The transfer matrix in four-dimensional Causal Dynamical Triangulations

Andrzej Görlich

Niels Bohr Institute, University of Copenhagen

Praha, June 25th, 2012

Introduction

Causal Dynamical Triangulations (CDT) is a background independent approach to quantum gravity.

- The partition function of quantum gravity is defined as a formal integral over all geometries weighted by the Einstein-Hilbert action.

$$Z = \int \mathrm{D}[g] e^{iS^{EH}[g]} \quad \rightarrow \quad \sum_{\mathcal{T}} e^{-S^R[\mathcal{T}]}$$

- To make sense of the gravitational path integral one uses the standard method of regularization - discretization.
- The path integral is written as a non-perturbative sum over all causal triangulations $\mathcal{T}$. (lattice regularization)
- Wick rotation is well defined due to global proper-time foliation. $(a_t \rightarrow i a_t)$

Introduction

Causal Dynamical Triangulations (CDT) is a background independent approach to quantum gravity.

- The partition function of quantum gravity is defined as a formal integral over all geometries weighted by the Einstein-Hilbert action.

$$Z = \int \mathrm{D}[g] e^{iS^{EH}[g]} \quad \rightarrow \quad \sum_{\mathcal{T}} e^{iS^{R}[\mathcal{T}]}$$

- To make sense of the gravitational path integral one uses the standard method of regularization - discretization.
- The path integral is written as a non-perturbative sum over all causal triangulations $\mathcal{T}$. (lattice regularization)
- Wick rotation is well defined due to global proper-time foliation. $(a_t \rightarrow ia_t)$

Introduction

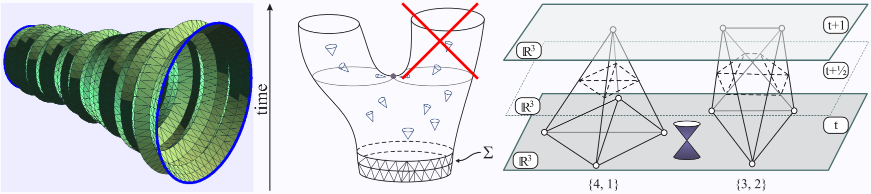
Causal Dynamical Triangulations (CDT) is a background independent approach to quantum gravity.

- The partition function of quantum gravity is defined as a formal integral over all geometries weighted by the Einstein-Hilbert action.

$$Z = \int \mathrm{D}[g] e^{iS^{EH}[g]} \quad \rightarrow \quad \sum_{\mathcal{T}} e^{-S^R[\mathcal{T}]}$$

- To make sense of the gravitational path integral one uses the standard method of regularization - discretization.
- The path integral is written as a non-perturbative sum over all causal triangulations $\mathcal{T}$. (lattice regularization)
- Wick rotation is well defined due to global proper-time foliation. $(a_t \rightarrow i a_t)$

Causal Dynamical Triangulation



- 4**D simplicial manifold** ($S^3 \times S^1$) is obtained by gluing pairs of 4-simplices along their 3-faces. The metric is **flat** inside each 4-simplex. Curvature is localized at triangles.
- **C**ausal **D**ynamical **T**riangulations assume global proper-time foliation. Spatial slices (leaves) are build from equilateral **tetrahedra**. They have fixed topology (S^3) and are not allowed to split in time.
- Foliation distinguishes between time-like (a_t) and spatial-like links (a_s).

Regge action

The Einstein-Hilbert action has a natural realization on piecewise linear geometries called Regge action

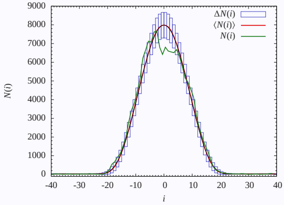
$$S^E[g] = -\frac{1}{G}\int \mathrm{d}t \int \mathrm{d}^D x \sqrt{g}(R - 2\Lambda)$$

