COUPLING MATTER TO QUANTUM GRAVITY

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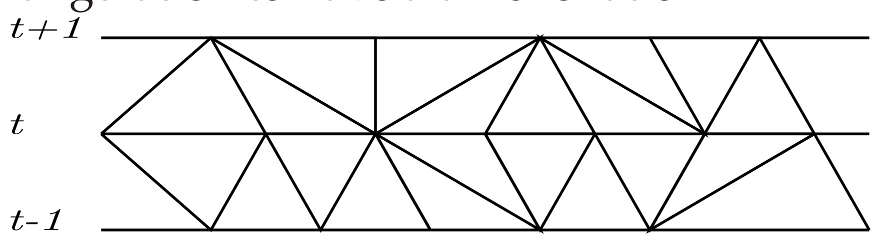
TAKE AWAY MESSAGE

We constructed a multicritical 2 dimensional causal dynamical triangulation. This means we were able to find a solvable model of matter coupled to quantum gravity in the context of CDT.

CAUSAL DYNAMICAL TRIANGULATIONS

Causal Dynamical Triangulations (CDT) regularize the gravitational path integral.
In CDT a causality condition is included in the sum over all triangulations to give a clear way how to wick rotate from lorentzian to euclidean signature. This is achieved by requiring the triangulation to have a time foliation.



This is an example of a CDT with three moments of time, $t-1, t, t+1$.

The triangulations contributing to CDT are more regular than those in euclidean dynamical triangulations. This leads to a more desirable behavior.

- Monte Carlo Simulations in 4d find *deSitter* space-time and a mini-superspace action
- the dominant contributions to the path integral are not degenerate
- it is very likely that there exists a 2nd order phase transition which allows for a continuum limit.

For a review on CDT see [1].

MULTICRITICAL BEHAVIOR

A critical point signals a phase transition. At a n-multicritical point the first $n-1$ derivatives of a coupling μ by the partition function Z are zero.

$$\left.\frac{\partial \mu}{\partial Z}\right|_{Z_c} = \cdots = \left.\frac{\partial^{n-1}\mu}{\partial Z^{n-1}}\right|_{Z_c} = 0 \qquad (7)$$

In 2d this has been examined analytically.

- the continuum limit at a critical point is a CFT
- the continuum limit of a triangulation with dimers is conformal matter coupled to quantum gravity [2]

MATRIX MODEL

Matrix models are a way to calculate the partition function for a triangulation as the Gaussian integral over a matrix.

$$Z(\lambda, g_s) = \int \mathrm{d}\phi \; e^{\frac{N}{g_s}\mathrm{tr}V(\phi)} . \qquad (8)$$

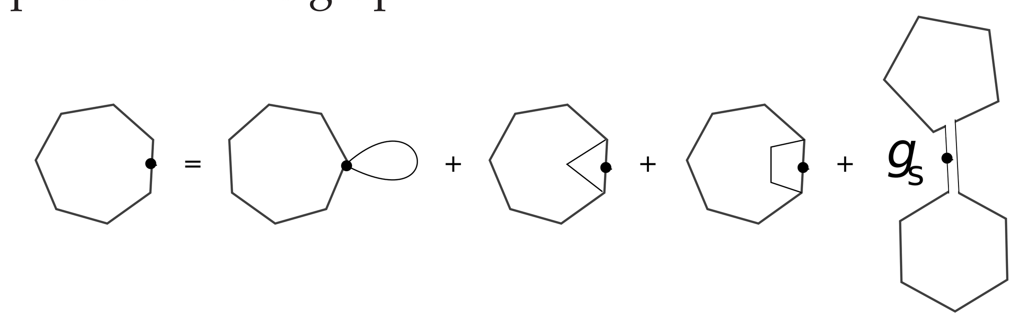
The potential for the multicritical model is given as

$$V(\phi) = \frac{1}{2}\phi^2 - \lambda\phi - \lambda\phi^3 - \frac{\lambda^3\xi}{2}\phi^4 . \qquad (9)$$

With $\xi = 0$ this is the normal CDT matrix model. The disc amplitude can be calculated from the self consistent equation

$$g_s W(x)^2 = V'(x)W(x) - Q(x) \qquad Q(x) = c_2x^2 + c_1x + c_0 , \qquad (10)$$

as expressed in this graph:



