# Shape Dynamics

## at Relativity and Gravitation, Prague 2012

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June 29, 2012

# What Is Gravity?

## General Relativity: Gravity is Spacetime Geometry

**field**: spacetime metric $g_{\mu\nu}$
**symmetry**: general covariance
**effective field theory**: low dimensional local terms
$\Rightarrow$ General Relativity as well posed effective field theory

## Machian Principles

**field**: spatial metric $g_{ab}$
**three symmetries**: local time reparametrization invariance, local spatial conformal invariance, spatial covariance
$\Rightarrow$ Barbour's principles for a gravity theory

# Machian Picture

## Two problematic Machian Principles

1. **relativity of clocks**: local time reparametrization invariance
   $\Rightarrow$ local Hamilton constraints
2. **relativity of rods**: local spatial conformal invariance
   $\Rightarrow$ local spatial conformal constraints

Generically, second class constraint system
$\Rightarrow$ can **not** simultaneously realized as phase space symmetries.

## Two possibilities:

1. retain local spatial conformal invariance (symmetry trading)
   $\Rightarrow$ Shape Dynamics
2. BRST-formulation: implement both as BRST differentials (symmetry doubling)
   $\Rightarrow$ Doubly General Relativity
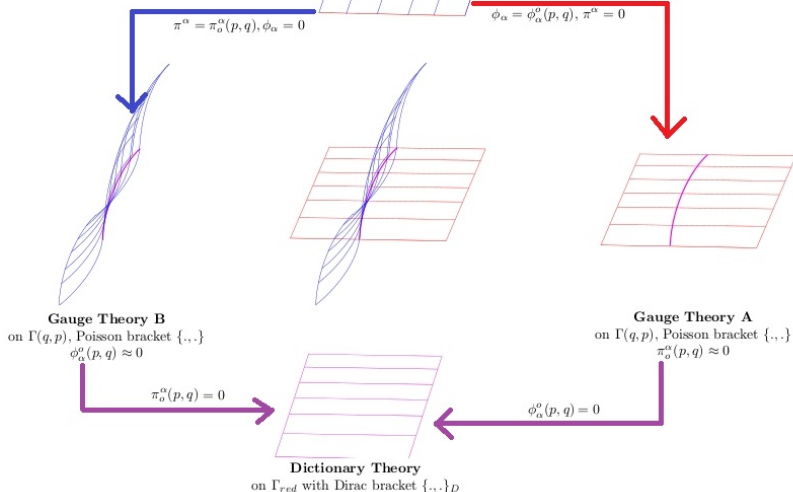
# Symmetry Trading Mechanism



**Linking Theory**
ext. phase space $\Gamma(q, p) \times \Gamma(\phi, \pi)$
ordinary Poisson bracket $\{.,.\}$
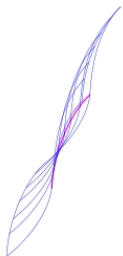ext. first class constraints:
$\chi_\alpha^1 = \phi_\alpha - \phi_o^\alpha(p, q) \approx 0$
$\chi_2^\alpha = \pi^\alpha - \pi_\alpha^o(p, q) \approx 0$

$\pi^\alpha = \pi_o^\alpha(p, q), \phi_\alpha = 0$

$\phi_\alpha = \phi_\alpha^o(p, q), \pi^\alpha = 0$

**Gauge Theory B**
on $\Gamma(q, p)$, Poisson bracket $\{.,.\}$
$\phi_\alpha^o(p, q) \approx 0$

**Gauge Theory A**
on $\Gamma(q, p)$, Poisson bracket $\{.,.\}$
$\pi_o^\alpha(p, q) \approx 0$

$\pi_o^\alpha(p, q) = 0$

$\phi_\alpha^o(p, q) = 0$

**Dictionary Theory**
on $\Gamma_{red}$ with Dirac bracket $\{.,.\}_D$
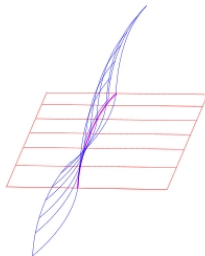
# Construction of Shape Dynamics

On ADM-phase space $(g_{ab}, \pi^{ab})$ with :



**ADM Gravity**

$$S(N) = \int N \left( \frac{G(\pi,\pi)}{\sqrt{|g|}} - (R - 2\Lambda)\sqrt{|g|} \right)$$
$$H(v) = \int \pi^{ab} \mathcal{L}_v g_{ab}$$

**Shape Dynamics**

$$H_{SD} = V - V_o$$
$$Q(\rho) = \int (\pi - \langle \pi \rangle \sqrt{|g|})$$
$$H(v) = \int \pi^{ab} \mathcal{L}_v g_{ab}$$

# Heuristic for Symmetry Doubling

## Constrained Hamiltonian path integral

$$Z = \int Dq\, Dp\, \delta[\chi]\, \delta[\sigma]\, |\{\chi, \sigma\}|\, \exp(i \int dt\, p.\dot{q})$$
$$= \int Dq\, Dp\, D\eta\, DP\, \exp(i \int dt(p.\dot{q} + P.\dot{\eta} - \{\Omega, \Psi\}))$$

with BRST-generator $\Omega$, gauge-fixing $\Psi$.

## Wish list

1. both Machian invariances implemented as invariances under
   $s_1. = \{\Omega, .\}$ and $s_2. = \{\Psi, .\}$

2. locality of $L = p.\dot{q} + P.\dot{\eta} - \{\Omega, \Psi\}$

3. intuitive interpretation

# From Symmetry Trading to Symmetry Doubling

## Symmetry Trading requires

two first class surfaces (original and equivalent gauge symmetry)
that **gauge fix** one another

## BRST-gauge-fixing

- $\Omega$ is nilpotent because orig. system is first class
- $\Psi$ can be chosen nilpotent because equiv. system is first class
- if $H_o$ (on shell) Poisson commutes with $\Omega$ and $\Psi$ then gauge fixed

$$H_{BRS} = H_o + \{\Omega, \Psi\}$$

is annihilated by both $s_\Omega$ and $s_\Psi$

## Symmetry Doubling:

Canonical action $S = \int dt (p_i \dot{q}^i + P_\alpha \dot{\eta}^\alpha - H_{BRS})$ is invariant under two BRST-transformations (up to a boundary term).

# Doubly General Relativity

## Hamiltonian of Doubly General Relativity

The Hamiltonian of Doubly General Relativity is

$$H_{DGR} = S(\frac{\pi}{\sqrt{|g|}} + \lambda) + H(F^a) + \mathcal{O}(\eta)$$

It is **local** and **invariant** under **two** BRST transformations:

1. ADM-BRST transformation encoding on-shell spacetime diffeomorphism symmetry
2. SD-BRST transformation encoding spatial conformal symmetry

# Summary

1. Implement **all** of Barbour's Machian principles in gravity theory
2. Symmetry trading is generic and gives equivalent gauge theories
3. Shape Dynamics is locally equivalent to General Relativity
4. Symmetry trading implies symmetry doubling in BRST formalism
5. Equivalence of Shape Dynamics and GR $\Rightarrow$ Doubly General Relativity
6. DGR implies a new **local** theory space for gravity. To explore:
   - are there semiclassical predictions (beyond E-H-action)?
   - universality classes on this revised theory space (FRGE methods)?
   - new bulk/bulk dualities?

# THANK YOU