Static, stationary and inertial Unruh-DeWitt detectors on the BTZ black hole

arXiv:1206.2055

Lee Hodgkinson [1]
University of Nottingham

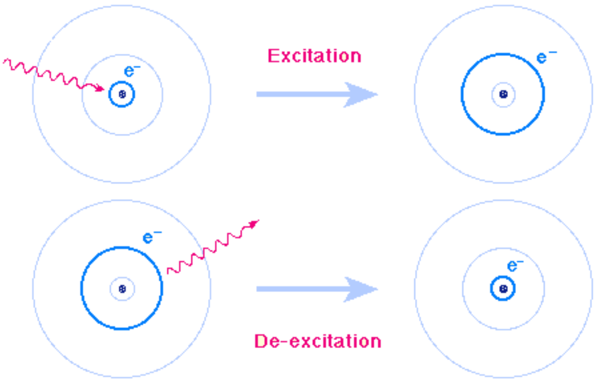
Relativity and Gravitation:100 years after Einstein in Prague
June 2012

[1](in collaboration with Jorma Louko)

Particle detectors

- Ambiguity in particle notion in QFT
- Operational definition of “particles”: Couple a simple QM system (detector) to the quantum field.

Detectors-Excitation and De-excitation

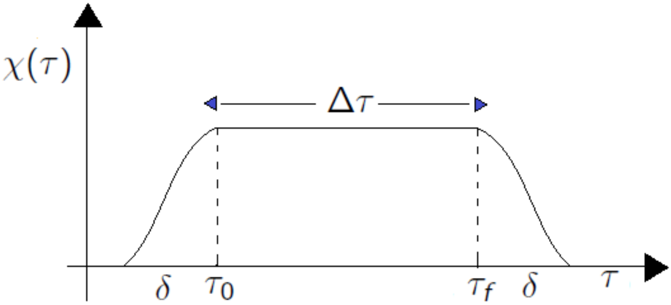


Detector response

- $|0\rangle_d \otimes |\Psi\rangle \rightarrow |E\rangle_d \otimes |\text{any}\rangle$
- What is probability of this transition? Use first-order perturbation theory; the interesting part of the probability proportional to
-
$$\mathcal{F}(E) = 2 \lim_{\epsilon \rightarrow 0_+} \text{Re} \int_{-\infty}^{\infty} \mathrm{d}u \, \chi(u) \int_0^{\infty} \mathrm{d}s \, \chi(u-s) \, \mathrm{e}^{-iEs} \, W_\epsilon(u, u-s)$$

The instantaneous transition rate

- Extreme care must be taken when obtaining the transition rate $\dot{\mathcal{F}}_\tau(E)$: Schlicht(2004), Satz(2007)
- Must switch smoothly, obtain response function in regulator free form, then take sharp switching limit.



Transition rate in three dimensions arbitrary Hadamard state

- $$W_\epsilon(\mathrm{x}, \mathrm{x}') = \frac{1}{(4\pi)}\left[\frac{U(\mathrm{x}, x')}{\sqrt{\sigma_\epsilon(\mathrm{x}, \mathrm{x}')}} + \frac{H(\mathrm{x}, \mathrm{x}')}{\sqrt{2}}\right]$$
- Hadamard characterises singularity in coincidence limit $\mathrm{x} \to \mathrm{x}'$
- Wightman function for BTZ spacetime has singularities even when $\sigma(\mathrm{x}, \mathrm{x}') \neq 0$.
- Result we obtained:

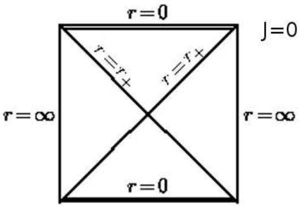
$$\dot{\mathcal{F}}_\tau(E) = \frac{1}{4} + 2\int_0^{\Delta\tau} \mathrm{d}s \ \mathrm{Re}\left[\mathrm{e}^{-iEs} W_0(\tau, \tau - s)\right] \ .$$

