Scalar fields on anti-de Sitter background

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Outline

1. Weak turbulence
   - transfer of energy to high frequency modes
   - self-gravitating scalar field (Bizoń and Rostworowski)
   - self-interacting scalar field on fixed AdS background ??
2. Localized time-periodic solutions
   - geons, boson stars and oscillatons
   - Klein-Gordon fields
   - Self-interacting scalar fields

Scalar field coupled to gravity

Negative cosmological constant – asymptotically AdS

P. Bizoń and A. Rostworowski: Weakly Turbulent Instability of Anti-de Sitter Spacetime, *Phys. Rev. Lett.* **107,** 031102 (2011)

evolution of spherically symmetric massless scalar field

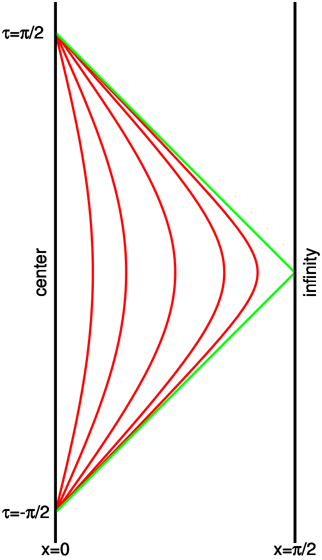
– energy is shifted to small wavelength high frequency modes
$\longrightarrow$ weak turbulence
$\longrightarrow$ black hole formation

What changes when the background is a fixed AdS spacetime?

– for a Klein-Gordon field equations are linear
$\longrightarrow$ no weak turbulence

– self-interacting scalar fields ??

Anti-de Sitter spacetime



Conformal coordinate system

$$ds^2 = \frac{1}{k^2 \cos^2 x}\left(-d\tau^2 + dx^2 + \sin^2 x \, d\Omega^2\right)$$

All timelike geodesics emanating from a point meet again at another point

A light ray can travel to infinity and back in a finite time
– infinity may be like a mirror for null rays

AdS background corresponds to an effective attractive force

Self-interacting scalar field on AdS background

$3+1$ dimensional spacetime, spherical symmetry

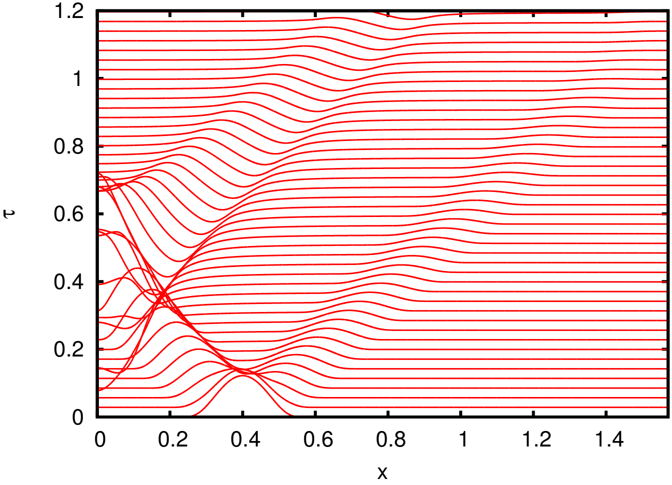
$$-\phi_{,\tau\tau} + \phi_{,xx} + \frac{4}{\sin(2x)}\,\phi_{,x} = \frac{U'(\phi)}{k^2\cos^2 x}$$

the cosmological constant is $\Lambda = -3k^2$

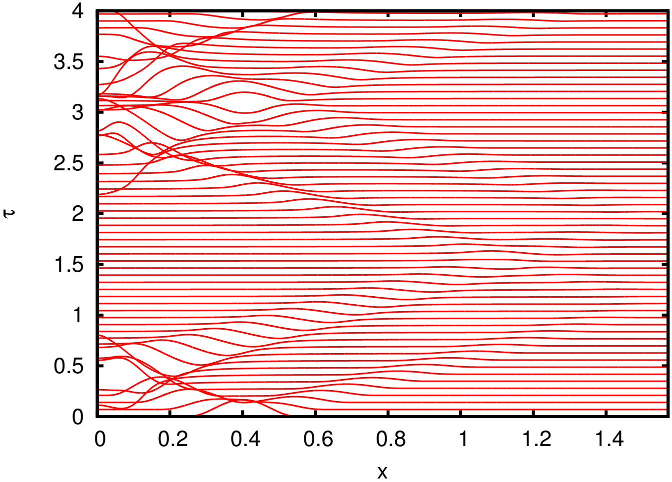
Example:

$k = 1\,, \qquad U(\phi) = \frac{1}{2}\phi^2 - \frac{1}{4}\phi^4 + \frac{1}{6}\phi^6$

initial data: a spherically symmetric shell (finite width)

