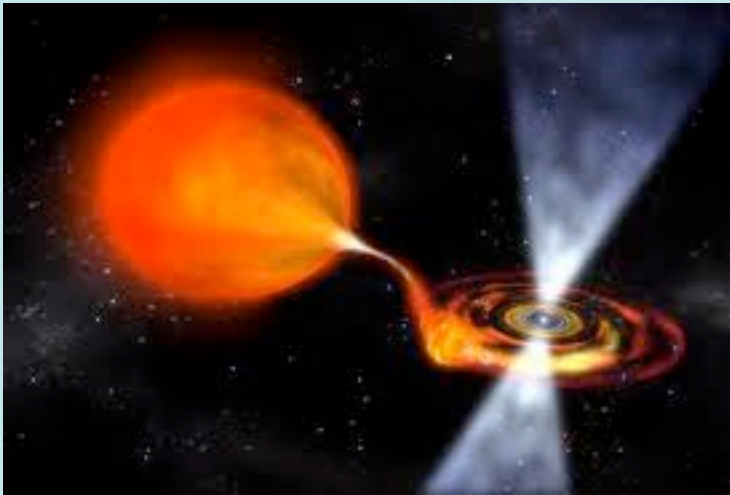


Hair of astrophysical black holes

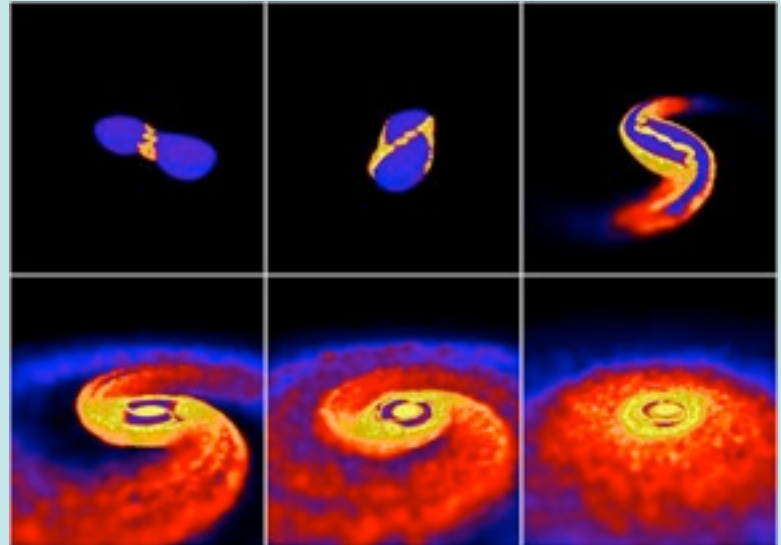
Maxim Lyutikov (Purdue U., Osservatorio Arcetri)

Collapse of a rotating NS into BH:

Accretion induced collapse of
a neutron star



NS-NS mergers



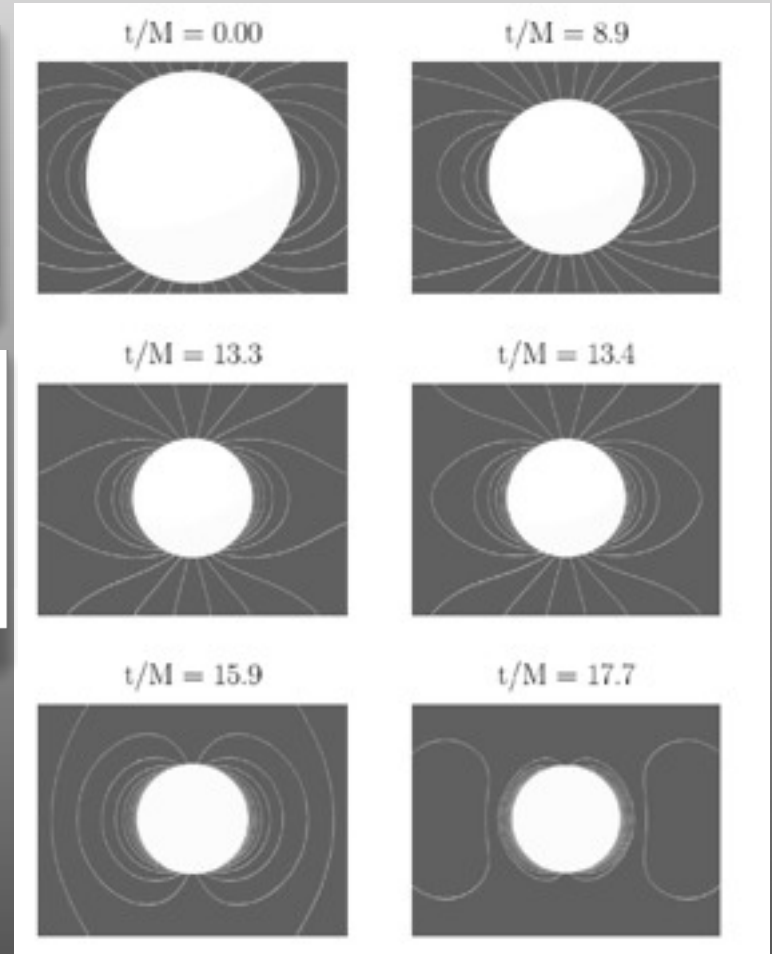
Transient NS ~ 100 msec

Rotating magnetized NS collapses into BH:
what's the behavior of the magnetic field?

The “No hair” theorem

“No hair theorem”: Isolated BH is defined by mass, angular momentum and electric charge.

Collapse of a magnetized NS into BH in vacuum: B-field is lost on ~ dynamic time



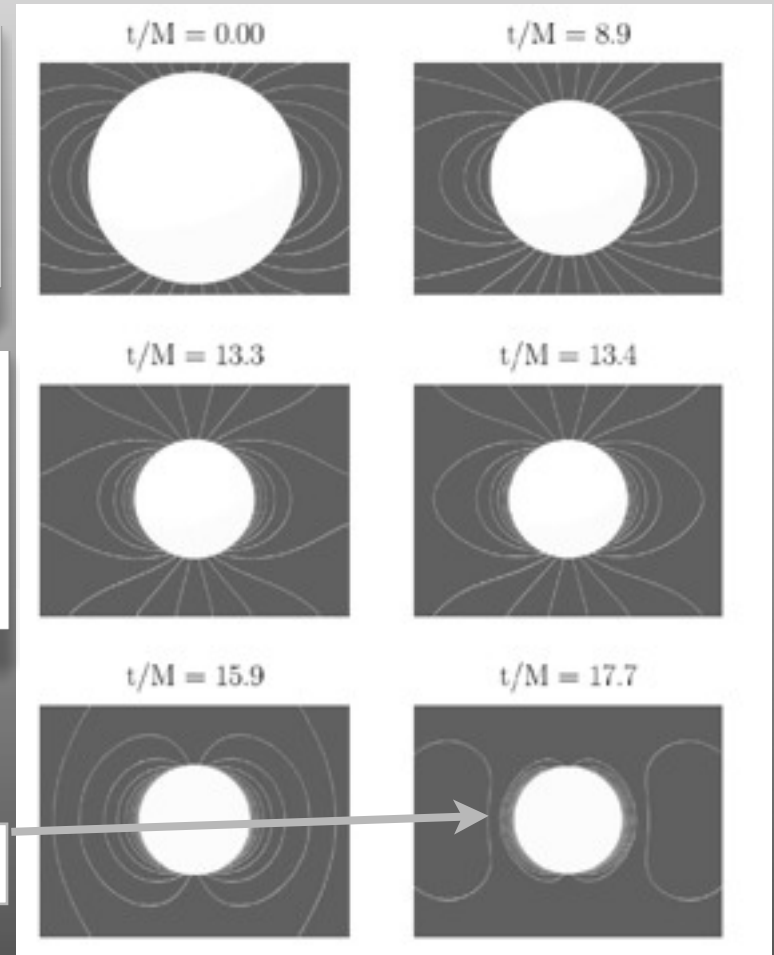
Baumgarte & Shapiro, 2003

The “No hair” theorem

“No hair theorem”: Isolated BH is defined by mass, angular momentum and electric charge.

Collapse of a magnetized NS into BH in vacuum: B-field is lost on ~ dynamic time

Such process is prohibited if outside is plasma



Baumgarte & Shapiro, 2003

The proof of “no Hair” theorem assumes outside vacuum.

