The Universal Features of the Lovelock gravity

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Plan of the Talk

I. Why Lovelock and higher dimensions?

II. Universal Features

(i) Lovelock characterization by Bianchi derivative

(*D, Pramana 74(2010)875, arxiv:0802.3034*)

(ii) Pure Lovelock gravity in $d = 2n$, $d = 2n + 1$ and $d = 2n + 2$ dimensions

(*D, Ghosh, Jhingan, Phys.Lett. B , arxiv:1202.4575*)

(iii) Pure Lovelock static Black hole: Characterization and Thermodynamical universality

(*D, Pons, Kartik, arxiv:1110.0673, 1201.4994*)

(iv) Uniform density sphere - The Schwarzschild interior solution

(*D, Molina, Khugaev, Phys. Rev. D81(2010)104026, arxiv:1001.3922*)

III. Lovelock vacuum as global monopole

IV. Discussion

I. Why Lovelock and higher dimensions?

Einstein gravity is linear in Riemann curvature, R_{abcd}

To probe high energy effects: Should include $R_{abcd}^2, ...$

Yet the character of the Equation should remain the same - Second order quasi-linear

This uniquely identifies Lovelock Lagrangian which is homogeneous polynomial in Riemann with specific coefficients.

Quadratic Gauss-Bonnet: $L_{GB} = R_{abcd}^2 - 4R_{ab}^2 + R^2$

Higher order terms make non-zero contribution in the Equation only for

$$d \geq 2n + 1$$

where n is the degree of polynomial; i.e. $d > 4$

Thus high energy effects appear in *higher dimensions*

II. Universal Features

Bianchi Derivative

$D^2 = 0$ Bianchi identity: $R_{ab[cd;e]} = 0$

On taking trace, multiplying by $g^{ac}g^{bd}$, this gives the

divergence free Einstein tensor: $\nabla_b G^{ab} = 0$, then

$$G_{ab} = \kappa T_{ab} + \Lambda g_{ab} \tag{1}$$

Thus Einstein's equation follows from the Riemann curvature and Λ as a true constant of spacetime structure

Question: *Is it possible to define analogue of Riemann, $R^{(n)}_{abcd}$ for nth order polynomial in Riemann such that the trace of its Bianchi derivative yields the corresponding divergence free Einstein tensor, $G^{(n)}_{ab}$?*

The answer is yes and it uniquely characterizes the Lovelock gravity

D, arxiv:0802.3034, Pramana 74(2010)875

The Riemann analogue is defined as

$$R^{(n)}_{abcd} = F^{(n)}_{abcd} - \frac{n-1}{n(d-1)(d-2)} F^{(n)} (g_{ac} g_{bd} - g_{ad} g_{bc}),$$
$$F^{(n)}_{abcd} = Q_{ab}{}^{mn} R_{cdmn} \tag{2}$$

where

$$Q^{ab}{}_{cd} = \delta^{aba_1b_1...a_nb_n}_{cdc_1d_1...c_nd_n} R_{a_1b_1}{}^{c_1d_1} ... R_{a_nb_n}{}^{c_nd_n},$$
$$\nabla_a Q^a{}_{bcd} = 0 \tag{3}$$

and

$$g^{ac} g^{bd} R^{(n)}_{ab[cd;e]} = 0 = \nabla_b G^{(n)b}_a \tag{4}$$

The analogue of n^{th} order Einstein tensor is given by

$$G^{(n)}_{ab} = n(R^{(n)}_{ab} - \frac{1}{2} R^{(n)} g_{ab}) \tag{5}$$

and

$$R^{(n)} = \frac{d-2n}{n(d-2)} F^{(n)} \tag{6}$$

Here $F^{(n)}$ is the Lovelock Lagrangian which is non-zero for $d = 2n$ while

$$R^{(n)} = R^{(n)}_{ab} g^{ab} = 0 \tag{7}$$

for arbitrary metric g_{ab}. Ricci involves first and second derivatives of the metric which are all arbitrary and hence

$$R^{(n)}_{ab} = 0 \tag{8}$$

identically for $d = 2n$.

Though Lovelock Lagrangian $F^{(n)}$ is non-zero in $d = 2n$ but its variation vanishes while $R^{(n)}_{ab} = 0$ directly for $d = 2n$.

Bianchi derivative yielding the equation of motion is universal for the Lovelock gravity

Lovelock gravity in the critical $d = 2n + 1$ dimension

Einstein gravity is kinematical in $d = 2 + 1 = 3$ dimension for $n = 1$

Vacuum is flat;i.e. $R_{ab} = 0$ implies $R_{abcd} = 0$

Question: Is it true for all n;i.e. $R^{(n)}_{ab} = 0$ **implies** $R^{(n)}_{abcd} = 0$**?**

Does the Lovelock gravity behave the same in all critical $d = 2n + 1$ dimensions?

