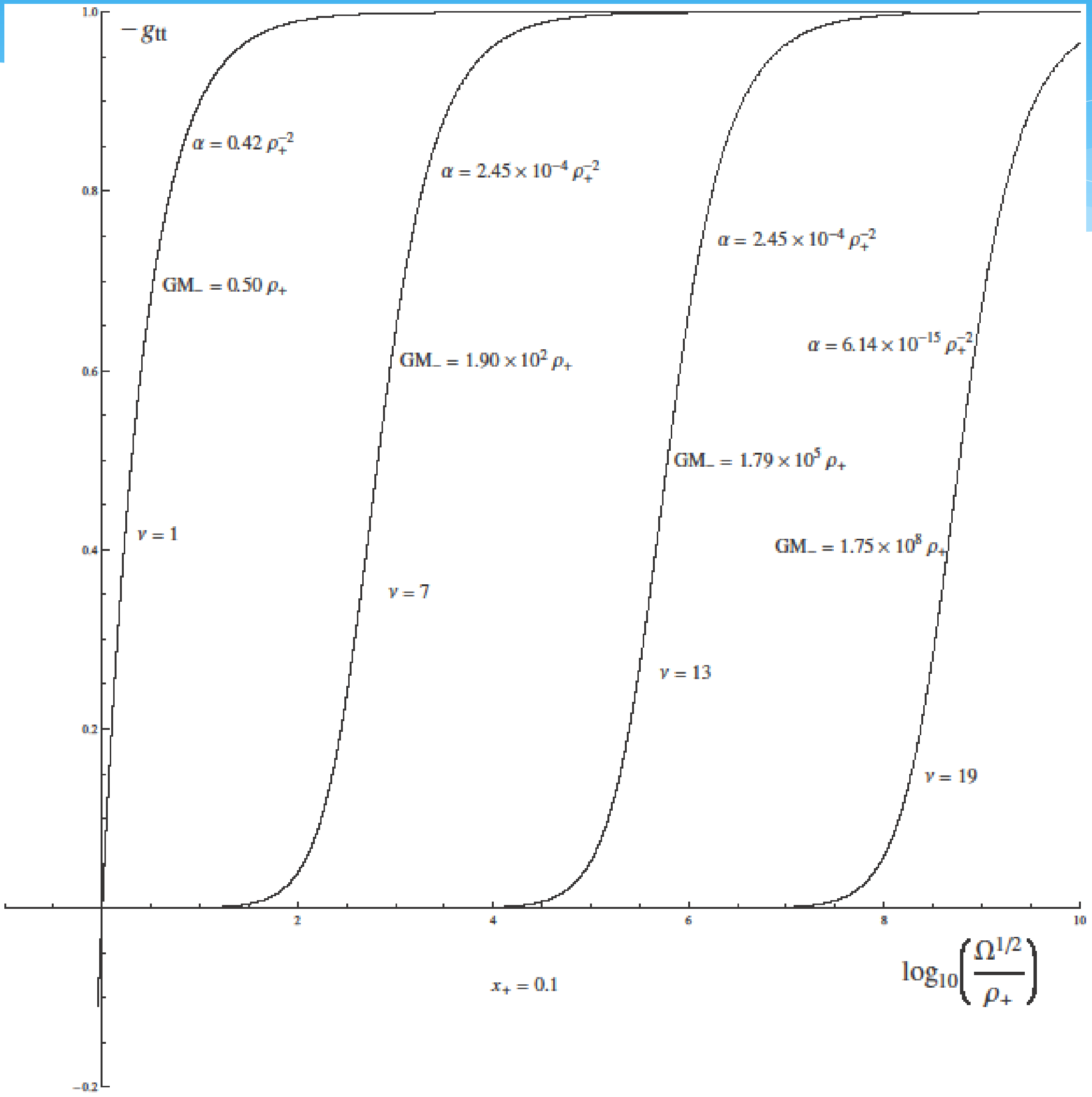
$$ds^2 = -(1 - \frac{2M}{\rho})dt^2 + \frac{d\rho^2}{1 - \frac{2M}{\rho}} + \rho^2 d\Omega.$$

The behavior at infinity of the AF BHS.

$$S(r) = \frac{1}{(r-\eta)^2} + \frac{1-\nu^2}{12\eta^2} - \frac{1-\nu^2}{12\eta^3}(r-\eta) + O(r-\eta)^2,$$

$$F_\nu(r) = (r-\eta)^2 - \frac{(6M\nu^2\eta^\nu + (\nu-1))}{3\eta}(r-\eta)^3 + O(r-\eta)^4.$$

The modified gravitational field.