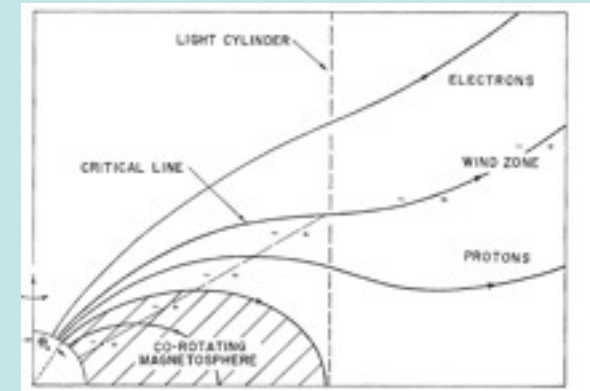
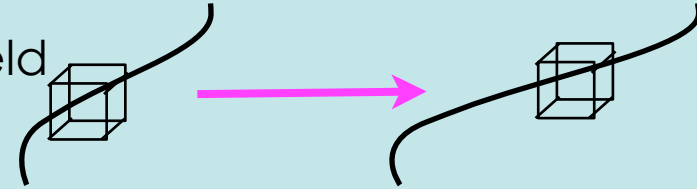
Outside plasma: $\mathbf{E} \cdot \mathbf{B} = 0$ - frozen-in B-field

Rotating NS:

- generate plasma out of vacuum
- generate currents that open fields to infinity

After NS collapse, the BH rotates with finite

$$\Omega_H \approx \frac{\chi}{5} \frac{c^4 R_{\text{NS}}^2}{(GM_{\text{NS}})^2} \Omega_{\text{NS}} = 2.9 \times 10^3 \text{rads}^{-1} \chi_{-1} P_{\text{NS},-3}^{-1}$$

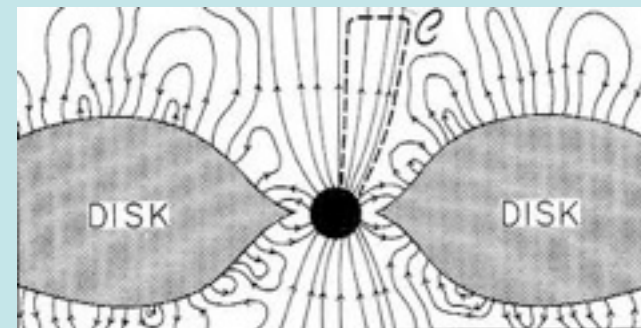


Goldreich & Julian, 1969

If a BH keeps producing plasma, like a NS, B-field cannot slide off.

Field lines that connected NS surface to infinity, has to connect horizon to infinity

This is different from Blandford-Znajek, where B-field is supplied by **external, independent currents**. Here **currents are self-produced**



The “no hair” theorem not applicable to collapse of rotating NSs: high plasma conductivity introduces topological constraint (frozen-in B-field).

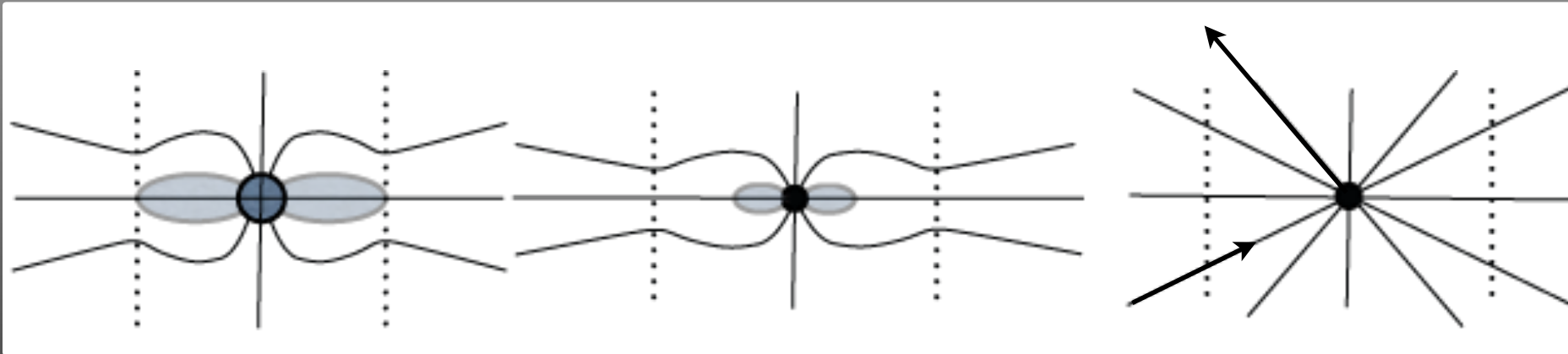
Conserved number: open magnetic flux:

$$N_B = e\Phi_\infty / (\pi c \hbar)$$

BH's hair!

$$\Phi_\infty \approx 2\pi^2 B_{NS} R_{NS}^3 / (P_{NS} c)$$

Can be measured at infinity: BH hair



Stationary axially symmetric B-field (Grad-Shafranov equation)

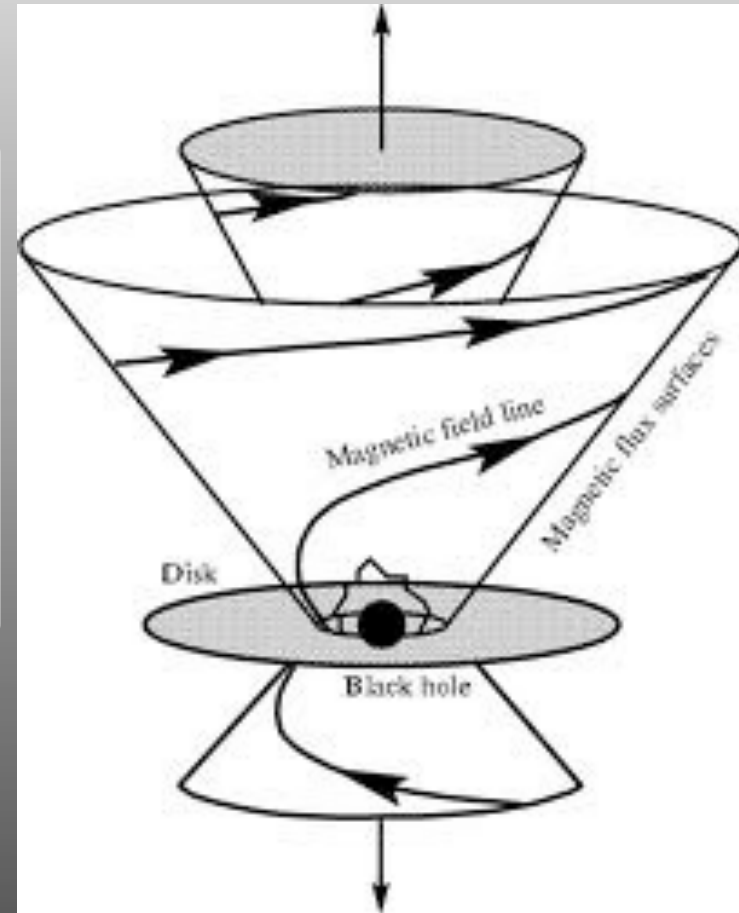
Stationary axisymmetric B-field.

Shape of flux surface $\Psi(r, \theta)$

Current enclosed by the flux surface $I(\Psi)$

Flux surface is at same pressure $P(\Psi)$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right) \right) \Psi = P(\Psi) r^2 \sin^2 \theta + 2 \partial_\Psi I^2(\Psi)$$



Time-dependent Grad-Shafranov equation

Lyutikov 2011b

- Two types of time-dependent:

- **variable current** for given shape of flux surfaces

$$\varpi^2 \nabla \left(\frac{1 - \varpi^2 \Omega^2}{\varpi^2} \nabla P \right) + \frac{4I(\nabla P \cdot \nabla I)}{(\nabla P)^2} + \varpi^2 \Omega (\nabla P \cdot \nabla \Omega) = 0$$

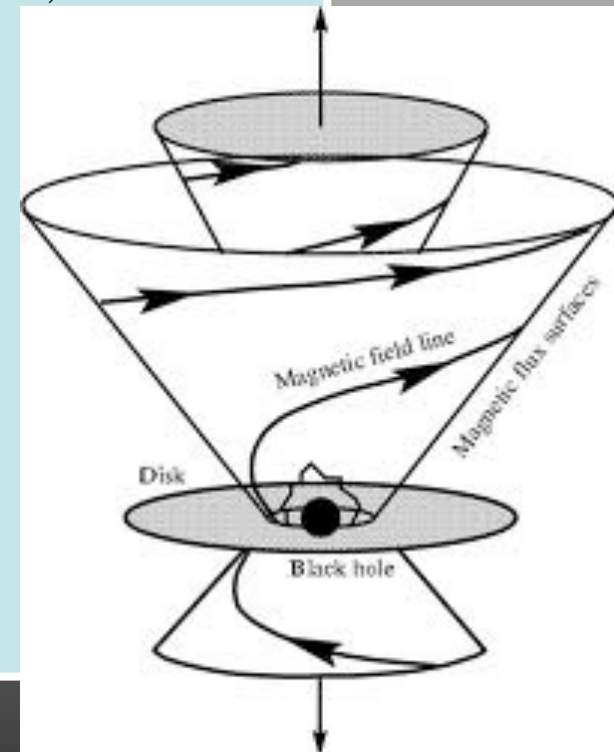
$$\partial_t^2 \Omega = \frac{\mathbf{B} \cdot \nabla (\mathbf{B} \cdot \nabla \Omega)}{B_p^2}$$

