Spontaneous breaking of Lorentz symmetry for canonical gravity

Steffen Gielen
Perimeter Institute for Theoretical Physics

27 June 2012

Based on joint work with Derek Wise
Phys. Rev. D **85** (2012) 104013; arXiv:1206.0658

Introduction

In first order formulations of general relativity one has a notion of local Lorentz invariance, which can be thought of as one way of implementing the equivalence principle.

It is crucial to understand the fate of Lorentz covariance in attempts to quantise gravity, both theoretically and with regard to a possible phenomenology of quantum gravity (including matter). There are strong experimental constraints on many possible types of violation of Lorentz covariance and any proposed theory of quantum gravity must prove itself consistent with such constraints.

In Hamiltonian formulations, particular the Ashtekar-Barbero connection formulation, the issue of Lorentz covariance has been the focus of some debate, since the Ashtekar-Barbero formulation naturally uses the gauge group $SU(2)$ or $SO(3)$.

Canonical First Order GR

Starting from the Lorentz covariant Palatini-Holst action

$$S[e,\omega]=\frac{1}{8\pi G}\int \kappa_{abcd}\, e^a\wedge e^b\wedge R^{cd}[\omega],$$

where κ_{abcd} is a non-degenerate $SO(3,1)$-invariant bilinear form on $\mathfrak{so}(3,1)$,

$$\kappa_{abcd}=\frac{1}{2}\epsilon_{abcd}+\frac{1}{2\gamma}(\eta_{ac}\eta_{bd}-\eta_{ad}\eta_{bc})\,,$$

one can perform the usual canonical analysis and find that the 18 momenta π^i_{ab} conjugate to the spatial components of the connection ω^{ab}_i are expressible in terms of only 12 tetrad components e^a_i. This leads to *second class constraints*.

Canonical First Order GR (II)

These provide an obstacle to quantisation and usually require introducing new variables which are harder to interpret in terms of spacetime geometry.
In Holst's analysis one avoids this by imposing 'time gauge' $e^0_i = 0$ and defining

$$A^{ab} = \omega^{ab} + \frac{\gamma}{2}\epsilon^{ab}{}_{cd}\omega^{cd}$$

Only the $\mathfrak{so}(3)$ part of A (the *Ashtekar-Barbero connection*) has nonvanishing conjugate momentum, and one avoids second class constraints. However the resulting theory is naturally an $SO(3)$ gauge theory.

In our formalism we replace time gauge by a condition involving a field of *internal observers* y which specifies a time direction locally, and leads to a spontaneous breaking of symmetry from $SO(3,1)$ to a subgroup $SO(3)_y$ depending on $y(x)$ at each spacetime point x.

GR with Local Observers

For a given spacetime manifold with metric g or frame field e, we think of a *field of observers* as a unit future-directed timelike vector field u. Using the frame field we can map it to a spacetime scalar $y = e(u)$ valued in the velocity hyperboloid $\mathrm{H}^3 = SO(3,1)/SO(3)$. This can then be defined without specifying the metric.

Our formalism uses the following variables:

- a field of internal observers y, valued in $\mathrm{H}^3 \subseteq \mathbb{R}^{3,1}$,
- a nowhere-vanishing 1-form $\hat{u}$, thought of as non-dynamical and generalising the normal to a foliation (if $\hat{u} \wedge d\hat{u} = 0$, $\hat{u}$ is of the form $\hat{u} = N\,dt$),
- an $\mathbb{R}^3_y$-valued 'triad' 1-form E, where $\mathbb{R}^3_y$ is the subspace of $\mathbb{R}^{3,1}$ orthogonal to y (this generalises time gauge).

GR with Local Observers (II)

The spacetime coframe field is then simply given by

$$e = E + \hat{u}\, y \tag{1}$$

anologous to how one reconstructs the spacetime metric in the ADM formulation using lapse and shift. The field of internal observers y defines a field of spacetime observers by $y = e(u)$, and one finds that $E(u) = 0$ so that E is actually *spatial*. Similarly, we can define spatial and temporal parts of the spin connection and

$$\omega = \Omega + \hat{u}\, \Xi \tag{2}$$

Substituting (1) and (2) into the Palatini-Holst action gives us a generalised Hamiltonian formulation of vacuum GR in terms of y, E, Ω and Ξ. Up to this stage everything is Lorentz covariant.

