Quasinormal modes from a naked singularity

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Plan of the talk

Naked Reissner-Nordström singularity

- Naked singularities and cosmic censorship conjecture
- Reissner-Nordström (RN) metric:

$$dS^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2 \quad \text{with} \quad f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2}$$

- Cauchy and event horizons $r_\pm$:

$$f(r) = 0 \quad \Rightarrow \quad r_\pm = m \pm \sqrt{m^2 - q^2} \quad \text{for} \quad q^2 < m^2$$

- Extremal Reissner-Nordström black hole: for $q^2 = m^2$, $r_- = r_+ = m$.
- For $q^2 > m^2$, there are no horizons $\Rightarrow$ naked Reissner-Nordström singularity

Klein-Gordon equation for the scalar field

From the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi)=0$$

with the decomposition in spherical harmonics

$$\Psi(t,r,\theta,\phi)=\sum_{\ell,m}\psi_\ell(t,r)Y_{\ell m}(\theta,\phi).$$

we obtain

$$\frac{d^2\psi_\ell(t,r)}{dt^2}=\frac{f(r)}{r^2}\frac{d}{dr}\left[r^2f(r)\frac{d\psi_\ell(t,r)}{dr}\right]-\frac{\ell(\ell+1)f(r)}{r^2}\psi_\ell(t,r).$$

We want to solve the initial value problem with compactly supported initial data.

Necessary conditions at the center

If we Laplace transform the last equation, the solution $\psi_\ell(t,r)$ is given by the inverse Laplace transform of $\tilde{\psi}_\ell(s,r)$, given as

$$\tilde{\psi}_\ell(s,r) = \int_0^\infty dr' \; G_\ell(s,r,r') I_\ell(s,r').$$

where $G_\ell(s,r,r')$ is the Green's function and $I_\ell(s,r')$ is the initial data term.

The Green's function is constructed from the two linearly independent solutions $U_{\ell 1,2}(s,r)$ of the homogeneous equation:

$$-\frac{f(r)}{r^2}\frac{d}{dr}\left[r^2 f(r)\frac{dU_{\ell 1,2}(s,r)}{dr}\right] + \left[s^2 + \frac{\ell(\ell+1)f(r)}{r^2}\right] U_{\ell 1,2}(s,r) = 0.$$

But for $r \to 0$, $U_{\ell 1,2}(s,r) = \exp(\beta_{1,2} \cdot r)$, with $\beta_{1,2} \in \mathbb{R}$.

Both solutions are regular $\Rightarrow$ no uniquely defined Green's function

We need one more condition at $r = 0$: $\psi_\ell(t,0) = 0$
(physical condition: "nothing falls in or out of the singularity")

From a RN black hole to a naked RN singularity

Fixed mass m, increasing charge q, $\ell = 2$
For $q^2/m^2 \gtrsim 9/8$ there is no secondary peak!

Defining

$$\phi_\ell(t,r) = \psi_\ell(t,r) \cdot r$$

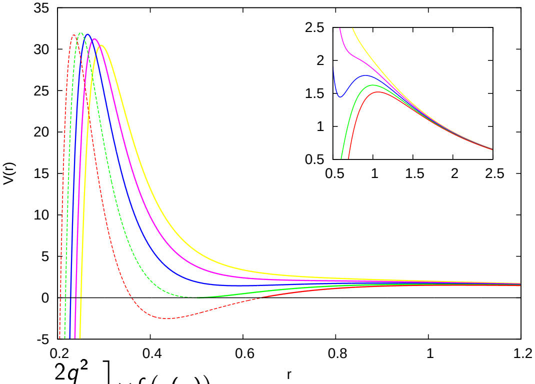
and using the tortoise coordinate x

$$\frac{dr}{dx} = f(r),$$

we rewrite the wave eq. as

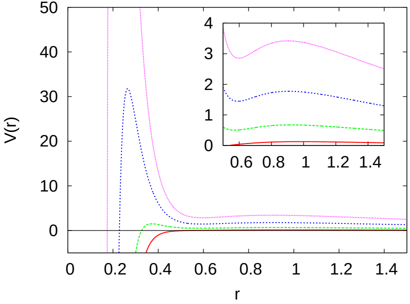
$$\partial_t^2 \phi_\ell - \partial_x^2 \phi_\ell = V(m,q,\ell,x)\phi_\ell$$

where the potential is

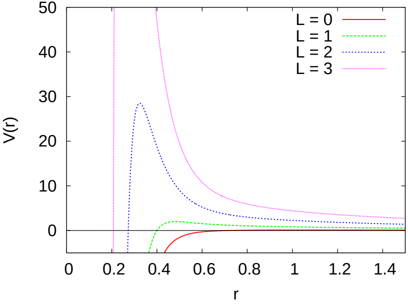


$$V(m,q,\ell,x) = \left[\frac{\ell(\ell+1)}{r^2(x)} + \frac{2m}{r^3(x)} - \frac{2q^2}{r^4(x)}\right] \times f\left(r(x)\right).$$

ℓ dependence of the potential (low ℓ)