The BTZ black hole

- $(2+1)$-dimensional black hole.
- Periodically identify AdS_3.
-

$$ds^2 = -(N^{\perp})^2 dt^2 + f^{-2} dr^2 + r^2 \left(d\phi + N^{\phi} dt \right)^2$$

with lapse and shift functions $N^{\perp} = f = \left(-M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2} \right)^{1/2}$,
$N^{\phi} = -\frac{J}{2r^2} \qquad (|J| \leq M\ell)$



Wightman function for BTZ

- Express in terms of AdS_3 Wightman by method of images

$$G_{\mathrm{BTZ}}(\mathrm{x},\mathrm{x}') = \sum_n G_A(\mathrm{x},\Lambda^n \mathrm{x}')$$

- For field in Hartle-Hawking Vacuum, AdS Wightman functions are (Carlip 1995)

$$G_A^{(\zeta)} = \frac{1}{4\pi}\left[\frac{1}{\sqrt{\Delta \mathrm{X}^2}} - \zeta\frac{1}{\sqrt{\Delta \mathrm{X}^2 + 4\ell^2}}\right], \quad \zeta \in \{0, 1, -1\}$$

-

$$\Delta \mathrm{X}^2 := \left[(X_1 - X_1')^2 - (T_1 - T_1')^2 + (X_2 - X_2')^2 - (T_2 - T_2')^2\right]$$

Transition rate for BTZ

- $$\Delta\tilde{X}_n^2 := \Delta X^2\left(x(\tau), \Lambda^n x(\tau - \ell\tilde{s})\right)/(2\ell^2)$$
$$= -1 + \sqrt{\alpha(r)\alpha(r')}\cosh\left[(r_+/\ell)\left(\phi - \phi' - 2\pi n\right) - (r_-/\ell^2)\left(t - t'\right)\right]$$
$$- \sqrt{(\alpha(r) - 1)(\alpha(r') - 1)}\cosh\left[(r_+/\ell^2)\left(t - t'\right) - (r_-/\ell)\left(\phi - \phi' - 2\pi n\right)\right],$$
- where,
$$\alpha(r) = \left(\frac{r^2 - r_-^2}{r_+^2 - r_-^2}\right)$$
- $$\dot{\mathcal{F}}_\tau(E) = \frac{1}{4} + \frac{1}{2\pi\sqrt{2}} \sum_{n=-\infty}^{\infty} \int_0^{\Delta\tau/\ell} \mathrm{d}\tilde{s}\, \mathrm{Re}\left[\mathrm{e}^{-iE\ell\tilde{s}} \left(\frac{1}{\sqrt{\Delta\tilde{X}_n^2}} - \frac{\zeta}{\sqrt{\Delta\tilde{X}_n^2 + 2}}\right)\right]$$

Detector co-rotating with horizon angular velocity

- Spacetime dragged along with hole's rotation
- Look at detector co-rotating with BH at the horizon velocity (for a non-spinning hole this reduces to static detector)
- Singularities arise that aren't characterised by Hadamard, but are integrable.
- $$\dot{\mathcal{F}}(E) = \mathrm{e}^{-E/T_{\text{loc}}}\dot{\mathcal{F}}(-E).$$

  where T_{loc}: local co-rotating Hawking temperature.
- (Stationary, NON-co-rotating detectors do NOT respond thermally)

Asymptotics co-rotating detector

- Large r_+/ℓ asymptotics (large black hole mass M in static case) for $E \neq 0$ (physically interesting case)