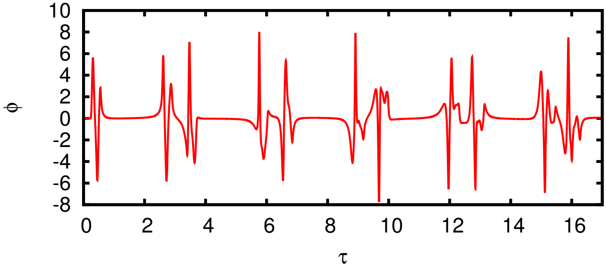


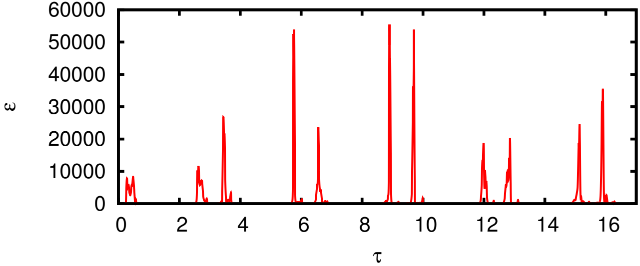
separates to an ingoing and an outgoing shell



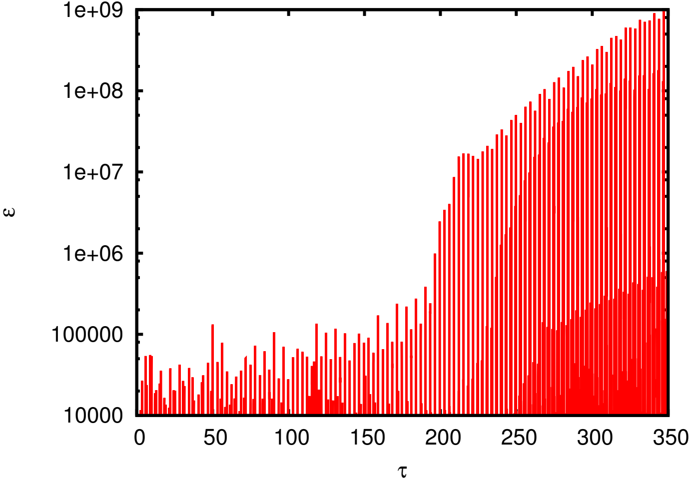
the shells get reflected back from infinity in finite time

time dependence of scalar field and energy density at center





time dependence of energy density at center



weak turbulence ??

O. J. C. Dias, G. T. Horowitz and J. E. Santos:
Gravitational Turbulent Instability of Anti-de Sitter Space,
arXiv:1109.1825v1 [hep-th] (2011)

– vacuum with negative cosmological constant

– use perturbation theory to show the existence of resonant modes $\longrightarrow$ turbulence

Nonlinear generalization of single perturbative mode: geon
– spatially localized periodic solutions
– not spherically symmetric

If there is a self-gravitating scalar field, there are spherically symmetric localized periodic solutions – oscillatons
– similar to boson stars, but in that case scalar field is complex and metric is static

Periodic localized solutions for a scalar field also exist on fixed AdS background: breathers (oscillons)

Breather solutions for Klein-Gordon field on AdS

Scalar potential $U(\phi) = \frac{1}{2}m^2\phi^2$, cosmological parameter k

There is a family of breather solutions labeled by $n > 0$ integer

$$\phi_n = \cos[(\mu + 2n)\tau]\,(\cos x)^\mu\, P_n^{(1/2,\mu-3/2)}\,(\cos(2x))$$

where

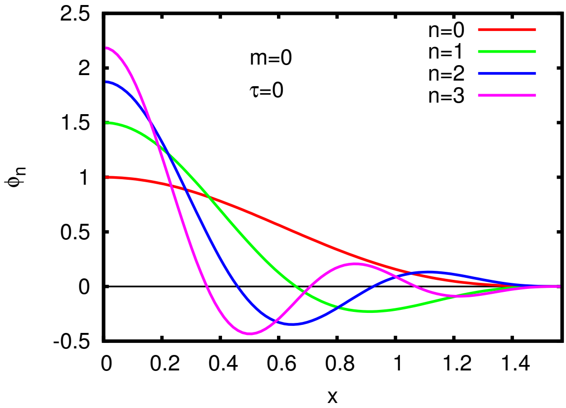
$$\mu = \frac{3}{2} + \sqrt{\frac{9}{4} + \frac{m^2}{k^2}}$$

and $P_n^{(a,b)}(x)$ is the Jacobi polynomial

Avis, Isham and Storey, PRD **18.** 3565 (1978)

