Variations on spacetimes with boost-rotation symmetry

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Relativity and Gravitation: 100 Years after Einstein in Prague

Abstract

Various aspects of boost-rotation symmetric spacetimes representing, in general, two rotating charged objects accelerated in opposite directions as, for example, the C-metric describing two accelerating black holes, are summarized and their limits are considered. A particular attention is paid (a) to the special-relativistic limit in which the electromagnetic field becomes the "magic field" of two oppositely accelerated, rotating charged relativistic discs [2]; (b) to the Newtonian limit which is analyzed using the Ehlers frame theory. In contrast to some previous discussions, our results are physically plausible in the sense that the Newtonian limit corresponds to the fields of classical point masses accelerated uniformly in classical mechanics. This corroborates the physical significance of the boost-rotation symmetric spacetimes [1].

The Ernst method of removing nodal singularities from the charged C-metric representing a uniformly accelerated black hole with mass m, charge q and acceleration A by "adding" an electric field E is generalized. Utilizing the new form of the C-metric found recently, Ernst's simple "equilibrium cond." $mA = qE$ valid for small accelerations is generalized for arbitrary A [3].

The electromagnetic and gravitational radiation of the charged C-metric is also analyzed (work in progress).

Newtonian limit

The Newtonian limit of a particular spacetime is important connection between the general relativistic solution and a Newtonian one. The limit is made within the framework of Ehler's frame theory (see [1] for more details and references). The key point of this framework is the causality constant $\lambda = c^{-2}$ which is 0 in the Newtonian limit. But for the limit itself we have to choose a suitable set of observers (and naturally adapted coordinate system).

Using the global coordinates with the functions μ and ν known, we have to introduce a shift of coordinate origin $z = \zeta + \lambda^{-1} g^{-1}$ otherwise the particles iself will disappear to infinity.

with (shift of coordinate origin) *without*

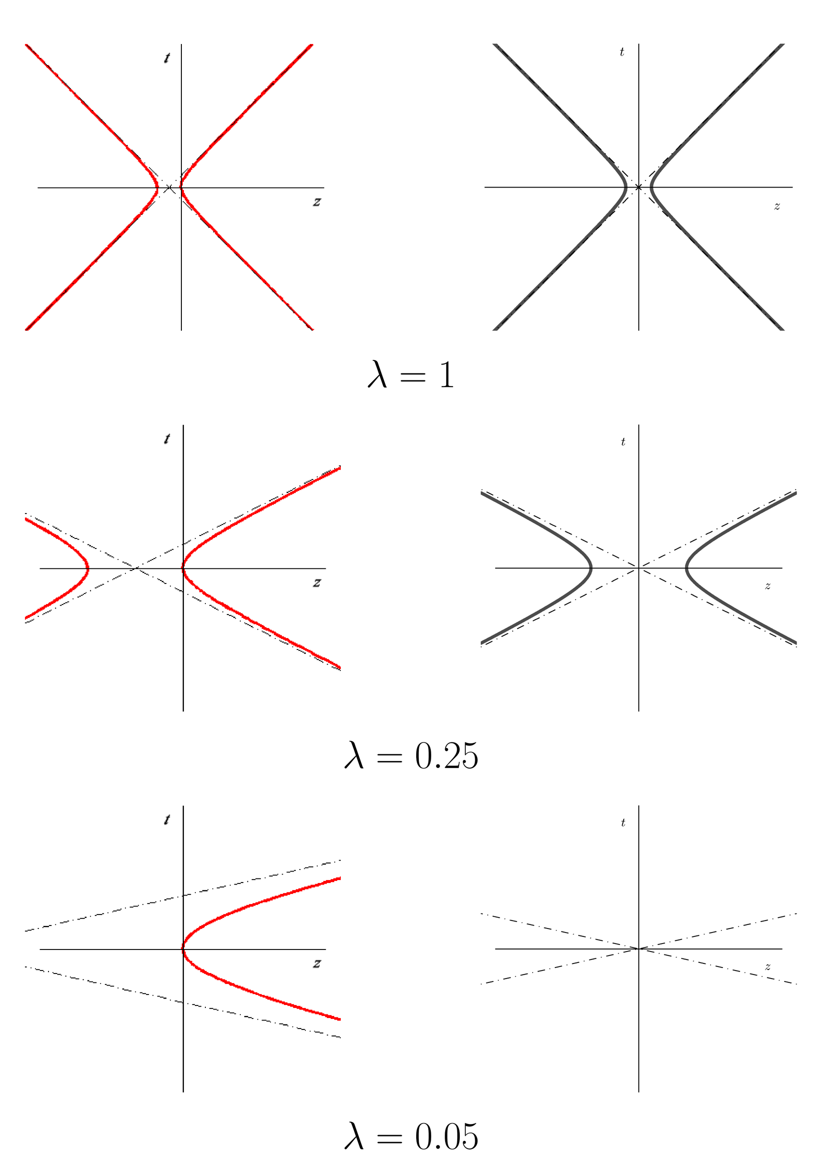


Figure: Sequence of spacetime diagrams with decreasing λ.