- **motion of flux surfaces**

$$\Delta^* P - \partial_t^2 P + \frac{4I(\nabla P \cdot \nabla I)}{(\nabla P)^2} - 2\partial_t \left(\frac{I^2 \partial_t P}{(\nabla P)^2} \right) = 0$$

$$F'(\nabla P)^2 = 2I\partial_t P$$

$$\partial_t I = \frac{1}{2} \Delta^* F$$



Time-dependent split-monopole solution in Schwarzschild metric

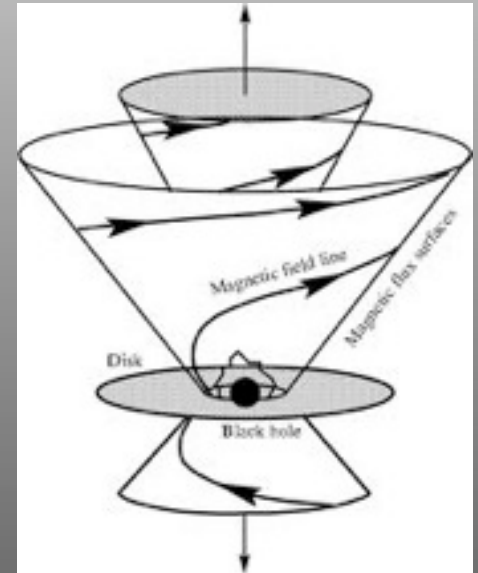
- Magnetosphere of collapsing NS:

$$B_\phi = -\frac{R_s^2 \Omega \sin \theta}{\alpha r} B_s, \quad B_r = \left(\frac{R_s}{r}\right)^2 B_s,$$

$$E_\theta = B_\phi, \quad j_r = -2 \left(\frac{R_s}{r}\right)^2 \frac{\cos \theta \Omega B_s}{\alpha}$$

$$\Omega \equiv \Omega (r - t + r(1 - \alpha^2) \ln(r\alpha^2)) \quad \alpha = \sqrt{1 - 2M/r}$$

$$B_s R_s^2 = \text{const}$$



Time-dependent split-monopole solution in Schwarzschild metric

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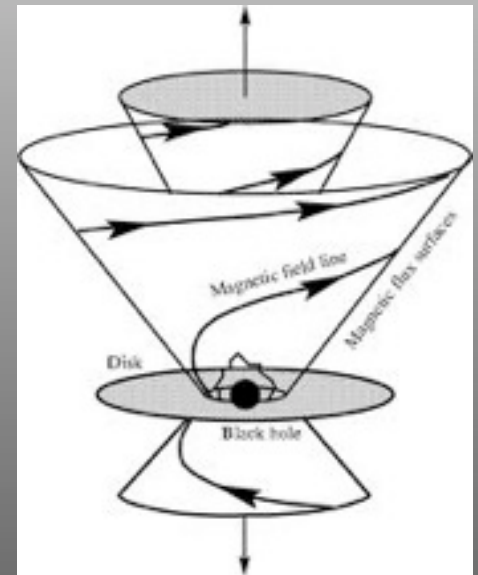
$$E_\theta = B_\phi, \quad j_r = -2 \left(\frac{R_s}{r}\right)^2 \frac{\cos \theta \Omega B_s}{\alpha}$$

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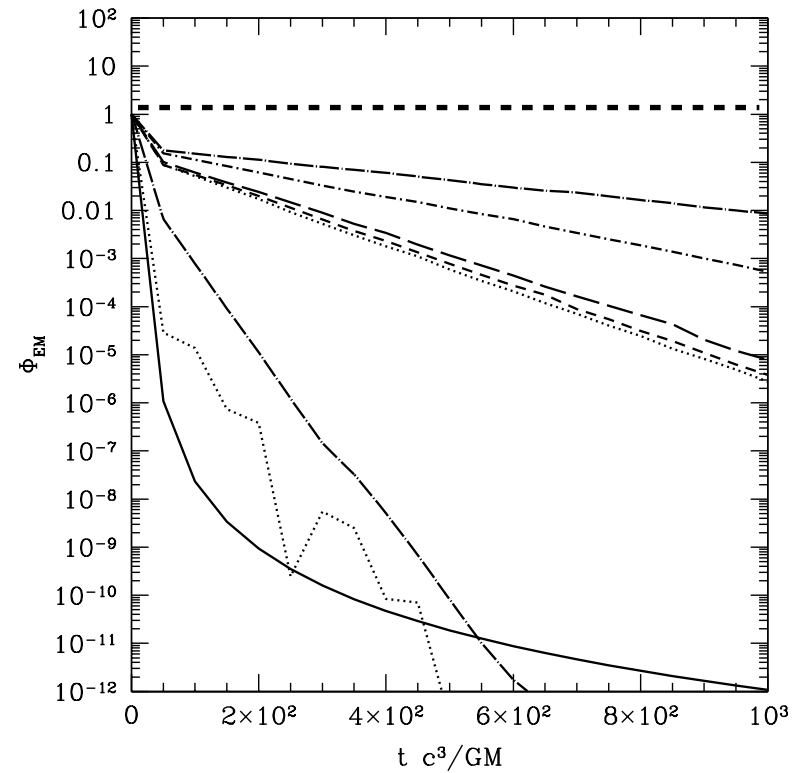
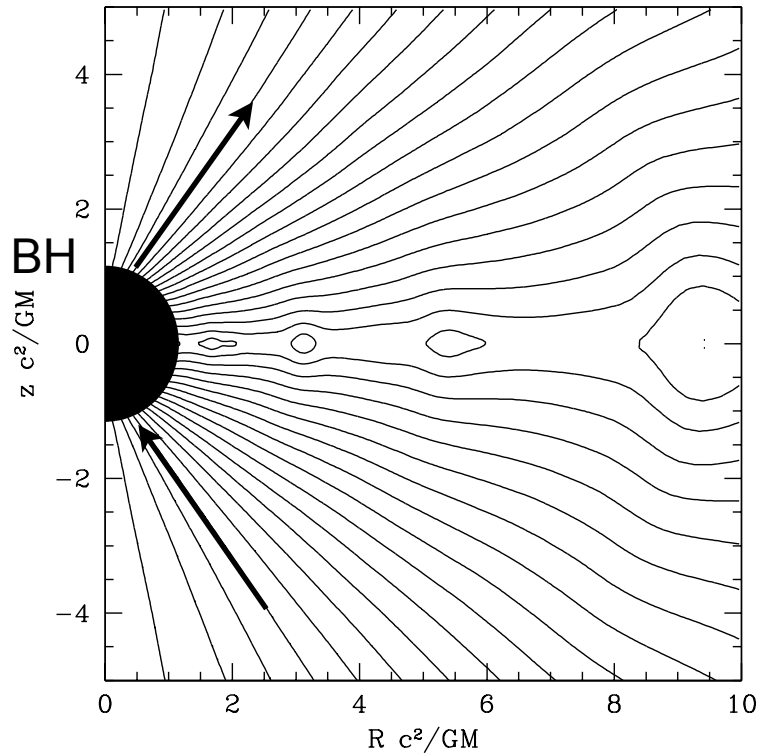
Take a relativistic object with monopolar B-field, rotate it arbitrarily. The field will remain monopolar.

Nothing “bad” happens to B-field as the NS collapses into BH



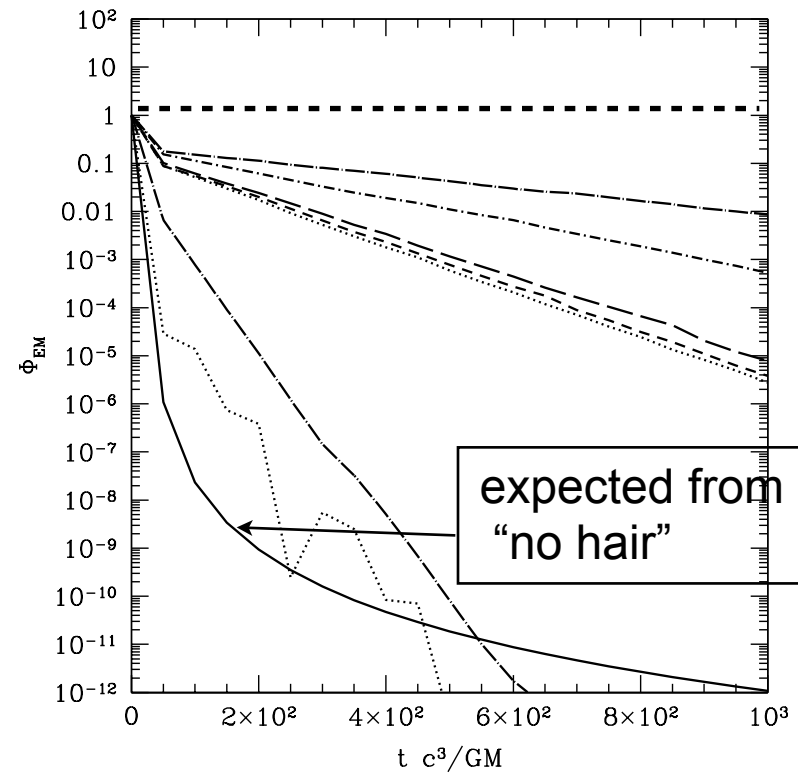
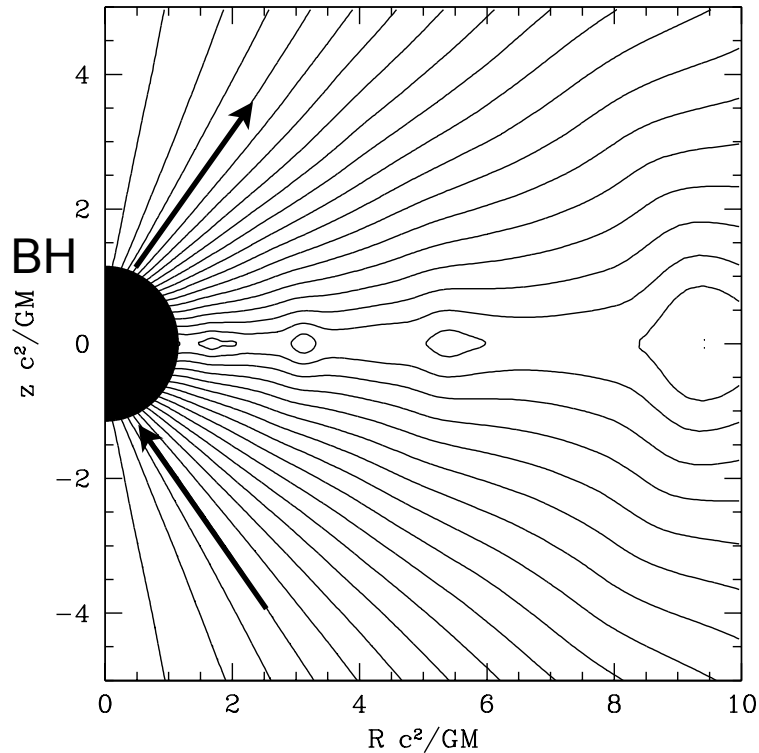
Simulations *(Lyutikov & McKinney, 2011)*