Yes, it does. It is universal in the critical $d = 2n + 1$ dimension

Consider the static pure GB vacuum spacetime, $G_{ab}^{(n)} = \Lambda g_{ab}$ and it is given by

$$ds^2 = V dt^2 - V^{-1} dr^2 - r^2 d\Omega^2_{(d-2)} \tag{9}$$

$$V = 1 - \left(\Lambda r^{2n} + \frac{M}{r^{d-2n-1}}\right)^{1/n}. \tag{10}$$

In the critical $d = 2n+1$ dimension with $\Lambda = 0$, $V = const. \neq 1$ $G_{ab}^{(n)} = 0$ implies

$$R_{abcd}^{(n)} = 0 \tag{11}$$

The pure GB/Lovelock vacuum is trivial - flat.

However it is not Riemann flat so long as $V = const. = 1 - K \neq 1$. It represents a global monopole with the Einstein stresses

$$G_t^t = G_r^r = 3G_\theta^\theta = -3\frac{K}{r^2}, \quad G_\theta^\theta = G_\phi^\phi = G_\psi^\psi. \tag{12}$$

This is the Barriola-Vilenkin global monopole.

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{4}\lambda(\phi^a\phi^a - \eta^2)^2 \tag{13}$$

where ϕ^a is a quadruplet of scalar fields ($a = 1, 2, 3, 4$). The field configuration describing monopole is

$$\phi^a = \eta f(r)\frac{x^a}{r} \tag{14}$$

where $x^a x^a = r^2$.

The energy momentum tensor of the global monopole then takes the form

$$\begin{aligned}
T_t^t &= \frac{1}{2A^2}\eta^2 f'^2 + \frac{3}{2}\frac{\eta^2 f^2}{r^2} + \frac{\lambda}{4}\eta^4(f^2-1)^2 \\
T_r^r &= -\frac{1}{2A^2}\eta^2 f'^2 + \frac{3}{2}\frac{\eta^2 f^2}{r^2} + \frac{\lambda}{4}\eta^4(f^2-1)^2 \\
T_\theta^\theta &= \frac{1}{2A^2}\eta^2 f'^2 + \frac{1}{2}\frac{\eta^2 f^2}{r^2} + \frac{\lambda}{4}\eta^4(f^2-1)^2
\end{aligned} \tag{15}$$

The equation of motion for the field ϕ^a reduces to the following equation for $f(r)$,

$$\frac{f''}{A} + \left[\frac{3}{rA} + \frac{1}{2B}\left(\frac{B}{A}\right)'\right] f' - \frac{3f}{r^2} - \lambda\eta^2 f(f^2-1) = 0\ . \tag{16}$$

In the asymptotic limit $f \approx 1$, we obtain

$$T_t^t = T_r^r = 3T_\theta^\theta = \frac{3\eta^2}{2r^2}. \tag{17}$$

$$V = 1 - \frac{8\pi G_5}{r}\int T_t^t r^2 dr = 1 - 12\pi G_5\eta^2. \tag{18}$$

This is the BV global monopole

The pure Lovelock vacuum which is flat in the critical 5-D in GB gravity manifests as global monopole in the Einstein gravity.

Lovelock-BTZ black holes in the odd critical $d = 2n + 1$ dimension

If we include Λ, then we shall have in the critical dimension, $d = 2n + 1$

$$V = 1 - \left(\Lambda r^{2n} + M\right)^{1/n}. \tag{19}$$

It is the analogue of the famous BTZ black hole in the critical $d = 2n+1$ dimension

BTZ black hole analogue thus exists in all critical dimensions, $d = 2n+1$

However the background is now not falt as is the case for $n = 1$ BTZ black hole but is instead that of a global monopole

Lovelock gravity in $d = 2n, 2n+1, 2n+2$ dimensions

In $d = 2n$,

$R^{(n)}_{abcd}$ and $R^{(n)}_{ab}$ have the same number of components and $G^{(n)}_{ab} = 0$ identically

In $d = 2n+1$,

$R^{(n)}_{abcd}$ is entirely determined in terms of $R^{(n)}_{ab}$, Vacuum is flat, Weyl $W^{(n)}_{abcd} = 0$.

No free propagation, BTZ black hole

In $d = 2n+2$,

Gravity is dynamic with free propagation.

Discussion

Universal behaviour of the Lovelock gravity in $d = 2n, 2n+1, 2n+2$ dimensions. A general universal feature

It is the degree n of the Lovelock polynomial that determines the relevant dimension $d = 2n+1, 2n+2$

Asymptotically it is always the Einstein gravity

The pure Lovelock black hole has the universal thermodynamics in terms of $T(r_h), S(r_h)$ in $d = 2n+1, 2n+2$ dimensions

For example, Entropy always goes as r_h^2 in $d = 2n+2$ dimension

This is also the characterizing property for this class of black holes - pure Lovelock

In the brane- bulk picture, bulk is though free of matter but gravity leaks into it from the brane

It is there governed by pure Gauss-Bonnet vacuum, which is the global monopole in the Einstein sector

Question: **Could we find a black hole on the brane matching with the "global monopole" in the bulk?**

Question: **Could this analysis be extended to rotating black hole?**

Question: **Could a 4-dimensional curved spacetime embeddable in 5-dimensional pure GB vacuum which is global monopole spacetime?**