GR with Local Observers (III)

The main role of the field of internal observers y is to give us a local embedding of $SO(3)$ into $SO(3,1)$ which can be freely changed by applying a Lorentz transformation $y \mapsto y' = \Lambda\, y$.
The spatial connection Ω can be projected to its $\mathfrak{so}(3)_y$ part $\mathbf{\Omega}$. Then under a local Lorentz transformation

$$\mathbf{\Omega} \mapsto \mathbf{\Omega}' = \Lambda^{-1}\, \mathbf{\Omega}\, \Lambda + \pi_{y'}(\Lambda^{-1}\, d^{\perp}\Lambda),$$

where $\pi_{y'}$ is a projector onto $\mathfrak{so}(3)_{y'}$ and $d^{\perp} = d - \hat{u} \wedge \pounds_u$ is a spatial exterior derivative. Therefore, if one only applies $SO(3)_y$ transformations which leave y invariant, $\mathbf{\Omega}$ transforms as an $SO(3)_y$ connection, while if one allows for transformations that rotate the local internal observer y to y' the transformed connection $\mathbf{\Omega}'$ is in $\mathfrak{so}(3)_{y'}$.

GR with Local Observers (IV)

To understand the dynamical structure of GR in this formalism, focus on the term in the action that determines the symplectic structure in Hamiltonian GR,

$$S = \frac{1}{8\pi G}\int \kappa_{abcd}\hat{u}\wedge E^a\wedge E^b\wedge \pounds_u \Omega^{cd} + \ldots$$

Since $E\wedge E$ is valued only in $\mathfrak{so}(3)_y$, only half of the components of Ω have nonvanishing conjugate momentum, and our formalism is analogous to Holst's derivation of the Ashtekar-Barbero formalism in time gauge.

One can make the splitting of $\mathfrak{so}(3,1)$ into a rotational subalgebra $\mathfrak{so}(3)_y$ and a complement $\mathfrak{p}_y$ explicit by choosing local bases J_I^{ab} and B_I^{ab} (depending on y). Then

$$A^I := \mathbf{\Omega}^I + \gamma K^I,$$

is conjugate to $(E\wedge E)^I$, where $\mathbf{\Omega}$ and K are the $\mathfrak{so}(3)_y$ and $\mathfrak{p}_y$ parts of Ω.

Summary

- We gave a formalism generalising the usual Hamiltonian form of GR leading to the $SO(3)$ Ashtekar-Barbero connection which relies on a field of local internal observers valued in H^3 which provide a local notion of time direction. It relates the (3+1) splitting in canonical GR to a splitting in the tangent space $\mathbb{R}^{3,1}$.

- The formalism is Lorentz covariant and avoids second-class constraints. Lorentz covariance is spontaneously broken by specifying a value for $y(x)$ locally, but can be recovered by allowing Lorentz transformations that change the local observer. It is analogous to MacDowell-Mansouri gravity where $SO(4,1)$ is locally broken to $SO(3,1)$.

- Holst's formalism in time gauge corresponds to a gauge-fixed version where one has fixed $y=(1,0,0,0)$ everywhere.

Outlook

We have given a reformulation of general relativity using local observers which define a local notion of time and give an embedding of the rotational subgroup $SO(3)$ into the Lorentz group that allows to reconstruct Lorentz covariance.

It would also be important to understand the coupling of matter and the role of the field of internal observers there. So far they have been treated like lapse and shift, as Lagrange multipliers.

The connection to similar constructions in spin foam models using local normals to identify an $SU(2)$ subgroup of the Lorentz group could be made more explicit.

Similar constructions could be useful in other circumstances in quantum gravity where local Lorentz covariance is not manifest, such as Hořava-Lifshitz gravity, shape dynamics or causal dynamical triangulations.

Thank you!