Gravitational Instabilities and Cosmic Censorship

Gustavo Dotti[1,3], Reinaldo Gleiser[1,3], I.Ranea-Sandoval[2,3],
J. Pullin [4], H. Vucetich[2]

[1] Universidad Nacional de Córdoba, Argentina
[2] Universidad Nacional de La Plata, Argentina
[3] CONICET, Argentina
[4] LSU, USA

100 years after Einstein in Prague - Prague, June 27, 2012

Black hole uniqueness: the Kerr-Newman family

- Stationary electrovac black hole solutions belong to the 3-parameter (M, a, Q) Kerr-Newman solution
- M, Q and a show up as constants of integration of Einstein's equations, found to be the mass, charge and angular momentum per unit mass.
- These spacetimes have a curvature singularity at (BL coordinates)

$$r^2 + a^2 \cos^2\theta = 0$$

$$\text{if } a = 0,\; 0 < r < \infty, \quad \text{if } a \neq 0, -\infty < r < \infty$$

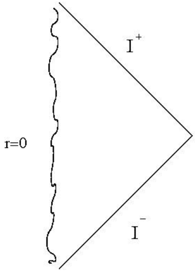
- There are horizon/s covering the singularity at r given by

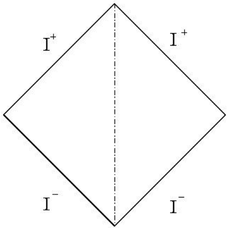
$$r^2 - 2Mr + a^2 + Q^2 = 0$$

For positive M there are three possibilities:

(i) two roots: $0 < r_i < r_o$, two horizons, Sub-extreme Black Hole
(ii) a double root, single horizon, Extreme Black Hole
(iii) no roots, no horizon, super-extreme case, Naked Singularity

Naked singularities in the KN family and WCC



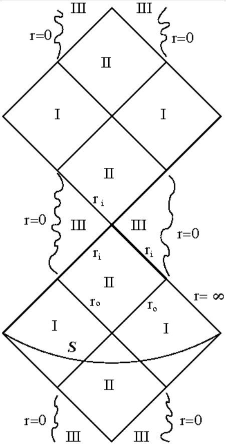


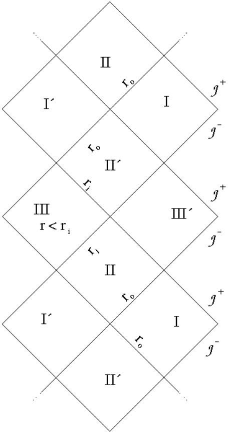
Left: CP diagram for the super-extreme ($Q^2 > M^2$) Reissner-Nordström ($a = 0$) spacetimes (also for the Schwarzschild naked singularity ($M < 0$)): These spacetimes admit a *partial Cauchy surface* $\mathcal{S}$ (intersected by every causal curve at most once).

Right: $a^2 > M^2$ Kerr (KNS): it is time orientable $V = (r^2 + a^2)\frac{\partial}{\partial t} + a\frac{\partial}{\partial \phi}$. Any two points can be connected by a future directed timelike curve!! In particular, there are CTCs through any point, there are no partial Cauchy surfaces.

WEAK COSMIC CENSORSHIP: Naked singularities do not arise in the collapse of "normal" matter

BH in the KN family: analytic extensions and SCC



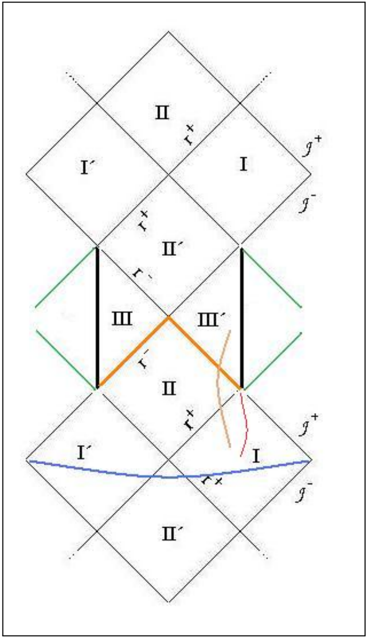


Block II absent in extreme BHs

Block III is beyond the CH for data on surface like S (this extension is non-unique)

STRONG COSMIC CENSORSHIP: For generic initial data in an appropriate class, the domain of development is inextensible.

