A lattice Universe: the fitting problem in cosmology.

100 years after Einstein in Prague. 25-29 June 2012.

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J.-P. Bruneton and JL, arXiv:1204.3433 [gr-qc]; accepted in CQG
J.-P. Bruneton and JL, in preparation.

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Why studying lattice models?

- Toy-model for the backreaction/averaging issue: What is the FLRW model emerging from smoothing the late-time Universe?
- Fitting problem: Link between observables in inhomogeneous Universe and in effective FLRW model.
- Hence, two aspects to this problem:
  - Kinematics: Metric of spacetime.
  - Observables: Under which conditions can we describe our Universe as FLRW on large scales?
- Improve on the Lindquist-Wheeler model (Problem at the boundaries of cells $\Rightarrow$ propagation of light and observables difficult to evaluate).[Clifton et al, PRD85, 2012].

A cubic lattice Universe (I)

- Aim: Look for a solution to field equations:
  - Regular cubic lattice of point masses M separated by an initial distance L: Solutions with cubic symmetries. Fourier expansion.
  - Dynamical solution: masses should separate from each other 'cosmologically'.
- Full solution impossible.
- For a lattice of 'galaxies': $M/L \sim 10^{-8}$: Power series expansion.
- Use synchronous coordinates: $ds^2 = -dt^2 + 2g_{0i}dx^i dt + g_{ij}dx^i dx^j$.

A cubic lattice Universe (II)

- Source term:

$$T_{00} \propto M \sum_{\mathbf{n}\in\mathbb{Z}^3} \delta^{(3)}(\mathbf{x} - L\mathbf{n}) = \frac{M}{L^3} \sum_{\mathbf{n}\in\mathbb{Z}^3} e^{\frac{2\pi \mathbf{n}.\mathbf{x}}{L} i}.$$

- Zero mode ($\mathbf{n} = \mathbf{0}$): constant comoving energy density: $\rho = \frac{M}{L^3}$.
- Periodic solution of the form $g_{ab} = \eta_{ab} + \frac{M}{L} h_{ab}$ impossible because in Fourier expansion:

$$G_{00}^{\text{zero mode}} = \mathcal{O}\left((M/L)^2\right) \neq T_{00} = \mathcal{O}(M/L).$$

- FLRW: $H^2 \propto \rho \Rightarrow$ metric contains $\sqrt{\rho} \sim \sqrt{M/L^3}$.
- Therefore: Expansion in powers of $\sqrt{\frac{M}{L}}$ natural.
- Metric: $g_{ab} = \eta_{ab} + \sqrt{\frac{M}{L}} h_{ab}^{(1)} + \frac{M}{L} h_{ab}^{(2)} + \mathcal{O}\left(\left(\frac{M}{L}\right)^{3/2}\right)$.

A cubic lattice Universe (III)

- Look for a solution in synchronous coordinates comoving with the masses. Only scalar modes considered.
- Fourier expansion of the metric quantities.
- Zero mode of order n fixed by zero mode of equations at order n+1.
- Solution diverges for Dirac Comb (in agreement with [Korotkin and Nicolai 1994, gr-qc/9403029])
- UV regularization: finite size objects. Peaked Gaussians:

$$\delta(x - nL) \sim \frac{1}{\eta\sqrt{\pi}} e^{-(x-nL)^2/\eta^2}.$$

A cubic lattice Universe (IV)

Solution at order $(M/L)^{3/2}$: $g_{0i} = 0$ and:

$$g_{ij} = \delta_{ij}\left[1 + 2\varepsilon\sqrt{\frac{GM}{Lc^2}}\sqrt{\frac{8\pi}{3}}\frac{ct}{L} + \frac{2GM}{Lc^2}\left(f_\eta(\mathbf{x}) + \frac{2\pi c^2 t^2}{3L^2}\right)\right.$$
$$\left. +2\left(\frac{GM}{Lc^2}\right)^{3/2}\left(2\epsilon\frac{ct}{L}\sqrt{\frac{8\pi}{3}}f_\eta(\mathbf{x}) - \frac{2\pi\epsilon}{9}\sqrt{\frac{8\pi}{3}}\frac{c^3t^3}{L^3}\right)\right]$$
$$+\frac{GM}{Lc^2}c^2t^2\partial^2_{ij}f_\eta(\mathbf{x}) + \left(\frac{GM}{Lc^2}\right)^{3/2}\varepsilon\sqrt{\frac{8\pi}{3}}\frac{c^3t^3}{3L}\partial_{ij}f_\eta(\mathbf{x}),$$

where:

$$f_\eta(\mathbf{x}) = \frac{1}{\pi}\sum_{\mathbf{n}\in\mathbb{Z}^3_*}\frac{e^{-\frac{\pi^2|\mathbf{n}|^2\eta^2}{L^2}}}{|\mathbf{n}|^2}e^{\frac{2\pi}{L}i\mathbf{n}.\mathbf{x}}.$$