An ansatz assuming one cut in the disc function leads to

$$W(x) = \frac{V'(x) - 2\lambda_c^2\zeta(x-b_c)^2\sqrt{(x-c_c)(x-a_c)}}{2g_s} , \qquad (11)$$

where all constants at their critical values are functions of g_s.
We now take a double scaling limit

$$g_s = G_s\epsilon^4 \quad \lambda = \lambda_* + \tilde{\Lambda}\epsilon^2 - \lambda\epsilon^3 \quad \zeta = \zeta_* - \frac{1}{2}\tilde{\Lambda}\epsilon^3 . \qquad (12)$$

These scaling relations can be motivated by the multicriticality conditions.
After a last redefinition

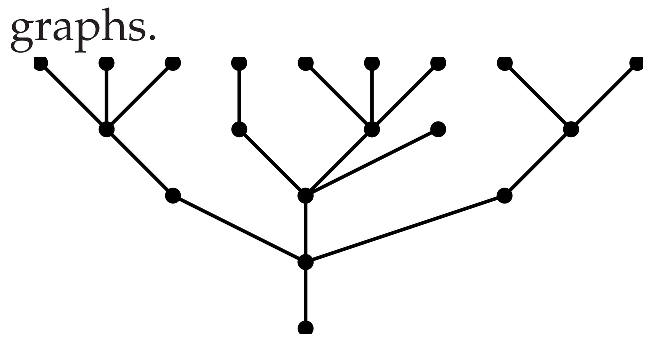
$$\Lambda_{cdt} = \Lambda + \frac{32\sqrt{3}5^{1/4}}{81}, \quad X_{cdt} = X + \frac{2}{\sqrt{3}5^{1/4}}G_s^{1/4} , \qquad (13)$$

we find the disc function for the multicritical CDT

$$W(x) = \frac{1}{\epsilon}\frac{1}{X_{cdt} + \Lambda_{cdt}^{1/3}} . \qquad (14)$$

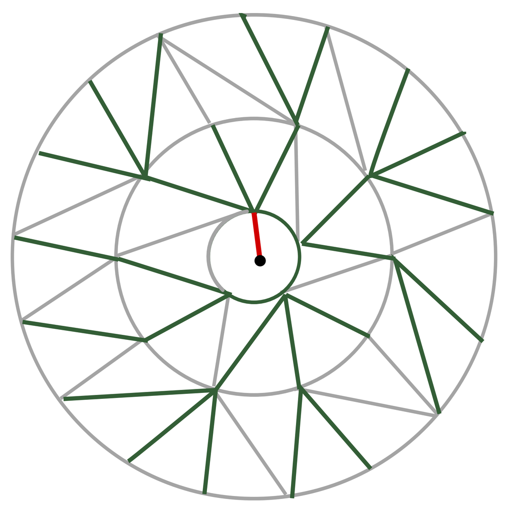
BRANCHED POLYMERS

Branched Polymer like configurations dominate in the euclidean path integral. They can be described mathematically as rooted tree graphs.



Ironically, although they are not prevalent in the path integral for CDT, there is a way to map a 2 dimensional CDT uniquely into a branched polymer.

- erase all space-like edges
  (all edges connecting vertices at the same time t)
- erase the leftmost edge at every vertex



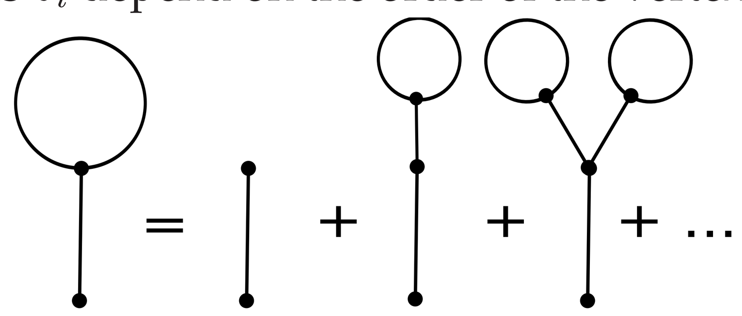
Green lines are the edges belonging to the branched polymer.