Strong CC: Cauchy Horizon instability



- Initial data surface (in blue) has a CH (in orange), at BH inner horizon.
- Uniqueness is lost for evolution beyond CH.
- Signals sent by radio station (red wordline) in an infinite interval of proper time reach observer (grey wordline) in an finite interval of its proper time.
- Divergency of energy of perturbations near the CH suggests that CH is unstable, probably a boundary for a perturbed spacetime near RN/Kerr

Stability issues

Efforts to prove the stability of interesting stationary solutions in the KN family:

BLACK HOLE EXTERIORS

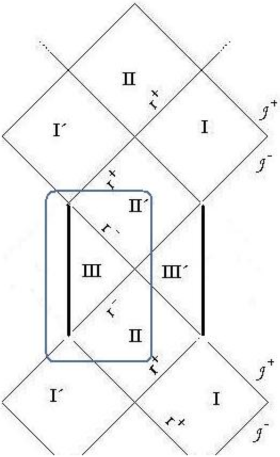
should be accompanied with similar efforts to find out whether the “undesirable” related stationary solutions:

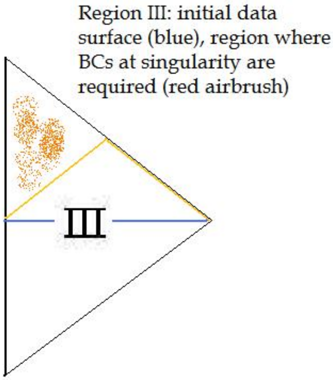
BH INNER STATIONARY REGIONS BEYOND CAUCHY HORIZONS, NAKED SINGULARITIES

are ruled out by stability arguments

The issue of non global hyperbolicity I:

Spherically symmetric case: evolution from a Partial Cauchy Surface





The CP diagram of a spherical naked singularity looks like that of region III of the (sub-extreme or extreme) RN BH, with the two copies of the inner horizon replaced with $\mathcal{I}^{\pm}$. The blue line represents a PCS, where initial data can be given, leading to a unique evolution only in its domain of dependence

- Gravitational perturbations admit a harmonic and parity decomposition. Odd and even (ℓ, m) perturbations obey a 1+1 wave equation (Regge, Wheeler, Zerilly, Moncrief, Vishveshwara, Gerlach, Sengupta, ...Kodama, Ishibashi...)

$$[\partial_t^2 - \partial_x^2 + V(x)]\Phi =: [\partial_t^2 + H]\Phi = 0, \; -\infty < t < \infty, \; x > 0 \qquad (1)$$

x is a “tourtoise" radial coordinate: $x : 0 \to \infty$ as $r : 0 \to \infty$.

- Equation (1) could be solved by separation of variables:

$$\Phi = \exp(\pm\sqrt{-E}t)\Psi(x), \; H\Psi = E\Psi \qquad (2)$$

if H admitted a complete set of eigenfunctions in the domain $x > 0$. Note that existence of modes with $E < 0$ implies instability.

- V is singular at $x = 0$. Two linearly independent local solutions of $H\Psi = E\Psi$ near $x = 0$ (say Ψ_1, Ψ_2). general local solution:

$$\Psi = A\cos(\gamma)\Psi_1 + A\sin(\gamma)\Psi_2$$

is parameterized by $\gamma \in S^2/\mathbb{Z}_2$ (set of b.c. at singularity.)

- A choice of γ fixes a set S_γ of functions where H in (1) is self-adjoint. Then (1) can be solved by linear superposition of modes (2), where $\Psi \in S_\gamma$

- This gives a unique evolution for the initial data on a PCS. Thus, ambiguity in evolution reduces to choice of γ (Gibbons, Hartnoll, Ishibashi, PTP 113 (2005) 963
- For gravitational perturbations of spherically symmetric naked singularities (negative mass Schwarzschild, $Q^2 > M^2$ Reissner-Nordström), there is a unique choice of γ consistent with the linear perturbation scheme, i.e., allowing *spatially uniformly* small perturbations at $t = 0$. For example, for the Schwarzschild NS, if ϵ is the amplitude of the perturbation, one gets:

$$R^{abcd}R_{abcd} = \frac{48M^2}{r^6} + \epsilon \left(\sin(\gamma) \left[\sim r^{-7} + ...\right] + \cos(\gamma) \left[\sim r^{-3} + ...\right]\right)$$

  which forces the choice $\gamma = 0$ ($\sim \pi$) (can prove that this choice fixes *all* scalar invariants)
- After having made a choice of γ, evolution of perturbations from a PCS, initially supported away the singularity, is unique
- Under the only consistent choice of γ, spherical NS are found to be (spectrum of self-adjoint H in Zerilli (even modes) equation contains exactly one negative energy (ie, unstable) eigenstate!)