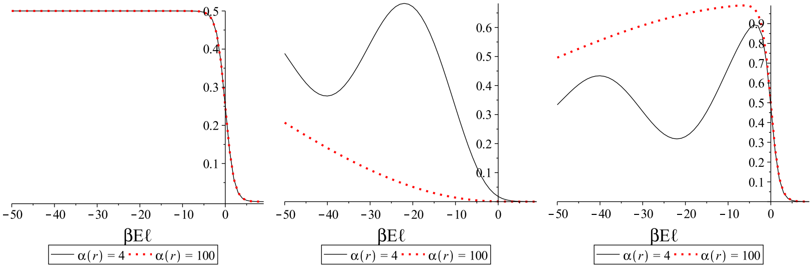
$$
\begin{aligned}
\dot{\mathcal{F}}(E) &= \frac{1}{2(\mathrm{e}^{\beta E \ell}+1)} - \frac{\zeta \mathrm{e}^{-\beta E \ell/2}}{2\pi} \int_0^\infty \mathrm{d}y \, \frac{\cos\left(y \beta E \ell/\pi\right)}{\sqrt{Q_0 + \cosh^2 y}} \\
&+ \frac{\mathrm{e}^{-\beta E \ell/2} \cos\left(\beta E r_-\right)}{\sqrt{\pi}\beta E \ell} \times \\
&\times \left\{ \operatorname{Im}\left[\left(\frac{(4K_1)^{i\beta E \ell/(2\pi)}}{\sqrt{K_1}} - \frac{\zeta (4Q_1)^{i\beta E\ell/(2\pi)}}{\sqrt{Q_1}} \right) \Gamma\left(1 + \frac{i\beta E \ell}{2\pi}\right) \Gamma\left(\frac{1}{2} - \frac{i\beta E\ell}{2\pi}\right) \right] \right. \\
&\left. + O\left(\mathrm{e}^{-2\pi r_+/\ell}\right) \right\}
\end{aligned}
$$

Asymptotics co-rotating detector

- small r_+/ℓ asymptotics (small black hole mass M in static case)

$$\dot{\mathcal{F}}(E) = \frac{\ell \mathrm{e}^{-\beta E \ell/2}}{\pi^2 r_+} \int_0^\infty \mathrm{d}v \int_0^\infty \mathrm{d}y \cos\left(\frac{v\beta E \ell r_-}{\pi r_+}\right) \cos\left(\frac{y\beta E\ell}{\pi}\right) \times$$
$$\times \left[\left(\frac{\alpha \sinh^2 v}{(\alpha - 1)} + \cosh^2 y\right)^{-1/2} - \zeta\left(\frac{1 + \alpha \sinh^2 v}{(\alpha - 1)} + \cosh^2 y\right)^{-1/2}\right] + O(1)$$

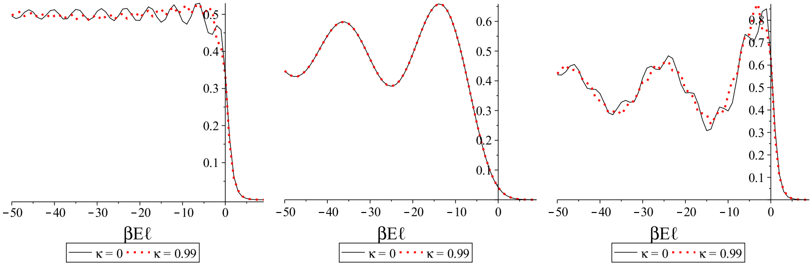
Co-rotating (static) plots



(a) $\zeta = 0$ (b) $\zeta = 1$ (c) $\zeta = -1$

Figure: $\dot{\mathcal{F}}$ as a function of $\beta E \ell$ for large non-spinning hole, with detector near (solid) and far from hole (dotted)

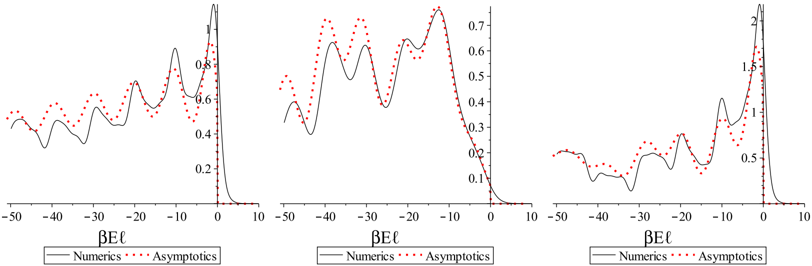
Co-rotating (static) sensitivity to BH rotation



(a) $\zeta = 0$ (b) $\zeta = 1$ (c) $\zeta = -1$

Figure: $\dot{\mathcal{F}}$ as a function of $\beta E \ell$ for smaller hole, non-spinning (solid) and spinning (dotted).