- Split-monopole
magnetosphere
- Slow balding



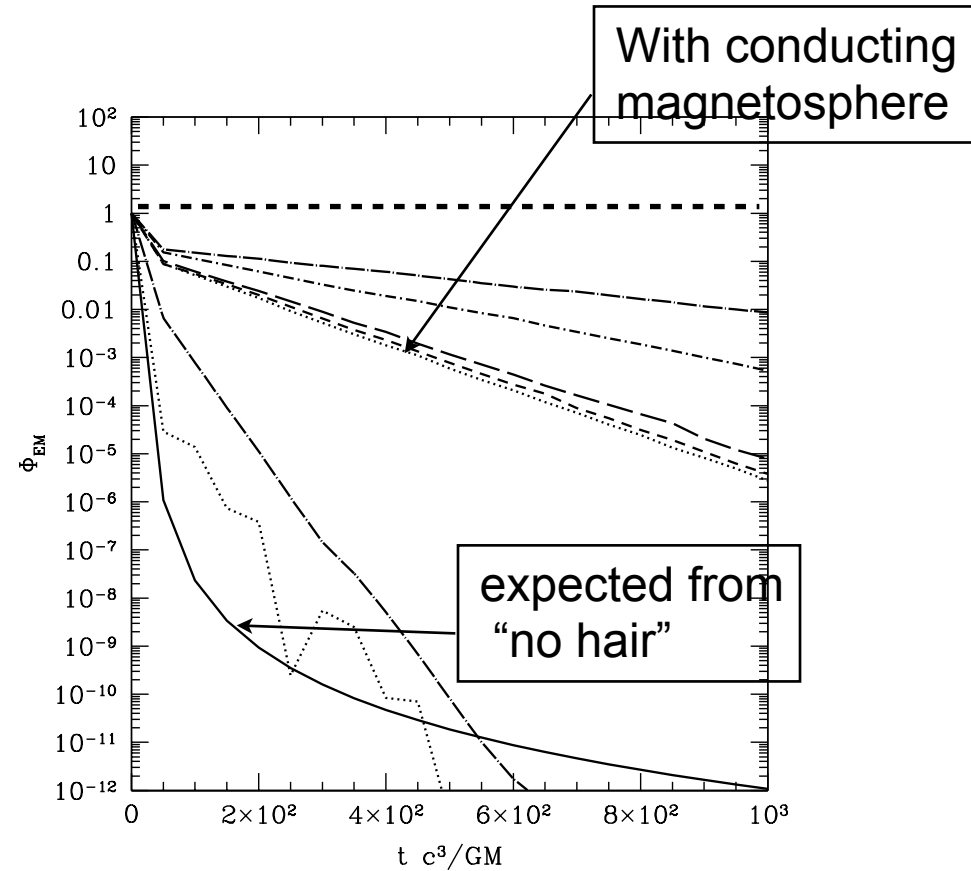
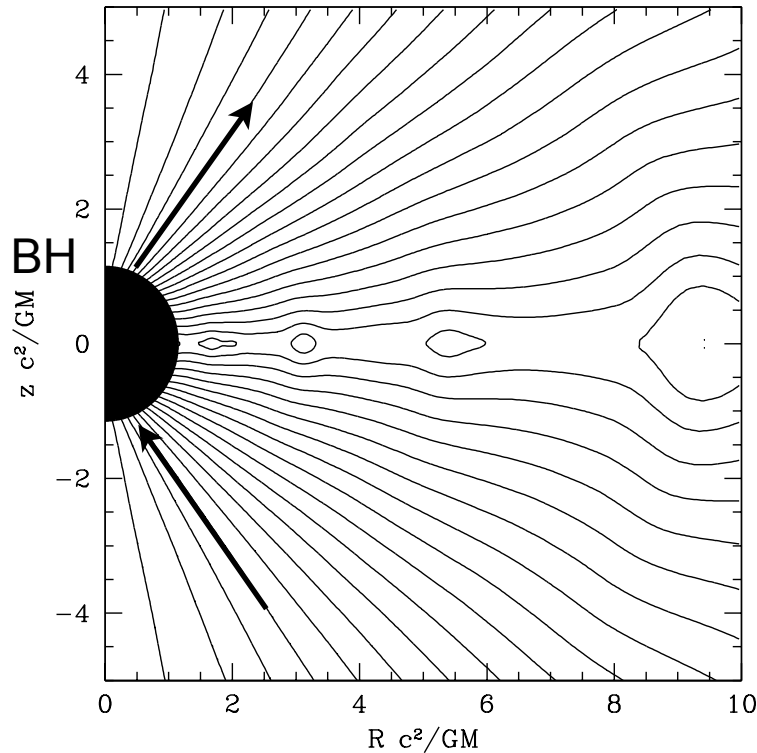
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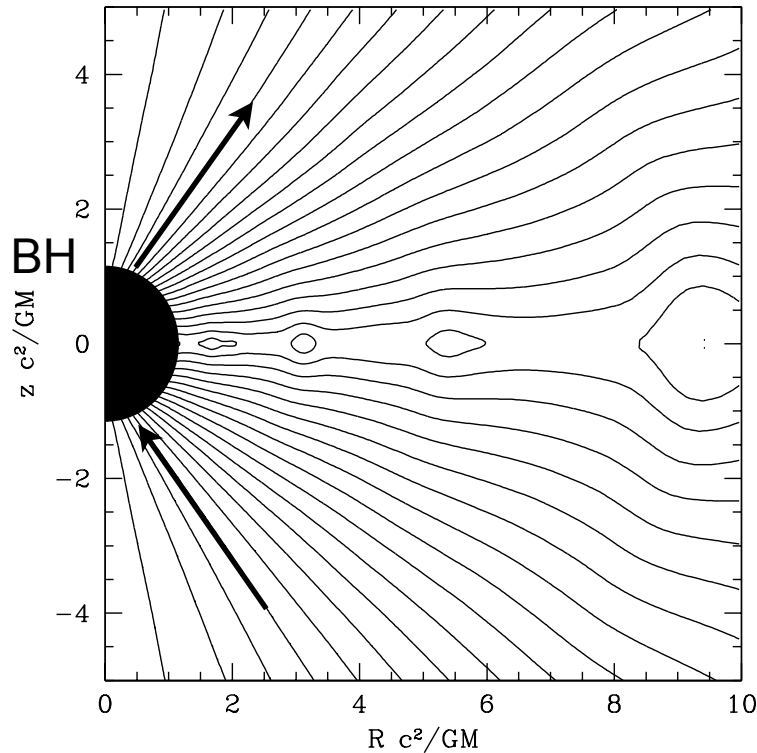
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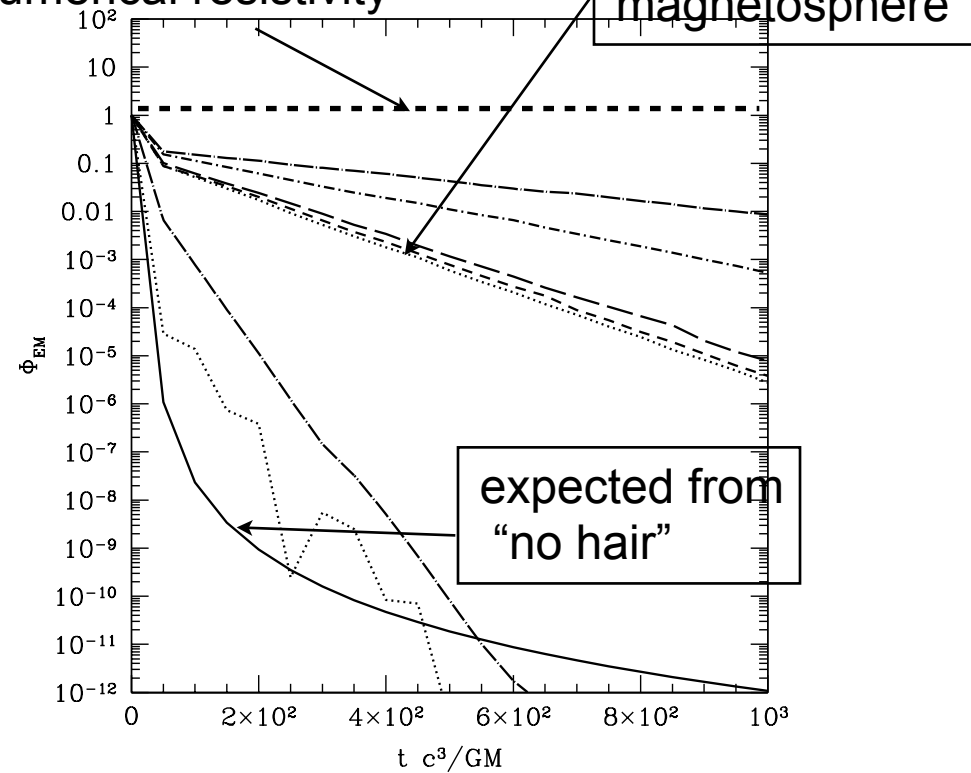


Simulations *(Lyutikov & McKinney, 2011)*

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Expected for
no numerical resistivity