Linear instability of the RN naked singularity and of RN BH inner static region beyond CH

Given generic initial data on a PCS for the linear perturbation problem for the Einstein-Maxwell equations, *compactly supported away from the singularity,* the (ℓ, m) even mode will evolve as $\exp(kt)$ for large t, with

$$k = \frac{(\ell+2)!}{2(\ell-2)!(\sqrt{9M^2+4Q^2(\ell-1)(\ell-2)}-3M)}$$

Signature of the instability (example); (ℓ, m) correction to Kretschman invariant (here $\beta_j = 3m + (-1)^j \sqrt{9M^2+4Q^2(\ell-1)(\ell-2)}$)

$$R_{abcd}R^{abcd} = \frac{48(Mr-Q^2)^2}{r^8} - \epsilon \left[\frac{24M\beta_2\, \ell(\ell+1)\, (Mr-Q^2)}{2\beta_1 r^7}\right] Y_{\ell m}(\theta,\phi)\, e^{k(t-x)}$$

Ref: G.D., R.J.Gleiser, Class.Quant.Grav., **27** 185007 (2010), gr-qc 1001.0152

(see G.D., R.J.Gleiser, Class.Quant.Grav., **26** 215002 (2009) for the Schwarzschild NS)

The issue of non global hyperbolicity II:

axially symmetric case, CTCs in Kerr spacetimes

- Kerr naked singularity and block III of a Kerr black hole have closed timelike curves through any point! This implies that there is no PCS.
- No notion of free data given on a spacelike surface, data has to be self-consistent.

Toy model without CTCs: scalar wave eqn $\Box\Phi = 0$ on $1+1$ Minkowski space $ds^2 = -dt^2 + dx^2$ admit left and right traveling wave solutions

$$\Phi(t,x) = f(t-x) + g(t+x)$$

where f and g are arbitrary 1-variable functions obtained from the initial data on Cauchy surface $t = 0$:

$$\{\Phi(t=0,x) = f(-x) + g(x), \dot{\Phi}(t=0,x) = f'(-x) + g'(x)\}$$

Toy model with CTCs: $1+1$ Minkowski space $ds^2 = -dt^2 + dx^2$ with periodic t, $t \sim t + \tau$. Still have traveling wave solutions but f and g have to be periodic in its -only- argument, with period τ. This means that the data $\{\Phi(t=0,x), \dot{\Phi}(t=0,x)\}$ are not two arbitrary functions, both must be periodic functions of x with period τ !

Linear instability of the Kerr naked singularity and of Kerr BH inner stationary region beyond CH (KIII)

- Massless scalar fields, Weyl spinors, Maxwell fields and linear gravity fields on Kerr Naked Singularities (KNS) and block III of Kerr BHs (KIII) can all be treated using Teukolsky master equation: ($\Delta = r^2 - 2Mr + a^2$)

$$
\begin{aligned}
T_s[\Psi_s] := & \left[\frac{(r^2+a^2)^2}{\Delta} - a^2 \sin^2\theta\right]\frac{\partial^2\Psi_s}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\Psi_s}{\partial t\partial\phi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\Psi_s}{\partial\phi^2} \\
& - \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\Psi_s}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi_s}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\Psi_s}{\partial\phi} \\
& - 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\Psi_s}{\partial t} + (s^2\cot^2\theta - s)\Psi_s = 0.
\end{aligned}
$$

  where Ψ_s is a spin-weight s null tetrad component of the relevant field (Weyl tensor in the case of gravity, for which $s = \pm 2$)

- A solution of Teukolsky equation can be used as a seed (potential) to generate a solution of the corresponding linear field equation on Kerr (Cohen & Kegeles PL 54A (1975) 5, Wald, Phys.Rev.Let 41 (1978), 203, e.g, a metric perturbation satisfying the linearized vacuum Einstein equations from an $s = \pm 2$ weight solution of Teukolsky equation.

- We proved that there are axially symmetric solutions of the Teukolsky equations for any s, that decay exponentially along radial directions, and that grow as $\propto \exp(kt), k > 0$.
- These can be used as seeds to construct Maxwell, scalar, spinor and linear gravitational fields that decay along spatial directions and grow exponentially with t.
- KIII and KNS are time orientable, since the field

$$V = (r^2 + a^2)\frac{\partial}{\partial t} + a\frac{\partial}{\partial \phi}$$

  is always timelike. The unstable axially symmetric solutions of the Teukolsky equation grow boundless along the integral lines of V (which is a congruence of timelike -accelerated- curves)

Numerical evidence: G.D., Gleiser and Pullin, Phys.Lett.B 644 (2007) 289
Proof grav. pert.: G.D, Gleiser, Ranea-Sandoval and Vucetich, C.Q.G 25 (2008) 245012

Otherl linear fields: G.Di, Gleiser, Ranea-Sandoval, C.Q.G 29(2012) 095017

Conclusions

- The static region beyond the Cauchy horizon of a Reissner-Nordström BH is linearly unstable. Reissner-Nordström and Schwarzschild naked singularities are also linearly unstable. In any of theses spaces, the evolution of data for linear gravity that is compactly supported on a PCS will grow exponentially in time.
- The stationary region beyond the Cauchy horizon of a Kerr BH is linearly unstable. The Kerr naked singularity is also linearly unstable: in these spaces there exist scalar, spinor, Maxwell and linear gravity fields satisfying:
  - fast decay along spatial directions,
  - exponential growth $\exp(kt/a), k > 0$
- Existence of these instabilities supports both forms of cosmic censor conjectures.