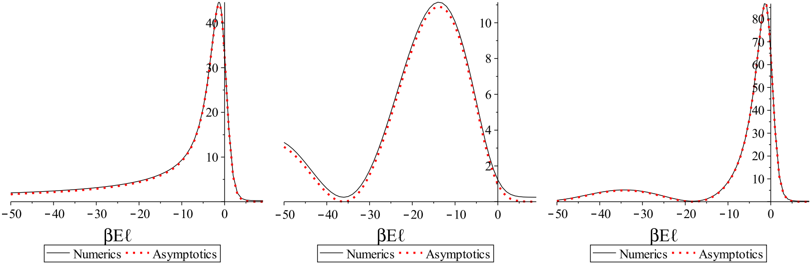
How are our asymptotics performing?



(a) $\zeta = 0$ (b) $\zeta = 1$ (c) $\zeta = -1$

Figure: $\dot{\mathcal{F}}$ as a function of $\beta E \ell$ for even smaller, highly spinning hole.

Domain of small (r_+/l) asymptotics



(a) $\zeta = 0$ (b) $\zeta = 1$ (c) $\zeta = -1$

Figure: $\dot{\mathcal{F}}$ as a function of $\beta E \ell$ for very small non-spinning hole

Inertial detector transition rate

- Considered detector radially in-falling on geodesic to $J = 0$ BTZ.
- Can observe numerically that detector does not respond thermally.
- Not surprising (Deser-Levin 1998): lack of real horizon in AdS_3 means detectors with $a < 1/\ell$, no well defined temp.

Large M asymptotics

- $$\dot{\mathcal{F}}_{\tau}^{n\neq 0}(E) = \frac{1}{\pi\sqrt{2\cos\tilde{\tau}}}\int_0^{\Delta\tilde{\tau}} \frac{\cos\left(\tilde{E}\tilde{s}\right)\,\mathrm{d}\tilde{s}}{\sqrt{\cos(\tilde{\tau}-\tilde{s})}}\left[(1-\zeta)\left(\frac{1}{\sqrt{K_1}}+\frac{1}{\sqrt{K_2}}\right)+\frac{\zeta f_+ - f_-}{2K_1^{3/2}}\right] + O\left(\mathrm{e}^{-5\pi\sqrt{M}}\right) ,$$
- where
$$f_{\pm} := \frac{\sin\tilde{\tau}\sin(\tilde{\tau}-\tilde{s})\pm 1}{\cos\tilde{\tau}\cos(\tilde{\tau}-\tilde{s})} .$$
- Detector operates for a finite time-can't push switch on to $-\infty$. Formula assumes the detector is switched on(off) before hitting singularities.

3D Plot of transition rate of inertial detector

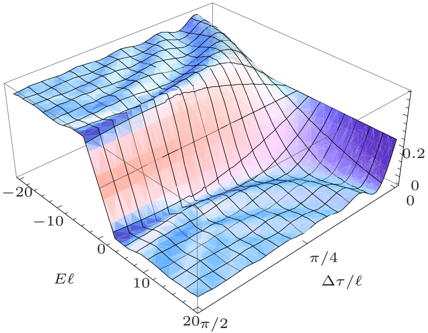


Figure: 3D plot of the the transition rate for a large hole

Conclusions and Summary

- Regulator-free transition probability and rate in arbitrary 3D Hadamard state-both well-defined and finite under sharp switching.
- Analysed response in the Hartle Hawking vacua on BTZ: found boundary conditions at infinity play a significant role.
- KMS exhibited for the co-rotating detector but not for radially in-falling (although nothing singular happens crossing horizon as expected for Hartle-Hawking vacuum).
- Analytic results obtained agree well with numerics.
- Future directions: 4D Schwarzschild (with Adrian Ottewill)