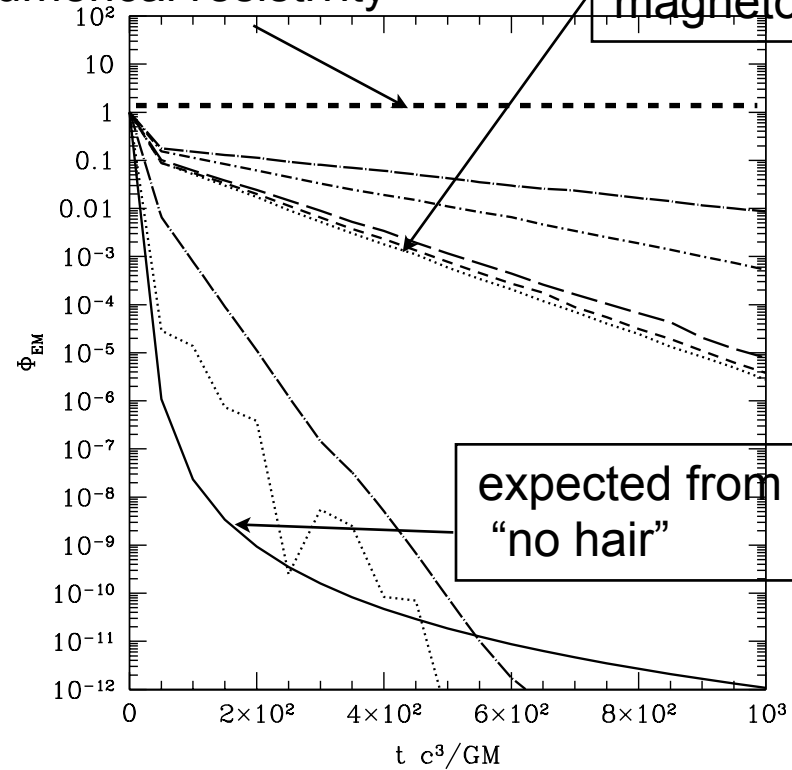
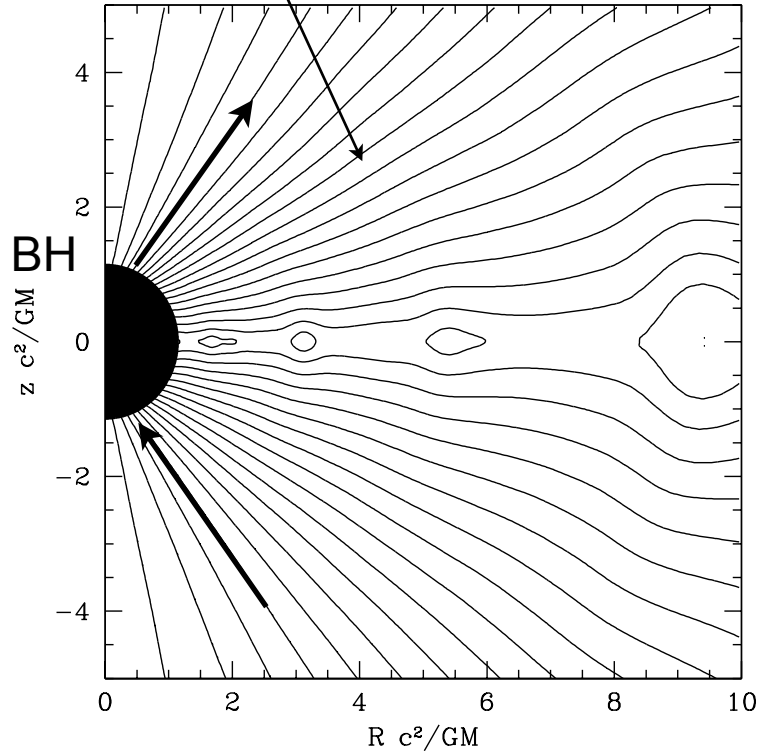
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Fields are NOT anchored
in heavy crust

Expected for
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With conducting
magnetosphere



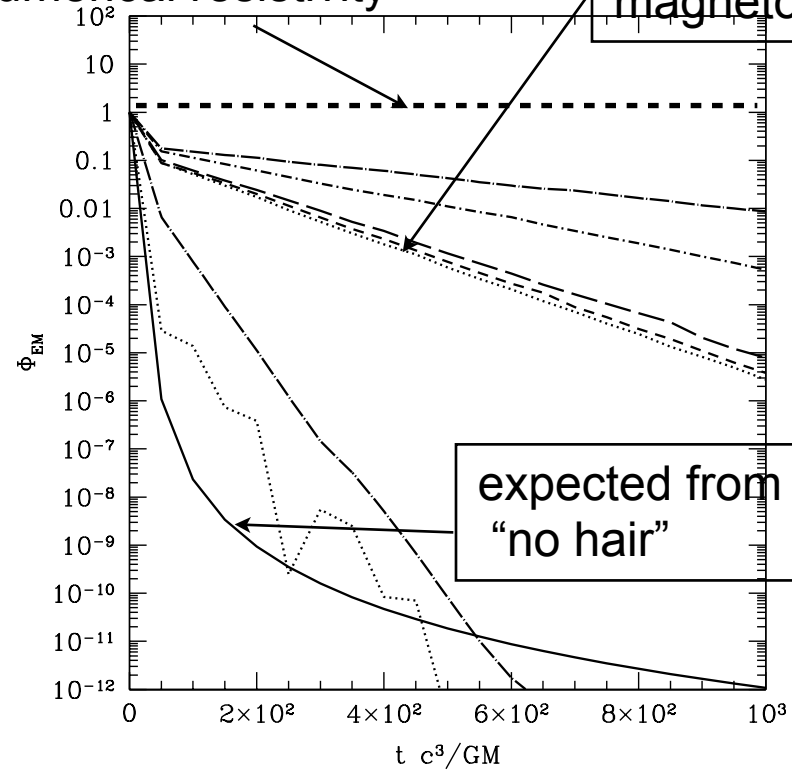
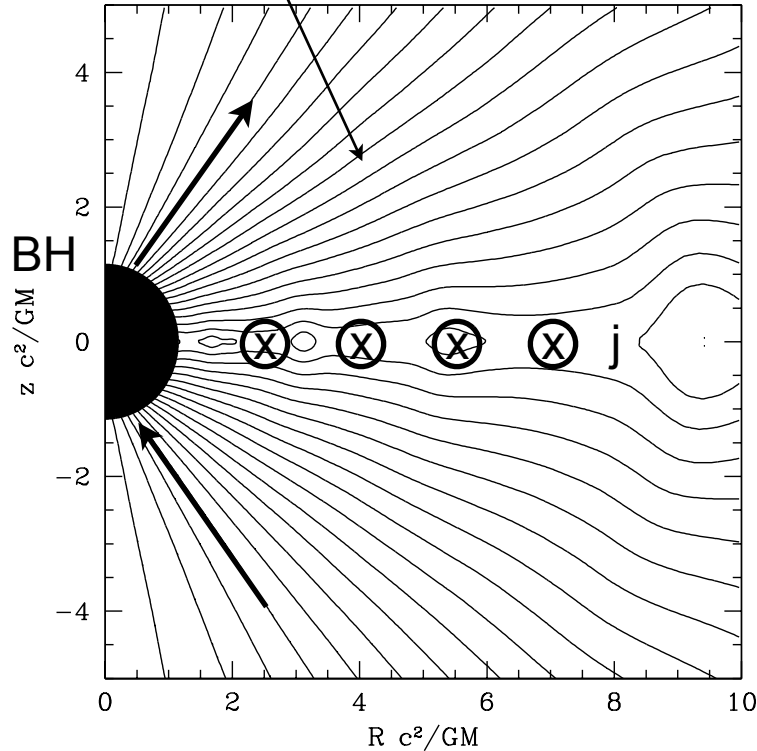
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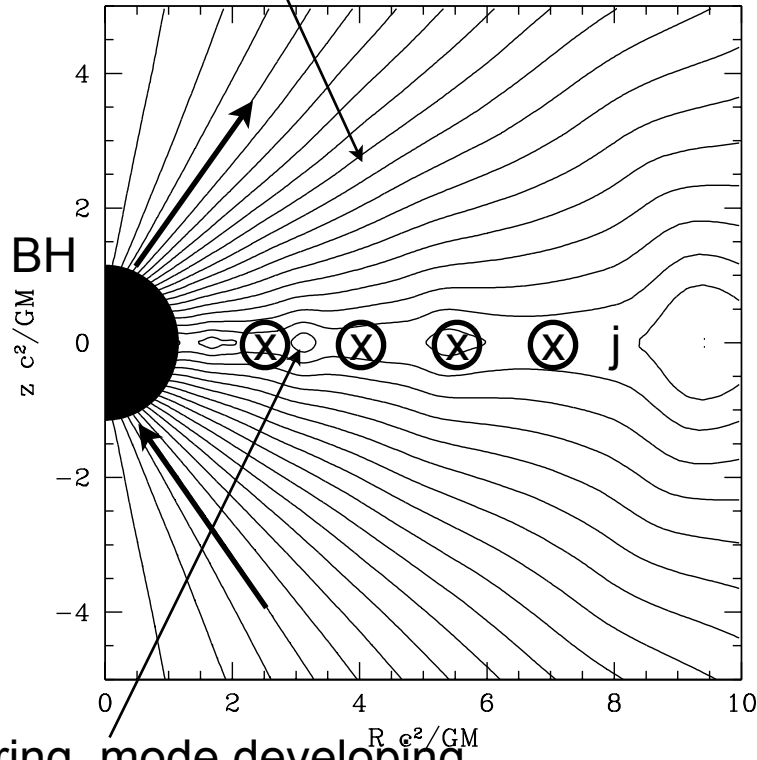