MULTICRITICAL BRANCHED POLYMERS

The partition function for branched polymers is

$$Z(\mu) = \sum_{BP}\prod_i v_i \prod_l e^{\mu} . \qquad (1)$$

The weights v_i depend on the order of the vertex. This graph



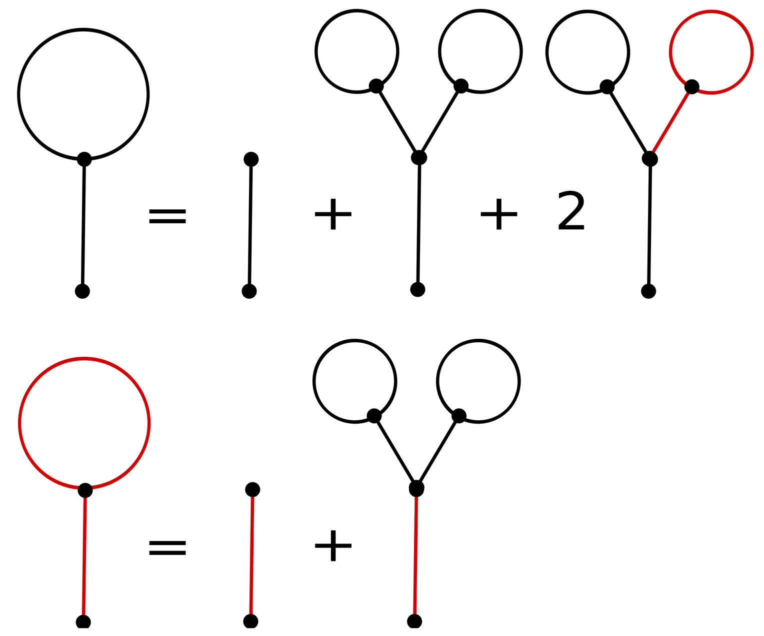
leads to a closed equation

$$e^{\mu} = \frac{1 + v_2Z + v_3Z^3 \ldots}{Z} := \frac{f(Z)}{Z} := F(Z) . \qquad (2)$$

We defined a partition function for a BP with dimers

$$Z(\mu, \zeta) = \sum_{BP}\prod_i v_i \prod_l e^{\mu} \sum_{HD(BP)} \zeta^{|HD(BP)|} , \qquad (3)$$

ζ is the dimer weight, $HD(BP)$ are all ways to place dimers on a given BP and $|HD(BP)|$ is the number of dimers in a given configuration. We make a split in two partition functions, W based in a dimer and a Z.



The dimer/ BP rooted in a dimer is marked red in this figure.
The graphic then corresponds to

$$e^{\mu} = \frac{1 + Z^2 + 2ZW}{Z}, \qquad e^{\mu} = \zeta\frac{1 + Z^2}{W} . \qquad (4)$$

For the multicritical point we now need

$$\left.\frac{\partial\mu(Z,\zeta_c)}{\partial Z}\right|_{Z_c} = \left.\frac{\partial^2\mu(Z,\zeta_c)}{\partial Z^2}\right|_{Z_c} = 0 . \qquad (5)$$

This can be solved and we find the critical exponents

$$\gamma = \frac{1}{3} \qquad d_H = \frac{3}{2} \qquad \sigma = \frac{1}{2} . \qquad (6)$$

REFERENCES

[1] J. Ambjorn, A. Goerlich, J. Jurkiewicz, et al. Nonperturbative quantum gravity. *arXiv:1203.3591*, March 2012. URL http://arxiv.org/abs/1203.3591.

[2] S. Matthias. The Yang-Lee edge singularity on a dynamical planar random surface. *Nuclear Physics B*, 336(3):pages 349–362, June 1990. ISSN 0550-3213. doi:10.1016/0550-3213(90)90432-D. URL http://www.sciencedirect.com/science/article/pii/055032139090432D.

[3] J. Ambjorn, L. Glaser, A. Gorlich, et al. New multicritical matrix models and multicritical 2d CDT. *arXiv:1202.4435*, February 2012. URL http://arxiv.org/abs/1202.4435.

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