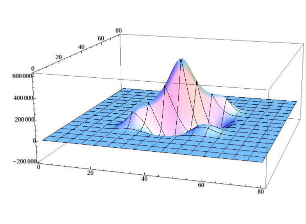
$$\frac{a_t}{a_s}$$

$$S^R[\mathcal{T}] = -K_0 N_0 + K_4 N_4 + \Delta(N_{14} - 6N_0)$$

N_0, N_4, N_{14} - number of vertices, simplices, simplices of type $\{1, 4\}$

De Sitter phase





In the de Sitter phase, the average volume is given by the formula

$$\bar{n}_t \equiv \langle n_t \rangle = H \cos^3\left(\frac{t}{W}\right).$$

It describes Euclidean **de Sitter** space (S^4), which is a maximally symmetric classical **vacuum solution** of the minisuperspace action

$$S[v] = \frac{1}{G} \int \frac{\dot{v}^2}{v} + v^{\frac{1}{3}} - \lambda v \mathrm{d}t, \quad \mathrm{d}s^2 = \mathrm{d}\tau^2 + a^2(\tau)\, \mathrm{d}\Omega_3^2, v = a^3$$

Quantum fluctuations

Correlations of spatial volume fluctuations around the classical solution $\bar{n}_t$ are given by the **semiclassical** expansion of the **effective action** describing quantum fluctuations,

$$\mathbf{C}_{ij} \equiv \langle (n_i - \bar{n}_i)(n_j - \bar{n}_j) \rangle, \quad [\mathbf{C}^{-1}]_{ij} = \left. \frac{\partial^2 S[n]}{\partial n_i \partial n_j} \right|_{n=\bar{n}}.$$

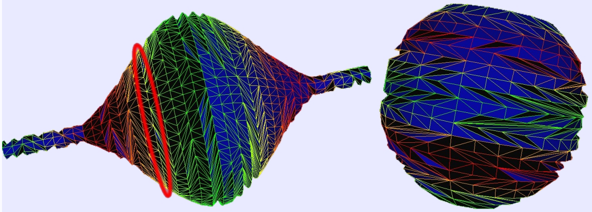
The effective action is a discretization of the minisuperspace action,

$$S[n] = \frac{1}{\Gamma} \sum_t \left(\frac{(n_{t+1} - n_t)^2}{n_{t+1} + n_t} + \mu n_t^{1/3} - \lambda n_t \right)$$

$$\Updownarrow$$

$$S[v] = \frac{1}{G} \int \frac{\dot{v}^2}{v} + v^{\frac{1}{3}} - \lambda v \mathrm{d}t$$

Transfer matrix



The model is completely determined by transfer matrix $\mathcal{M}$ labeled by 3D triangulations τ.

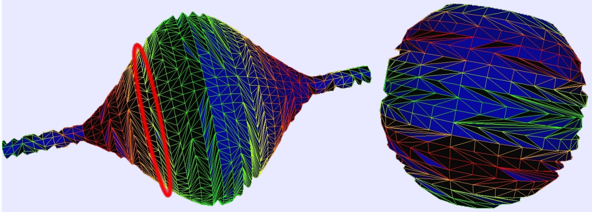
$$Z = \sum_{\mathcal{T}} e^{-S^R[\mathcal{T}]} = \mathrm{Tr}\mathcal{M}^T$$

$$P^{(T)}(\tau_1, \ldots, \tau_T) = \frac{1}{Z} \langle \tau_1 | \mathcal{M} | \tau_2 \rangle \langle \tau_2 | \mathcal{M} | \tau_3 \rangle \ldots \langle \tau_T | \mathcal{M} | \tau_1 \rangle$$

$$|n\rangle = \sum_{\tau \sim n} |\tau\rangle \quad \longrightarrow \quad \rho(n) = |n\rangle\langle n| \equiv \sum_{\tau \sim n} |\tau\rangle\langle\tau|$$

$$P^{(T)}(n_1, \ldots, n_T) = \frac{1}{Z} \langle n_1 | M | n_2 \rangle \langle n_2 | M | n_3 \rangle \ldots \langle n_T | M | n_1 \rangle$$

Transfer matrix



The model is completely determined by transfer matrix $\mathcal{M}$ labeled by 3D triangulations τ.