All finite energy solutions can be expressed as sums of ϕ_n

The number of nodes is given by n



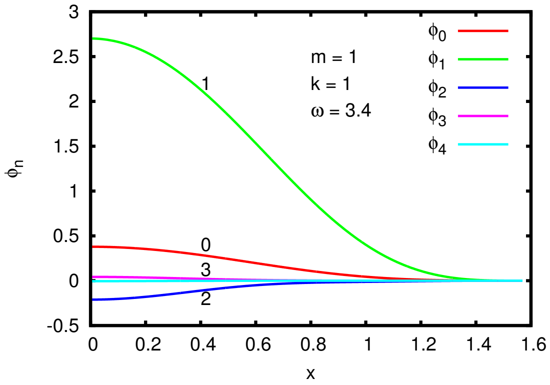
Solutions for $m > 0$ are similar, but more compact

Field equations are linear $\longrightarrow$ stable configurations

Self-interacting scalar fields

Periodic solutions: $\phi = \sum_{n=0}^{N} \phi_n \cos(n\omega\tau)$

Solve the system of ordinary differential equations by the spectral code Kadath of Philippe Grandclément



Small-amplitude small-k expansion

Use Schwarzschild area coordinates

$$ds^2 = -(1 + k^2 r^2)dt^2 + \frac{dr^2}{1 + k^2 r^2} + r^2 d\Omega^2$$

To leading order breathers are described by a single ordinary differential equation

$$S_{,\rho\rho} + \frac{2}{\rho} S_{,\rho} + (\omega_2 - \rho^2) S \pm S^3 = 0$$

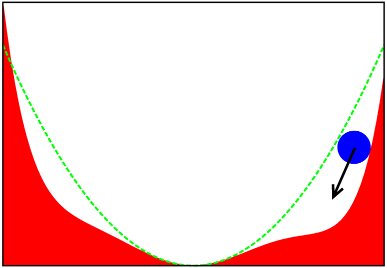
where $\rho = \sqrt{mk}\, r$

The choice $\pm$ is determined by the sign of $\lambda = \frac{5}{6} g_2^2 - \frac{3}{4} g_3$ where g_2 and g_3 are defined by

$$U(\phi) = m^2 \left(\frac{1}{2}\phi^2 + \frac{g_2}{3}\phi^3 + \frac{g_3}{4}\phi^4 + \dots \right)$$

The behavior of the potential near its minimum determines λ

If $\lambda > 0$ we call it an “attractive potential”



it is more flat than the same mass harmonic potential

$\longrightarrow$ oscillation period becomes longer

To leading order the scalar field is given by

$$\phi = \sqrt{\frac{k}{m|\lambda|}}\, S \cos(m\omega\, t)$$

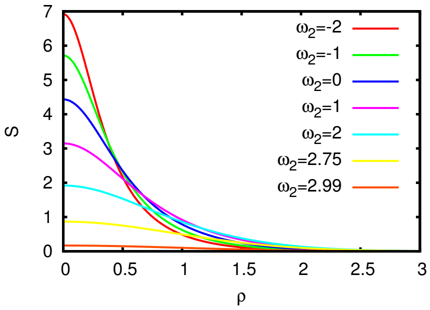
where the frequency is $m\omega$, where

$$\omega = \sqrt{1 + \omega_2 \frac{k}{m}}$$

In order to obtain the breather solution with a given frequency one has to solve the equation for S with the corresponding ω_2

$$S_{,\rho\rho} + \frac{2}{\rho} S_{,\rho} + (\omega_2 - \rho^2) S \pm S^3 = 0$$

If $\lambda > 0$ then $S_{,\rho\rho} + \frac{2}{\rho} S_{,\rho} + (\omega_2 - \rho^2) S + S^3 = 0$

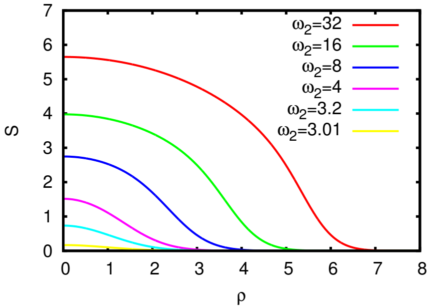


There are localized nodeless solutions for any $\omega_2 < 3$

Attractive potential

Higher amplitude breathers are more localized because of the attraction represented by the scalar potential

If $\lambda < 0$ then $$S_{,\rho\rho} + \frac{2}{\rho} S_{,\rho} + (\omega_2 - \rho^2) S - S^3 = 0$$



There are localized nodeless solutions for any $\omega_2 > 3$

Repulsive potential

Higher amplitude breathers have larger size