$q^2/m^2 < 9/8$



$q^2/m^2 > 9/8$

Large ℓ limit - the particle picture

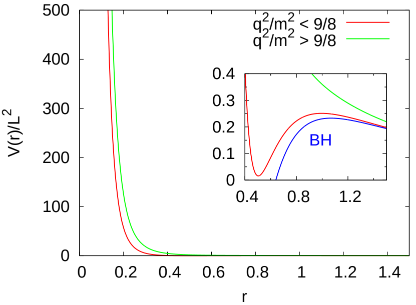
The effective potential is

$$V(r) = \frac{\ell^2}{r^2} f(r).$$

and its extremes are given by

$$r_{1,2} = \frac{3m \pm m\sqrt{9 - 8\frac{q^2}{m^2}}}{2},$$

so there is one maximum and one minimum for



$$\frac{q^2}{m^2} < \frac{9}{8} \Rightarrow \text{infinite barrier} + \text{small peak} \Rightarrow 2 \text{ effective regimes} \Rightarrow$$

lowest energy modes depend only on the small peak: continuity of the modes from BH to NS

and there are no extrema for

$$\frac{q^2}{m^2} > \frac{9}{8} \Rightarrow \text{infinite barrier} \Rightarrow \text{no low damped modes}$$

Numerical setup

using light-cone variables $u = t - x$ and $v = t + x$, the wave equation is

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = -4\frac{\partial^2 \phi}{\partial u \partial v} = V\phi\,,$$

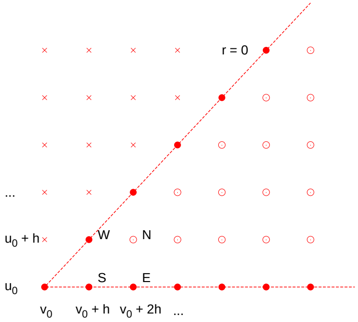
with the boundary conditions in the grid

$$\phi(r=0,t) = \phi(u, v = u + 2x_0) = 0\,,$$

$$\phi(u=0,v) = e^{-\frac{(v-v_c)^2}{2\sigma^2}}\,,$$

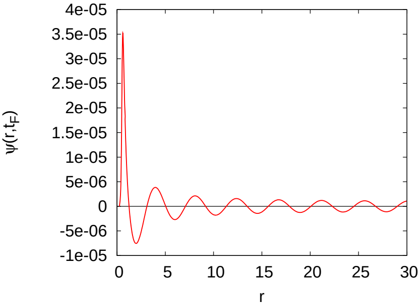
and the algorithm

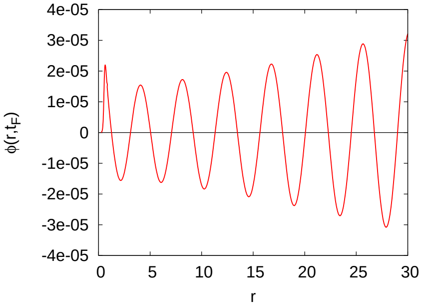
$$\phi_N = \phi_W + \phi_E - \phi_S - \frac{\phi_W + \phi_E}{8} V\Delta_v\Delta_u\,,$$



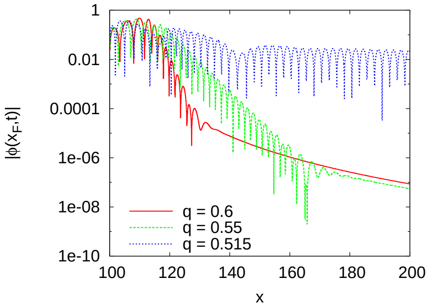
Conditions at the center

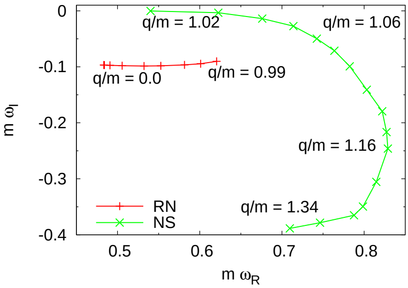
$\psi(t,0) = 0 \Rightarrow \phi(t,0) = 0$ and $\phi'(t,0) = 0$
guaranteed by the numerical boundary conditions





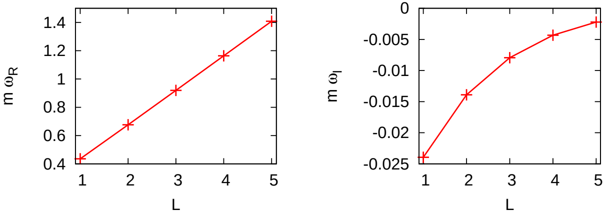
From a RN black hole to a naked RN singularity: discontinuity in the QNMs for low ℓ



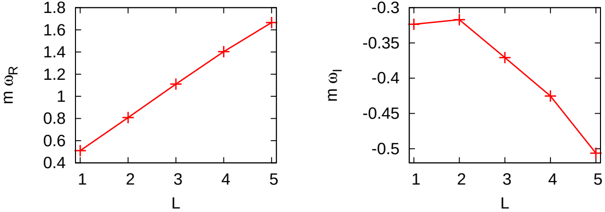


ℓ dependence of the QNMs (low ℓ)

$q^2/m^2 < 9/8$



$q^2/m^2 > 9/8$



Conclusions

- the evolution of the scalar field on the naked RN singularity is non-unique unless an additional boundary condition is specified at the singularity
- 4 qualitatively different cases for the low damped modes: low or large ℓ, and q^2/m^2 less or greater than $\sim 9/8$
- in the large ℓ limit
  - ($q^2/m^2 \lesssim 9/8$) there is a continuous transition from BH to NS
  - ($q^2/m^2 \gtrsim 9/8$) the low damped modes do not exist in the NS
- for low values of ℓ: the modes face a discontinuous transition from BH to NS
  - ($q^2/m^2 \lesssim 9/8$) $|\omega_I|$ decreases with ℓ
  - ($q^2/m^2 \gtrsim 9/8$) $|\omega_I|$ increases with ℓ, matching the behaviour for large ℓ