Finally, we will recover the Newtonian potential of a point particle undergoing uniform acceleration $z = \frac{1}{2} g t^2$.

References

[1] Bičák J., Kofroň D.: *The Newtonian limit of spacetimes for accelerated particles and black holes*, Gen. Rel. Gravit. **41** (2009)

[2] Bičák J., Kofroň D.: *Accelerating electromagnetic magic field from the C-metric*, Gen. Rel. Gravit. **41** (2009)

[3] Bičák J., Kofroň D.: *Rotating charged black holes accelerated by an electric field*, Phys. Rev. D **82** (2010)

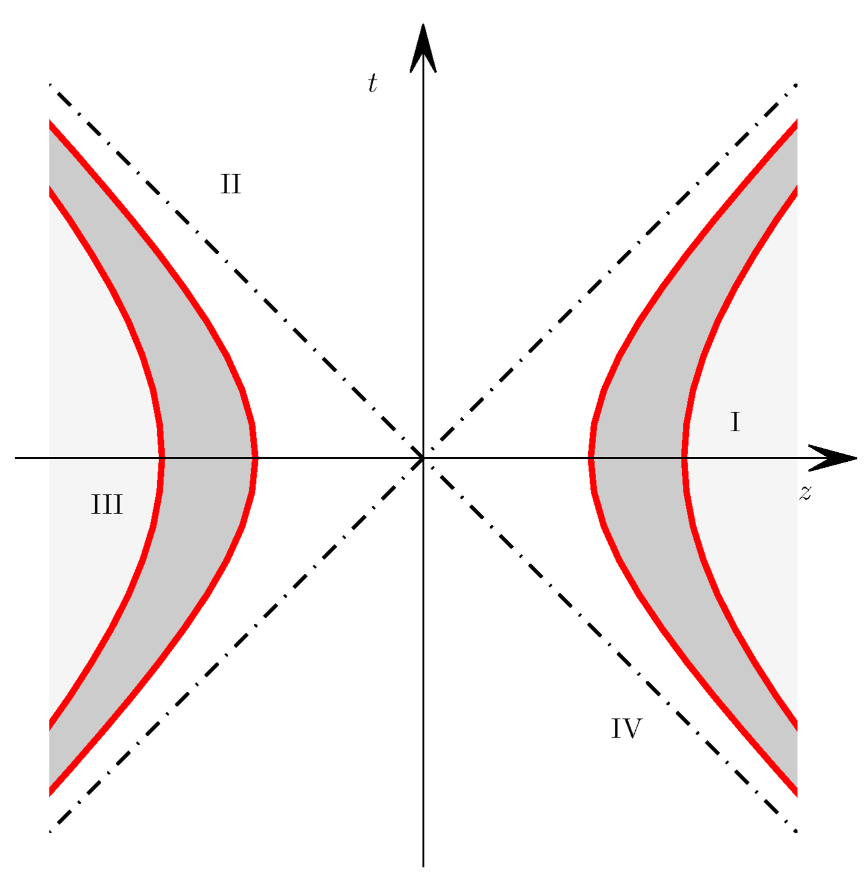
Boost-rotation symmetric spacetimes

Boost-rotation symmetric spacetimes are important examples of exact solutions of Einstein field equations which are known explicitly and which are interpreted as to describe non-trivially moving sources of gravitation field. The only assumptions that are made is the existence of two Killing vectors: boost Killing vector ξ_{B} whose orbits are hyperbolaes and axial Killing vector ξ_ϕ with closed orbits. Thus the metric of electrovacuum rotating boost-rotation symmetric spacetimes reads:

$$\mathrm{d}s^2 = \frac{e^{\mu}\left(z\mathrm{d}t - t\mathrm{d}z + \Omega\mathrm{d}\phi\right)^2 - e^{\nu}\left(z\mathrm{d}z - t\mathrm{d}t\right)^2}{z^2 - t^2} - e^{\nu}\mathrm{d}r^2 - e^{-\mu} r^2 \mathrm{d}\phi^2 . \tag{1}$$

The function μ, ν and Ω are functions of $a = z^2 - t^2$ and $b = r^2$ only and are to be determined from Ernst equations and the displacement of sources. Ernst equations are difficult to solve, but if we let the rotation and electromagntic field vanish, then then the Einsteins field equations are reduced to the wave equation in auxiliar flat spacetime $\Box\mu = 0$.