Simulations *(Lyutikov & McKinney, 2011)*

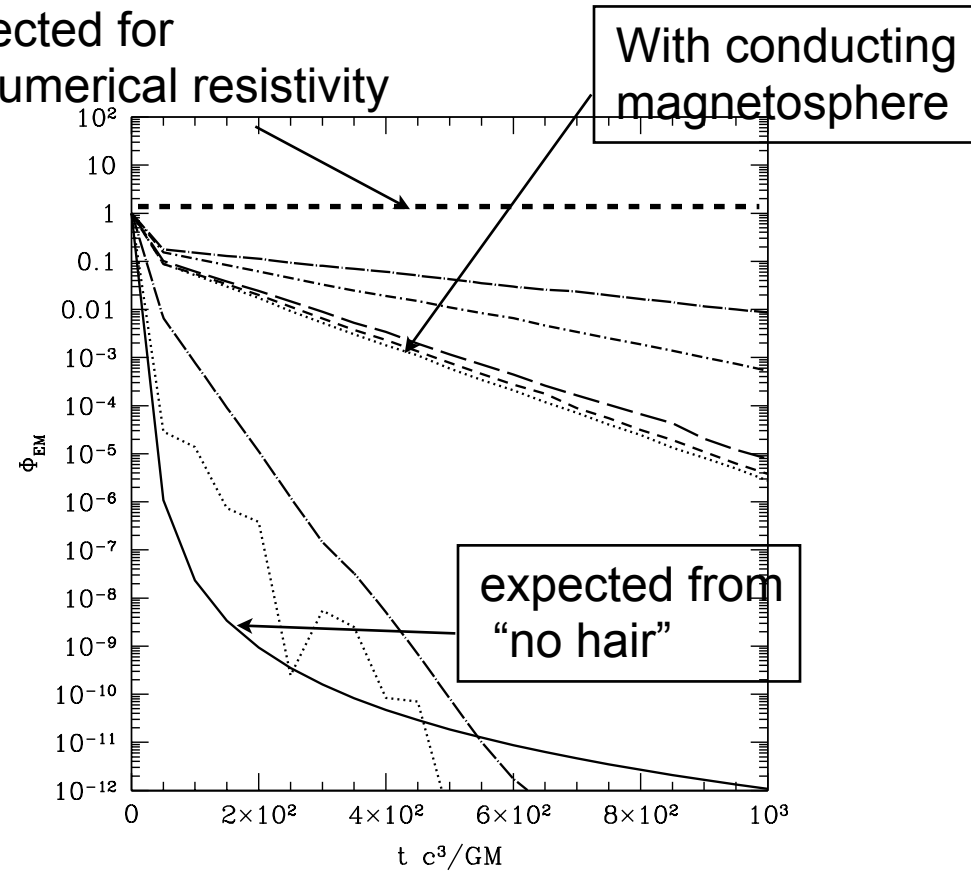
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Tearing mode developing



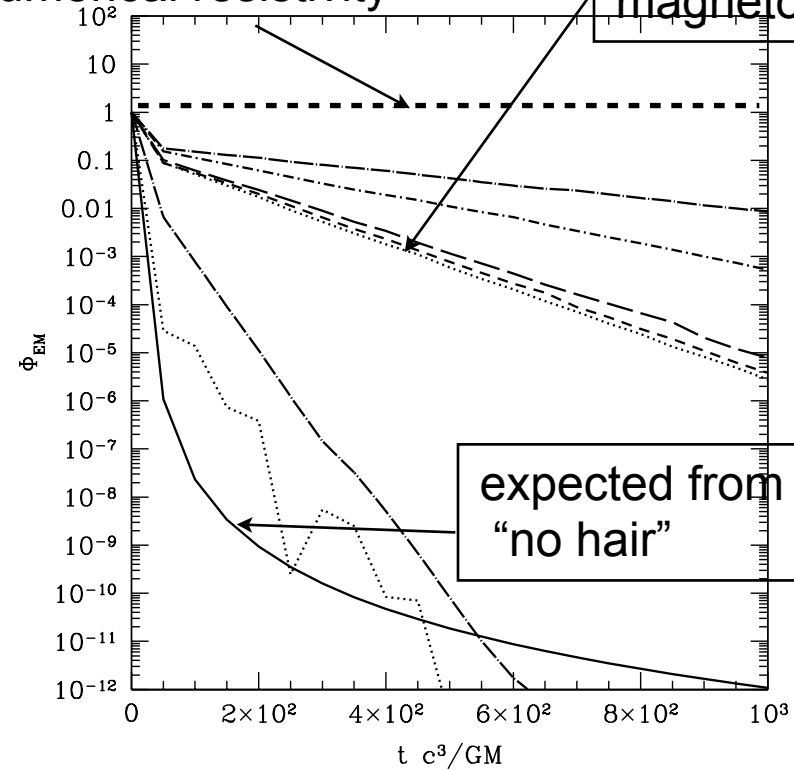
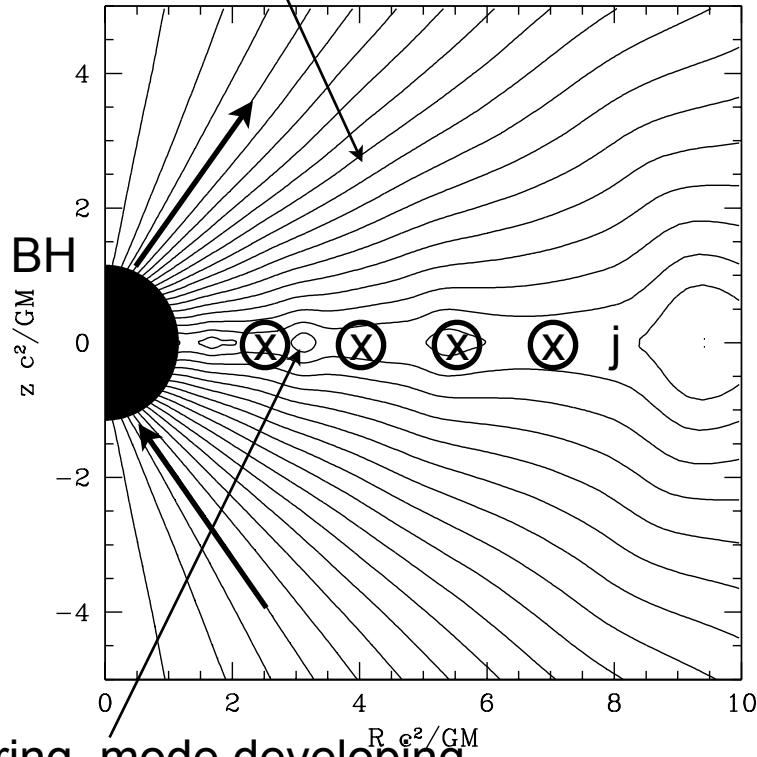
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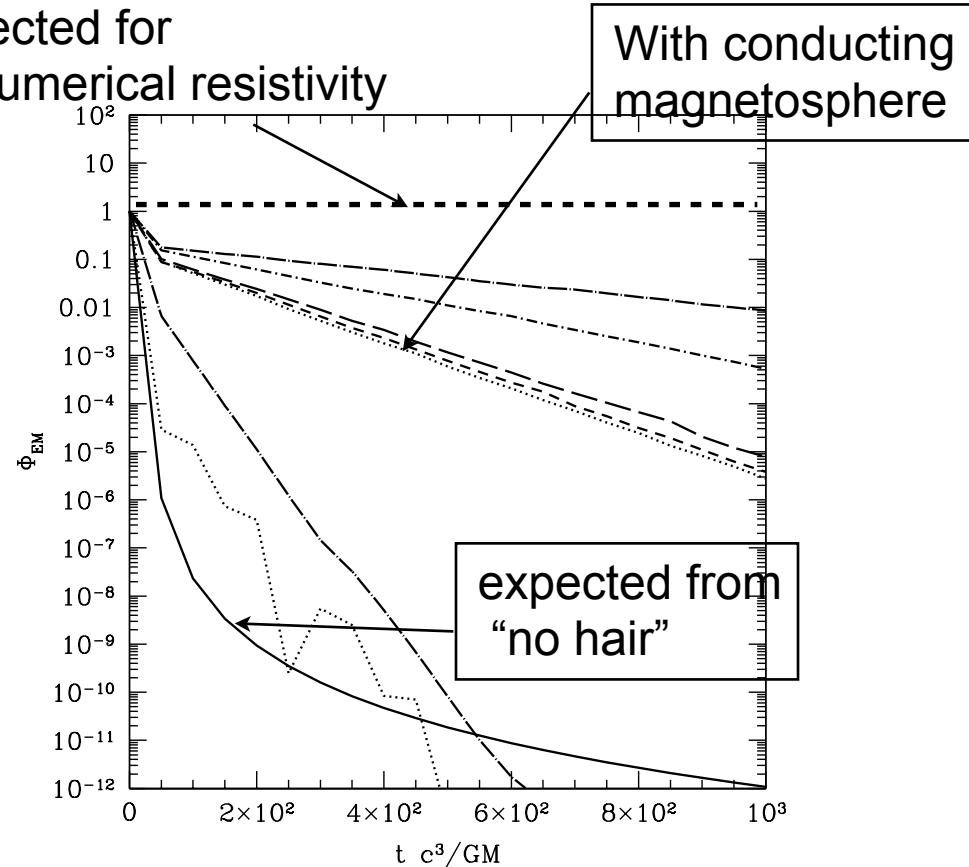
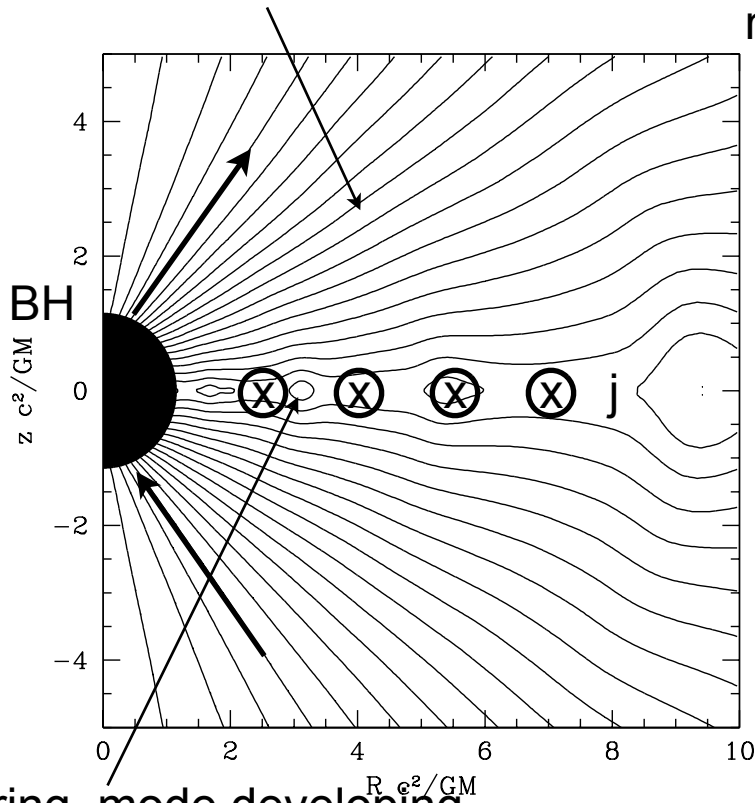
Fields are contained by the equatorial current,
just like in BZ, but this current is self-produced