The Potential and non-perturbative stability in AdS. (Slow fall off BC)

$$V(\phi) = \frac{\alpha}{\nu^2\kappa}\left(\frac{\nu-1}{\nu+2}\sinh((1+\nu)\,\phi l_\nu) + \frac{\nu+1}{\nu-2}\sinh((1-\nu)\,\phi l_\nu) + 4\frac{\nu^2-1}{\nu^2-4}\sinh(\phi l_\nu)\right)$$
$$+ \frac{\Lambda(\nu^2-4)}{6\kappa\nu^2}\left(\frac{\nu-1}{\nu+2}e^{-(\nu+1)\phi l_\nu} + \frac{\nu+1}{\nu-2}e^{(\nu-1)\phi l_\nu} + 4\frac{\nu^2-1}{\nu^2-4}e^{-\phi l_\nu}\right).$$

$$V(\phi) = (d-2)P'^2 - (d-1)P^2$$

$$m^2 = m_{BF}^2 + \frac{1}{4l}$$

The gravitational field is not uniquely determined by the mass of the spacetime, however the first law is satisfied.

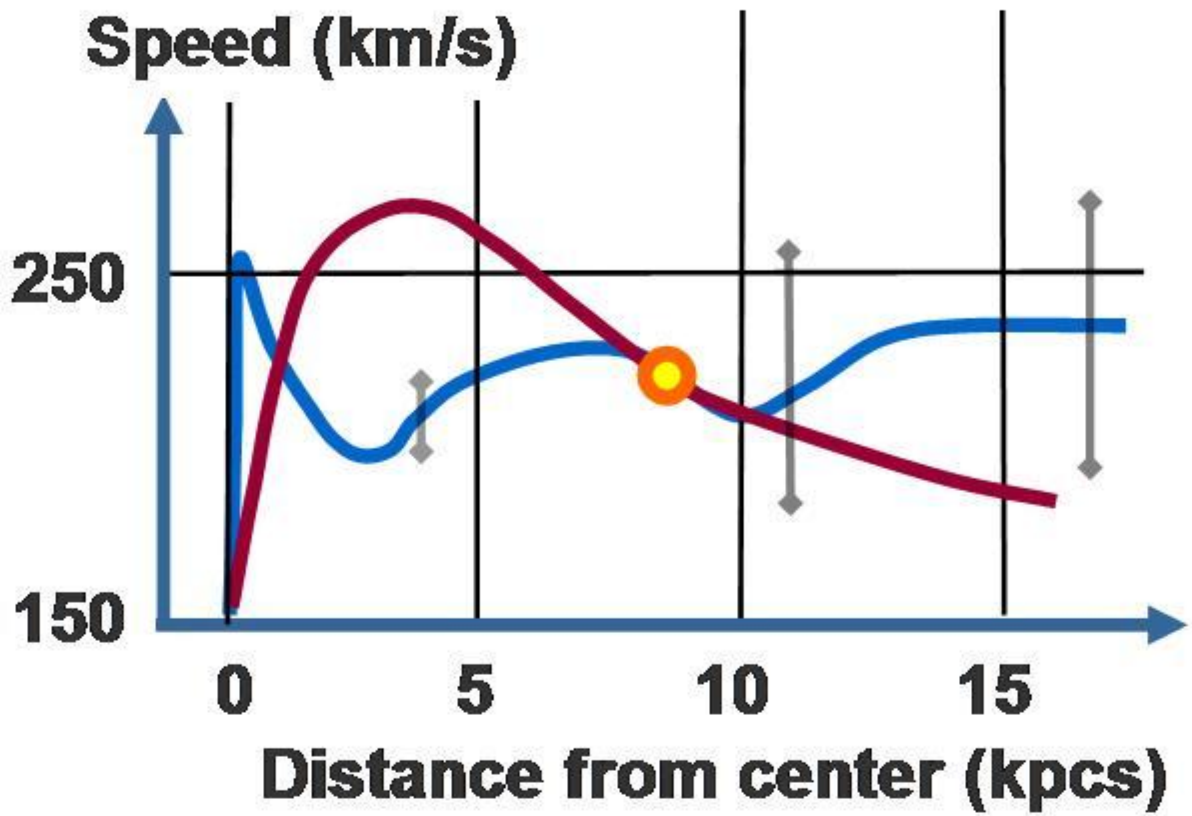
$$M_K = \frac{6\eta^\nu \nu^2 M + \nu - 1}{6\eta G}.$$