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Comments on the solution

- Expanding/contracting solution: Cosmological solution.
- Valid for finite interval of time: $\delta t \ll L\sqrt{L/M} \sim 1Gyr$ for $M/L \sim 10^{-8}$.
- Hubble flow after averaging out the periodic inhomogeneities:

$$H(t) = \varepsilon\sqrt{\frac{8\pi}{3}}\sqrt{\frac{GM}{L^3}} - \frac{4\pi GMt}{L^3} + 4\sqrt{6}\pi^{3/2}\varepsilon\left(\frac{GM}{L^3}\right)^{3/2} t^2.$$

- Identical to FLRW with $\rho = M/L^3$: average density.
- Fluid approximation is exactly valid.
- Comes from the zero mode: result of the toroidal symmetry (infinite lattice).

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Redshift at order M/L

- Solve the geodesic equations order by order for observer located at origin.
- Affine parameter: $\lambda < 0$ ($\lambda = 0$ at observation).
- Look back time and position along the ray determined via geodesic equation. $t = t(\lambda)$ and $\mathbf{x} = \mathbf{x}(\lambda)$.
- Redshift:

$$
\begin{aligned}
z(\lambda) &= -\sqrt{\frac{GM}{Lc^2}}\sqrt{\frac{8\pi}{3}}\frac{\lambda}{L} \\
&+ \frac{GM}{Lc^2}\left(\frac{14\pi\lambda^2}{3L^2} + \left[f_\eta(\mathbf{x}(\lambda)) - \lambda\partial_i f_\eta(\mathbf{x}(\lambda))v^i\right]_0^\lambda\right) + \mathcal{O}\left(\frac{M^{3/2}}{L^{3/2}}\right).
\end{aligned}
$$

- Exactly FLRW with tiny corrections.

Angular diameter distance (I)

- Solve Sachs optical equations at order M/L: $r_A(\lambda) = r_A^{(0)} + \sqrt{\frac{M}{L}} r_A^{(1)} + \frac{M}{L} r_A^{(2)}$.
- Orders 0 and 1 trivial.
- Ricci source at order M/L.
- Weyl curvature is of order $M/L \Rightarrow$ Shear of order M/L.
- Equations decouple at order M/L:

$$\frac{1}{\lambda}\frac{d^2 r_A^{(2)}}{d\lambda^2} = \frac{4\pi}{L^2} - \Delta f_\eta(\mathbf{x}(\lambda)).$$

- Source purely due to Ricci curvature. No Weyl focussing!
- Solution can be found analytically.
- Exactly FLRW with tiny corrections at order M/L.
- Inhomogeneous corrections suppressed wrt zero mode (FLRW homogeneous expansion).

Angular diameter distance (II)

It seems fine, but:

- Previous analysis only partial:
- Inhomogeneous part of r_A is $\frac{M}{L}S(\lambda)$. $S(\lambda)$ series that depends on v^i, direction of propagation of photons.
- For any choice of v^i, some terms in the series become big and:

$$\frac{M}{L}S(\lambda) = \mathcal{O}(1)!!!$$

- Failure of power series expansion except if:

$$\boxed{\frac{M}{L} \ll 100\left(\frac{\eta}{L}\right)^5}.$$

- If objects too compact, Weyl curvature kicks in: $|\sigma|^2 = \mathcal{O}(M/L)$ at least.
- For $M/L \sim 10^{-8}$, transition occurs at $\eta/L \sim 0.01$: $\eta \sim 10\text{kpc}$: galactic scale.
- Need to solve system without power series expansion: Numerical solution.

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Conclusion

- Universe made of a regular lattice of equal masses.
- Exact solution to EFE at order $(M/L)^{3/2}$.
- Kinematics is exactly the one of FLRW with corresponding dust density:
  Supports the dust approximation.
- Redshift is only modified marginally by anisotropies.
- Distances can be modified drastically! It depends on hierarchy of scales between:
  - Schwarzchild radius of objects, M,
  - Distance between objects, L,
  - Extend of objects, η.

  Small correctins only if:

$$\boxed{\frac{M}{L} \ll 100 \left(\frac{\eta}{L}\right)^5 .}$$

- Compactness is key to understand effect of Weyl curvature.
- Numerical computation needed.

THANK YOU