$$Z = \sum_{\mathcal{T}} e^{-S^R[\mathcal{T}]} = \mathrm{Tr}\mathcal{M}^T$$

$$P^{(T)}(\tau_1, \dots, \tau_T) = \frac{1}{Z}\langle \tau_1|\mathcal{M}|\tau_2\rangle\langle\tau_2|\mathcal{M}|\tau_3\rangle \dots \langle\tau_T|\mathcal{M}|\tau_1\rangle$$

$$\xcancel{|n\rangle = \sum_{\tau\sim n}|\tau\rangle} \quad \longrightarrow \quad \rho(n) = |n\rangle\langle n| \equiv \sum_{\tau\sim n}|\tau\rangle\langle\tau|$$

$$P^{(T)}(n_1, \dots, n_T) = \frac{1}{Z}\langle n_1|M|n_2\rangle\langle n_2|M|n_3\rangle \dots \langle n_T|M|n_1\rangle$$

Effective transfer matrix

The effective action obtained from the covariance matrix

$$S[n] = \frac{1}{\Gamma} \sum_t \left(\frac{(n_{t+1} - n_t)^2}{n_{t+1} + n_t} + \mu n_t^{1/3} - \lambda n_t \right)$$

suggests, that the measurements for aggregate states $|n\rangle$ are well described by an effective transfer matrix M labeled by the scale factor,

$$S_{eff} = \sum_t L_{eff}(n_t, n_{t+1})$$

$$L_{eff}(n, m) = \frac{1}{\Gamma} \left[\frac{(n-m)^2}{n+m} + \mu \left(\frac{n+m}{2} \right)^{1/3} - \lambda \frac{n+m}{2} \right]$$

$$\langle n|M|m\rangle = \mathcal{N} e^{-L_{eff}(n,m)}$$

Measurements

Assuming that

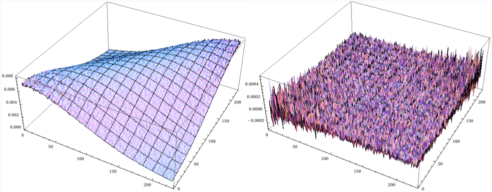
$$P^{(T)}(n_1,\ldots,n_T)=\frac{1}{Z}\langle n_1|M|n_2\rangle\langle n_2|M|n_3\rangle\ldots\langle n_T|M|n_1\rangle,$$

we can measure elements of M

$$\langle n|M|m\rangle=\sqrt{P^{(2)}(n,m)}\ \text{or}\ \langle n|M|m\rangle=\frac{P^{(3)}(n_1=n,n_2=m)}{\sqrt{P^{(4)}(n_1=n,n_3=m)}}$$

and check that it is consistent with the minisuperspace model

$$\langle n|M|m\rangle=\mathcal{N}e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m}+\mu\left(\frac{n+m}{2}\right)^{1/3}-\lambda\frac{n+m}{2}\right]}$$



Kinetic part

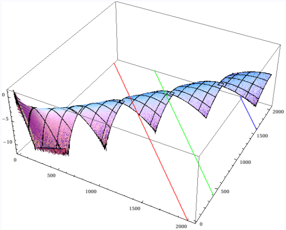
The kinetic term

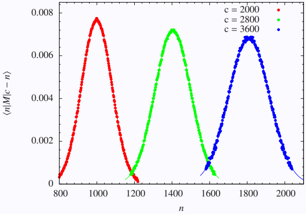
$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\frac{n+m}{2}\right]}$$

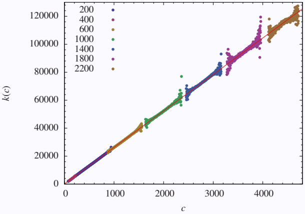
causes a Gaussian behaviour for $n + m = c$

$$\langle n|M|c-n\rangle = \mathcal{N}(c) e^{-\frac{(2n-c)^2}{k(c)}}, \quad k(c) = \Gamma \cdot c$$

$\Gamma \approx 26.1$ is constant for all ranges of n.







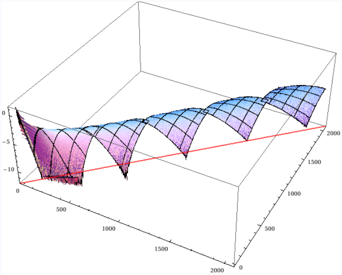
Potential part

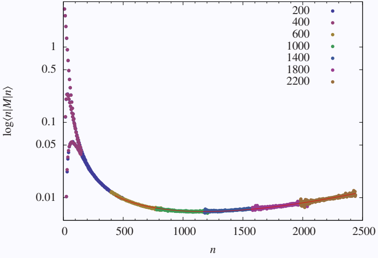
The potential term

$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda \frac{n+m}{2}\right]}$$

can be extracted from gathered data for $n = m$

$$\log\langle n|M|n\rangle = -\frac{1}{\Gamma}\left(\mu n^{1/3} - \lambda n\right) + \mathrm{const}$$