$$S = \frac{A}{4G} = \frac{\pi S(r_+)}{G} = \frac{\pi}{G}\frac{\nu^2\eta^{\nu-1}r_+^{\nu-1}}{\left(r_+^\nu - \eta^\nu\right)^2},$$

$$T = \frac{r_+\left(r_+^\nu - \eta^\nu\right)\left((\nu-2)\left(\eta^{\nu+2} + r_+^{\nu+2}\right) + (\nu+2)\left(r_+^\nu\eta^2 - r_+^2\eta^\nu\right)\right)}{4\pi\left(\eta^\nu r_+^{\nu+2}\left(\nu^2-4\right) - \nu^2\eta^{\nu+2}r_+^\nu + (\nu+2)\, r_+^2\eta^{2\nu} + (2-\nu)\, r_+^{2\nu+2}\right)}.$$

$$\delta M_K = T\delta S,$$

An old problem.



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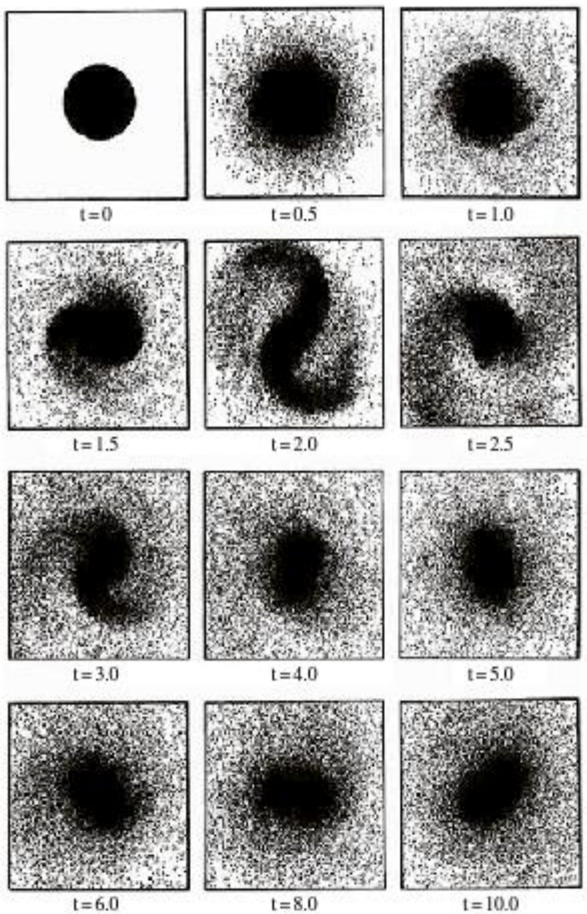

Final Remarks.

These are the first, exact, asymptotically flat Black Holes in four dimensions, with a minimally coupled scalar field.

They can be regarded as physically relevant if they are stable (de Sitter).

If these objects are stable they would represent the first, analytic, continuous, alternative for the gravitational field outside of a static, uncharged BH in four dimensional GR.