Simulations *(Lyutikov & McKinney, 2011)*

- Split-monopole magnetosphere
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Tearing mode developing

Fields are contained by the equatorial current, just like in BZ, but this current is self-produced

BZ parabolic solution: switch-off the disk \rightarrow relaxes to split monopole

Slowly balding black holes

As long as BH can produce pairs, open B-field lines do not slide off.

Field structure relaxes to split monopole

No need to anchor B-field into the heavy crust

Isolated BH acts as a pulsar, spins down electromagnetically, generates Poynting wind.

$$L \sim \frac{2}{3c} \left(\frac{\Phi \Omega_{BH}}{4\pi} \right)^2$$

Slow hair loss on **resistive** time scale - hard to predict

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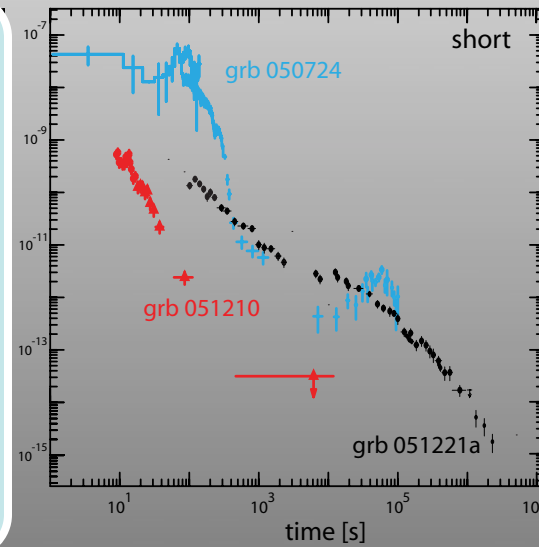
NB: Pair production by rotating BH on field lines penetrating the horizon is the key assumption of the Blandford-Znajek mechanism

NS-NS merger as central engine of short GRBs, prompt tails, supernova-less long GRBs and late flares

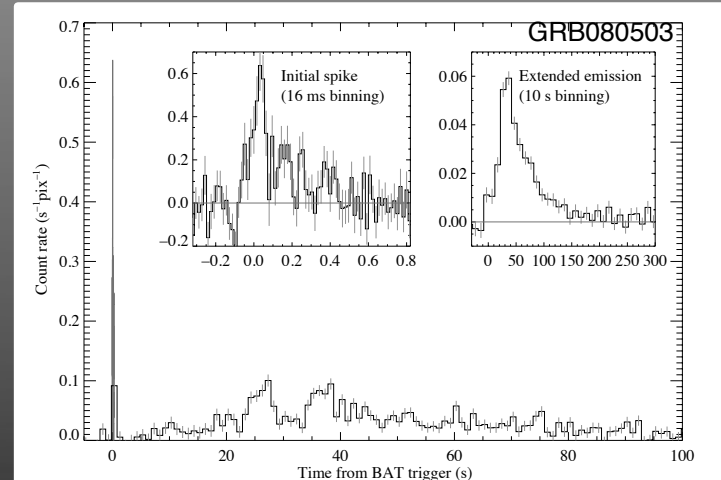
Active stage of NS-NS merger takes 10-100 msec, then collapse into BH. Very little mass is ejected.

Many short GRBs have long 100 sec tails, energetically comparable to the prompt spike.

