From ‘nothing’ to inflation and back again

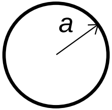
Vladimír Balek

Comenius University, Bratislava

Relativity and Gravitation, *100 years after Einstein in Prague*
Prague, 25-29th June 2012

playground

- closed universe



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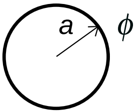

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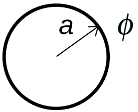

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ϕ in false vacuum (min/max of V) $\overset{eff}{=}$ pure gravity with Λ

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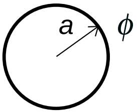

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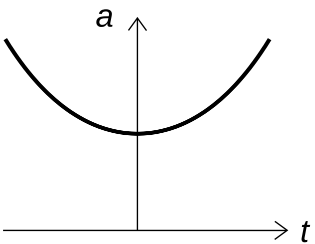

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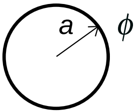

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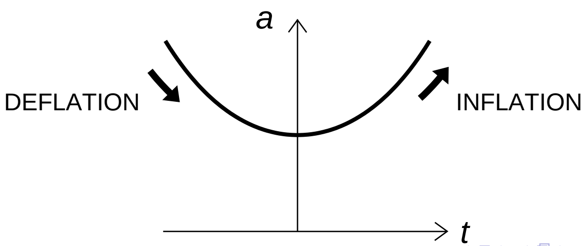

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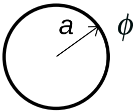

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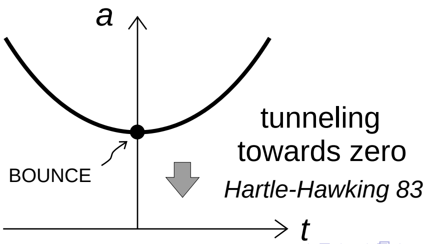

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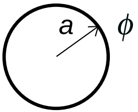

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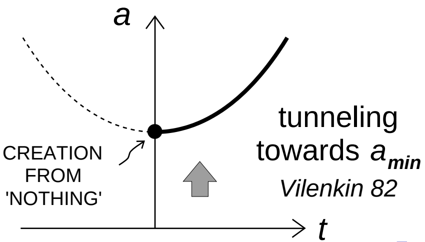

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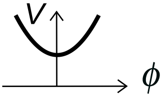

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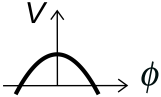

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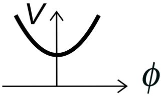

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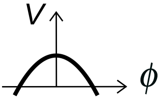

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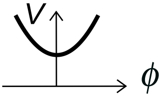

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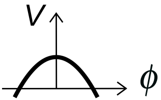

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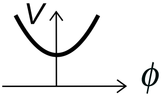

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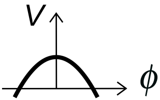

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aim of present work: to supply this bit of theory

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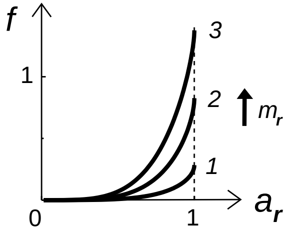

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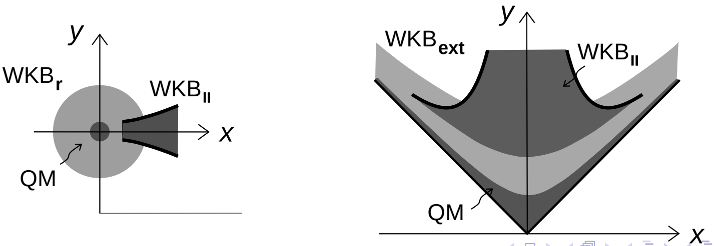

possible extensions: description of HM/CdL instanton; as for now: mathematics works, solution looks as expected

THANK YOU