Spacetime diagram



red lines – world lines of sources

gray regions – different conicity regions of the axis, or world-sheet of the black-hole horizon (in the case of the C-metric)

Properies

Boost-rotation symmetric spacetimes

- describe uniformly accelerated sources
- are asymptotically flat (for spatially bounded sources)
- can be transformed in global coordinates (1)
- hypersurface of null norm of the boost Killing vector ξ_{B} invariantly divide the spacetime in four quadrants
  - below the roof – locally Weyl spacetimes
  - above the roof – locally cylindrical waves
- are of algebraic type I, in general
- radiative!
- the interpretation is based upon special-relativistic limit in which the worldlines of sources are orbits of boost Killing vectors
- the axis itself suffers conical singularities – the remnants of the fact that the physical source of acceleration of the objects is not considered. Thus there are strings and struts along the axist that causes the acceleration

The rotating charged C-metric

The rotating charged C-metric is special case of boost-rotation symmetric spacetimes which could have been found due to its algebraical speciality (it is of algebraic type D). It describes two BHs with regular horizons.

$$\mathrm{d}s^2 = \frac{1}{A^2(x-y)^2}\Bigg\{ \frac{\mathcal{G}(y)}{1+(aAxy)^2}\left[\mathrm{d}t + aA\left(1-x^2\right)K\mathrm{d}\phi\right]^2 - \frac{1+(aAxy)^2}{\mathcal{G}(y)}\mathrm{d}y^2 + \frac{1+(aAxy)^2}{\mathcal{G}(x)}\mathrm{d}x^2 + \frac{\mathcal{G}(x)}{1+(aAxy)^2}\left[\left(1+a^2A^2y^2\right)K\mathrm{d}\phi + aAy^2\mathrm{d}t\right]^2\Bigg\}, \tag{2}$$

The function $\mathcal{G}(\xi)$ is polynomial and recently has been factorized by Hong and Teo. The transformation from (2) to (1) is explicitly known.

Minkowskian limit

The flat spacetime limit $G \to 0$ of the charged rotating C-metric leads to an electromagnetic field of two counterrotating bend charged discs undergoing uniform acceleration (see [2]).

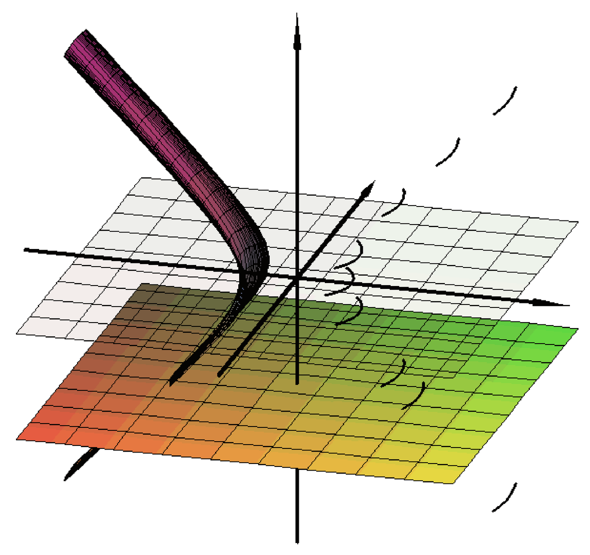


Figure: The space-time diagram of accelerated magic electromagnetic field.

Gravitational radiation

Gravitational radiation is investigated using the Bondi's approach to it. There are some non-trivial problems including:

- *simultaneity notion* – how to choose appropriate cuts of $\mathscr{I}^+$ when we are dealing with gravitational radiation from spatially extended sources? Or even black holes?
- motion of the sources
- presence of the conical singularities
- utilization of principal null geodesic congruences
- gravity – elecromagnetic interaction
- ...
- (work in progress)

Removing the conical singularities

By submersing the charged rotating black holes in "homogeneous" external electromagnetic field (a simple trick – actualy utilization of generating metods for Ernst equations) it is possible to "cure" the axis which suffers conical singularities. This is because by adding this external field we include the the physical source of the motion in the spacetime. It is the electromagnetic force then. The equilibrium condition is at the first order simply

$$mA = qE$$

(plus some corrections of higher order in m). See [3] for details.

But, unfortunatelly, this external electromagnetic field breaks the asymptotical flatness of spacetime.