Many GRBs have late time flares, 10^5 sec

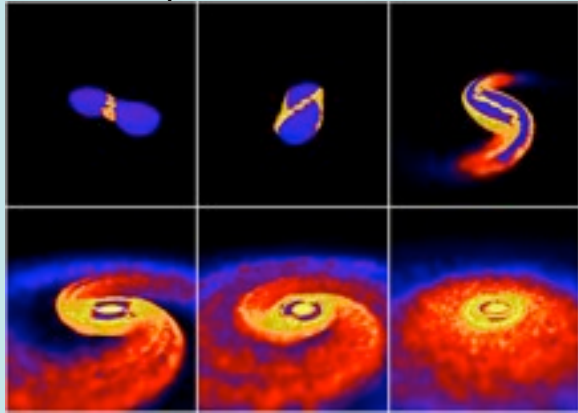


100 sec tail has ~ 30 times more energy than the prompt spike



B-field in NS-NS mergers: super-massive NS and/or BH-torus

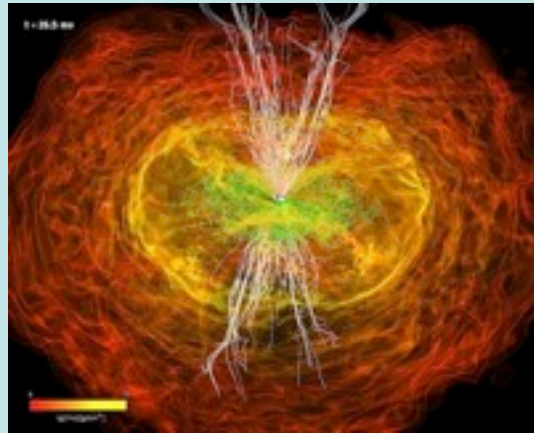
super-massive NS



Transient NS ~ 100 msec

Price & Rosswog

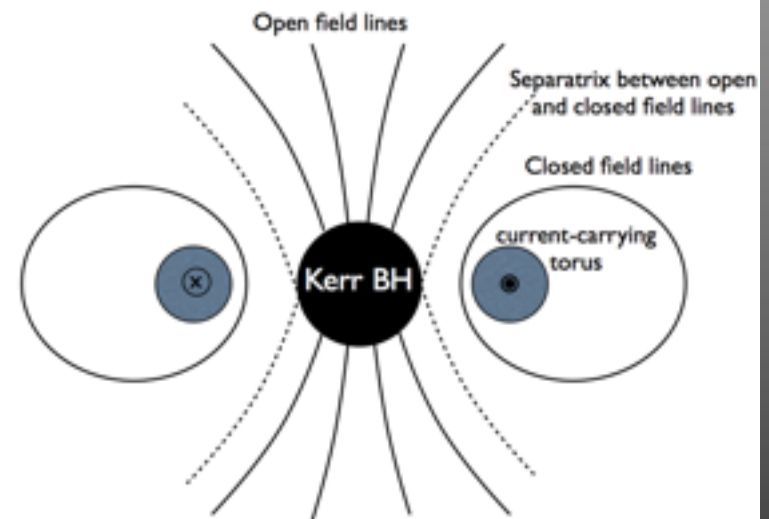
BH-torus, ~ 100 msec



Rezzolla et al

$$B \sim 10^{15} - 10^{16} \text{ G}$$

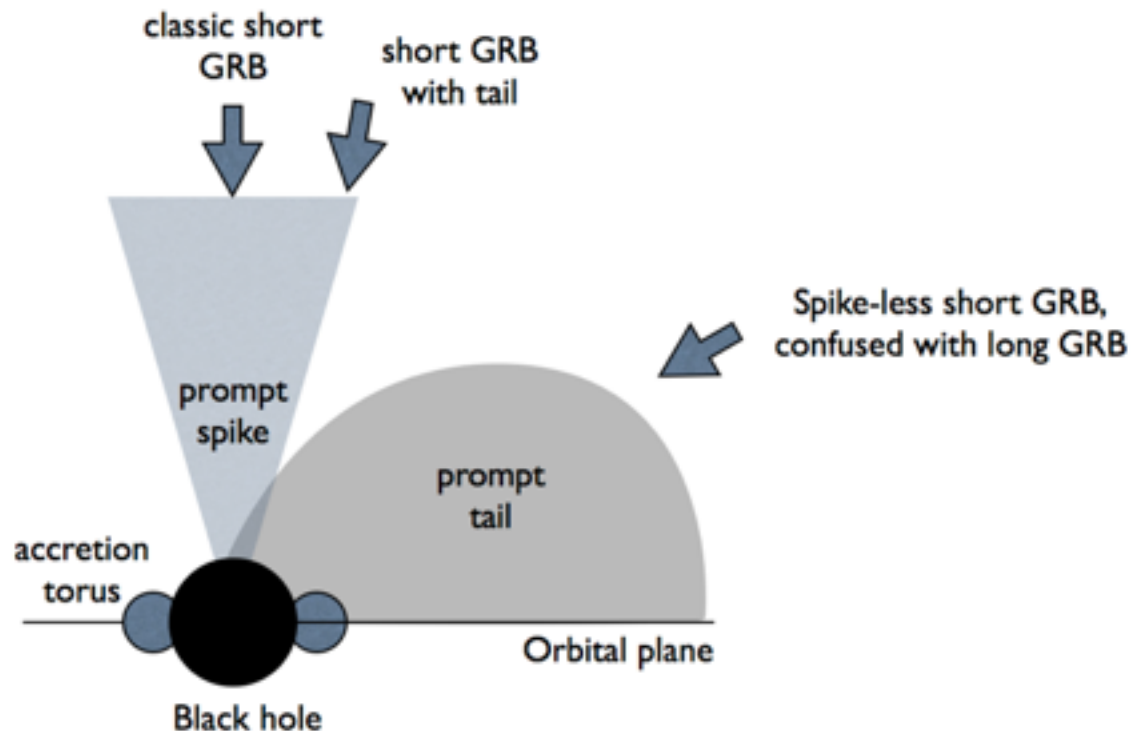
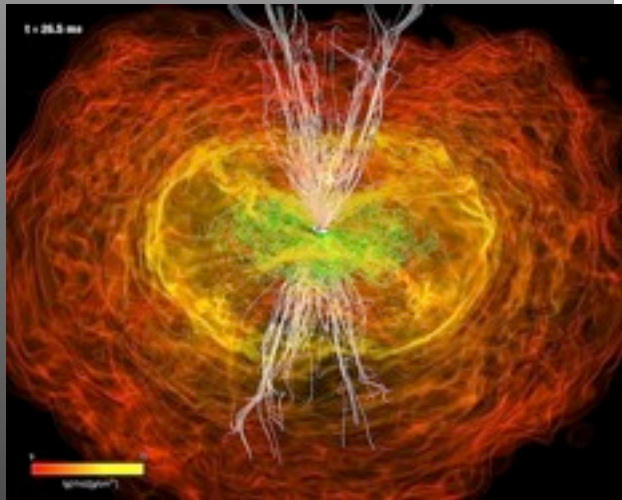
What happens after NS and disk collapse into BH?
Open magnetic flux is conserved.



The electromagnetic model of short GRBs

- NS-NS merger generates $B \sim 10^{15}$ G in the torus around BH (Rezzolla et al.)
- BH-torus launches a jet along the axis: prompt spike
- After torus collapse, isolated BH spins down electromagnetically, produces **equatorially-collimated** flow, $L \propto \sin^2 \theta$: prompt tail
- **Tail is more energetic**, but de-boosted for axial observer

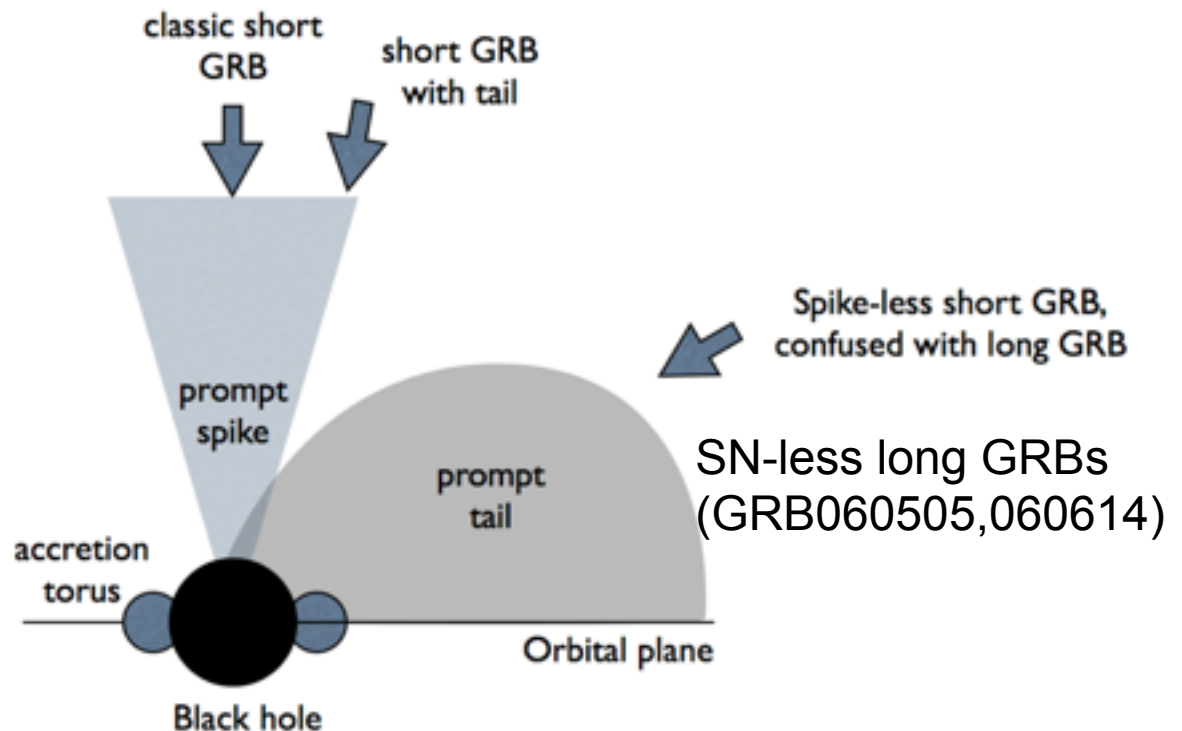
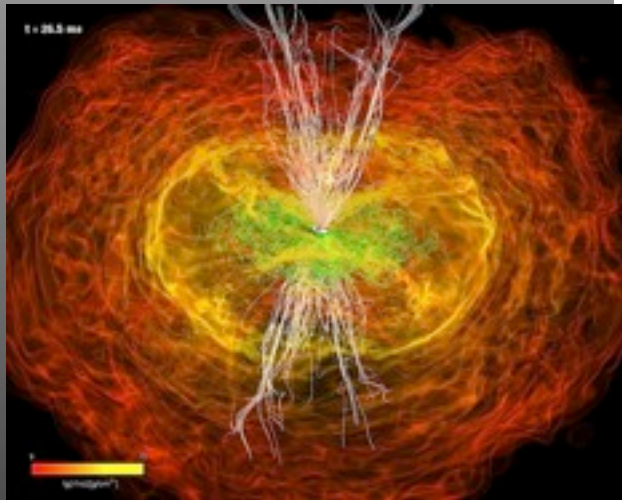
Rezzolla et al



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Rezzolla et al



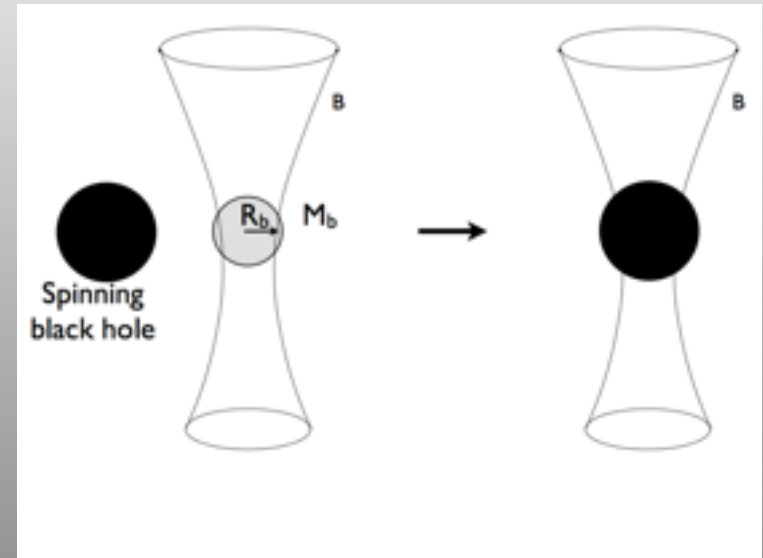
Episodic accretion: flares

Accretion of magnetized blobs

$$\frac{E_{EM}}{M_b c^2} = \frac{2}{3c} \left(\frac{\Phi_b \Omega_{BH}}{4\pi} \right)^2 \frac{\tau_{rec}}{M_b c^2} \geq 1$$

Accretion can be super-efficient (for long retention time-scales)

Need $10^{-5} M_{\text{Sun}}$ blob to produce
 $L \sim 10^{48}$ erg/s flare



Episodic accretion: average power

$$\frac{L_{EM}}{\dot{M} c^2} = \frac{2}{3c} \left(\frac{\Phi_b \dot{n} \tau_{rec} \Omega_{BH}}{4\pi} \right)^2 \frac{1}{\dot{M} c^2} \geq 1$$

Steady-state accretion can be super-efficient

Conclusion: BH hair

- The “no hair theorem” is not applicable to rotating magnetized NSs collapsing into BH: open magnetic field lines are retained
- “Balding”, loss of open magnetic field lines, occurs on long **resistive**, not short dynamical, time scales
- Isolate BHs spin down electromagnetically
- May explain long prompt tails and late flares in short GRBs
- Some long GRBs are mis-identified short, SN-less ones