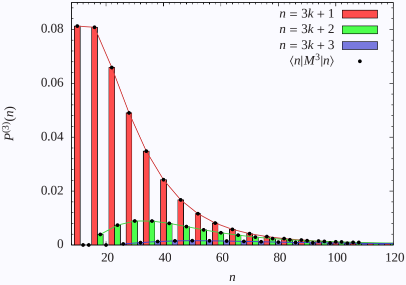


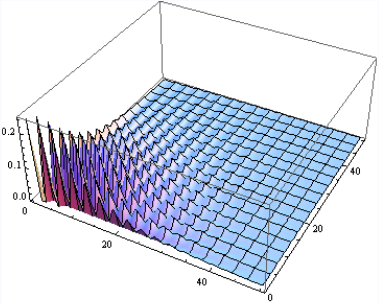
The stalk

For small volumes n we observe strong discretization effects,

$$P^{(3)}(n) = \frac{1}{\mathrm{Tr} M^3} \langle n | M^3 | n \rangle.$$

Split into three families. We smooth out M by summing over the families.

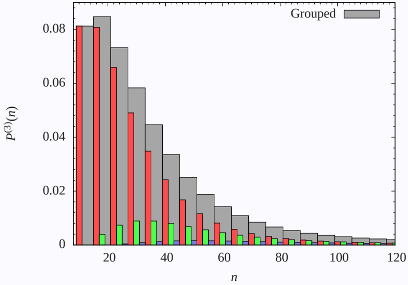


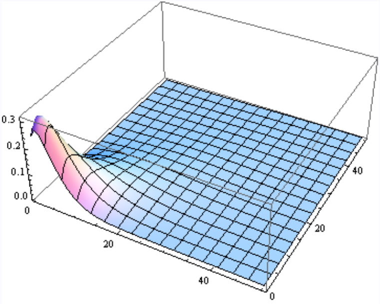


For small volumes n we observe strong discretization effects,

$$P^{(3)}(n) = \frac{1}{\mathrm{Tr} M^3} \langle n | M^3 | n \rangle .$$

Split into three families. We smooth out M by summing over the families.





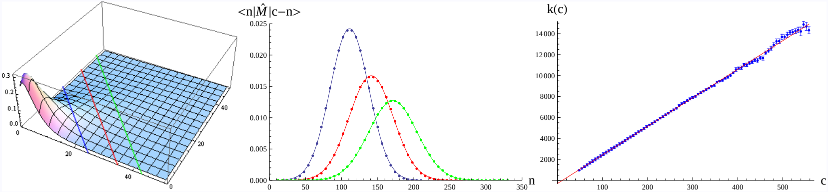
The stalk

The effective action for the stalk has the same form as for the blob

$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + v\left(\frac{n+m}{2}\right)\right]}, \quad v(x) = \mu x^{1/3} - \lambda x + \delta x^{-\rho}$$

Gaussian
for $n+m=c$

The kinetic term is in complete agreement, $k(c) = \Gamma \cdot c$



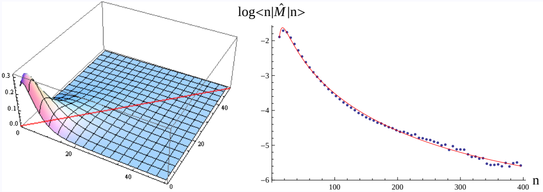
The effective action for the stalk has the same form as for the blob

$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + v\left(\frac{n+m}{2}\right)\right]}, \quad v(x) = \mu x^{1/3} - \lambda x + \delta x^{-\rho}$$

minisuperspace potential

possible curvature corrections

The potential term is slightly modified for small volumes



Conclusions

- The transfer matrix allows to directly measure the effective action
- Measurement of the transfer matrix is much faster then of the covariance matrix
- The effective action is fully consistent with the minisuperspace model, although in CDT we do not freeze any degrees of freedom
- For small volumes we observe strong discretization effects. Despite different nature, after the *smoothing* procedure, the effective action for small volumes is basically the same as for large volumes, with a small modification in the potential.