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Exact dynamical AdS black holes and wormholes with a Klein-Gordon field

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Based on
HM, arXiv:1204.4472 [gr-qc], submitted

Motivation

- Few (No?) exact dynamical AdS BH solutions to study AdS/CFT duality in the dynamical context
    - Perturbative solution used in Kinoshita, Mukohyama, Nakamura & Oda, PRL `09, PTP `09
- Non-linear instability of AdS vacuum
    - Spherical collapse of a massless-scalar (Klein-Gordon) field
        - Bizon & Rostworowski, PRL `11, Jalmuzna, Rostworowski & Bizon, PRD `11
    - Final fate in far future?
        - Static BH with no hair? (i.e. no scalar-hair theorem for static BH)
        - Dynamical BHs (with hair)?
- Exact dynamical AdS solutions may be helpful for giving suggestions or further investigations

System

- Einstein-Klein-Gordon-Λ system

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa_n^2 T_{\mu\nu} \text{ and } \Box\phi = 0,$$

$$T_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\rho\phi\nabla^\rho\phi.$$

- n-dim (spherical, plane, hyperbolic) symmetric spacetime $M^2 \times K^{n-2}$
  - M^2 is a Lorentzian, K^{n-2} is maximally symmetric (curvature k=1,0,-1)
  - Metric: $$ds^2 = g_{AB}(y)dy^A dy^B + R(y)^2\gamma_{ij}(z)dz^i dz^j,$$

M^2 K^{n-2}

- Generalized Misner-Sharp quasi-local mass

$$m := \frac{(n-2)V_{n-2}^{(k)}}{2\kappa_n^2}R^{n-3}\left[-\tilde{\Lambda}R^2 + k - (DR)^2\right]$$

where $(DR)^2 := g^{AB}(D_A R)(D_B R)$

D_a: covariant derivative on M^2

$$\tilde{\Lambda} := 2\Lambda/[(n-1)(n-2)]$$

Metric ansatz

- Metric: $ds^2 = H(\rho)^{-2}\left[-dt^2 + d\rho^2 + S(t)\gamma_{ij}(z)dz^i dz^j\right]$
  - 3-classes of solutions:

$$H(\rho) = \begin{cases} \sqrt{-\tilde{\Lambda}}\sin\rho & [\text{class-I } (\Lambda < 0)], \\ \sqrt{-\tilde{\Lambda}}\rho & [\text{class-II } (\Lambda < 0)], \\ \sqrt{-\tilde{\Lambda}}\sinh\rho & [\text{class-III } (\Lambda < 0)], \\ \sqrt{\tilde{\Lambda}}\cosh\rho & [\text{class-III } (\Lambda > 0)]. \end{cases}$$

<= We focus on this

- Conformal Killing vector: $\mathcal{L}_\xi g_{\mu\nu} = 2\psi g_{\mu\nu}, \qquad \psi := -\frac{H'}{H}.$
- H(ρ)=0 is AdS infinity: $\lim_{\rho\to\rho_\infty} R^{\mu\nu}{}_{\rho\sigma} = \tilde{\Lambda}(\delta^\mu_\rho\delta^\nu_\sigma - \delta^\mu_\sigma\delta^\nu_\rho)$
- The system reduces to a master equation for S(t)(>0)

Master equation

- Master equation:

$$E = \frac{1}{2}\dot{X}^2 + V_{(\mathrm{k})}(X),$$

$$V_{(\mathrm{k})}(X) := \frac{(n-2)^2}{2}\left(kX^{2(n-3)/(n-2)} + wX^2\right)$$

where $X := S^{(n-2)/2}$

  - 1-dim potential problem (with constant E)
    - w=1 (class-I), 0 (class-II), -1 (class-III)
  - Exact solutions in a closed form obtained for n=4 or for k=0
    - n=4, k=1 by Lake '83, n=4, general k by Collins & Lang '87, Shaver & Lake '88 (obtained as a stiff-fluid solution)

- Physical quantities (vacuum for n=3, locally AdS)

$$\phi = \pm\sqrt{\frac{2(n-3)E}{(n-2)\kappa_n^2}}\int^t \frac{d\bar{t}}{S(\bar{t})^{(n-2)/2}},$$

$$\mu = \frac{(n-3)EH^2}{(n-2)\kappa_n^2 S^{n-2}},$$

$$m = \frac{EV_{n-2}^{(k)}}{(n-2)\kappa_n^2(\varepsilon H)^{n-3}S^{(n-3)/2}}.$$

ϕ is homogeneous

Class-I solution for n=4

- One-parameter family (parameter C_1)

$$S(t) = \frac{1}{2}(-k + 2C_1 \sin 2t)$$

$$\mu = \frac{(4C_1^2 - k^2)H^2}{4\kappa_4^2 S^2}, \quad m = \frac{V_2^{(k)}(4C_1^2 - k^2)}{4\kappa_4^2 \varepsilon H S^{1/2}}.$$

  - S(t)=0: curvature singularity
  - At AdS infinity (H(ρ)=0), μ converges to zero while m blows up (locally AdS)
- Real scalar field (positive energy density)

$$\pm(\phi - \phi_0) = \begin{cases} \sqrt{\frac{1}{2\kappa_4^2}} \ln\left|\frac{\sqrt{4C_1^2 - k^2} + (-k\tan t + 2C_1)}{\sqrt{4C_1^2 - k^2} - (-k\tan t + 2C_1)}\right| & [\text{for } k = 1, -1], \\ \sqrt{\frac{1}{2\kappa_4^2}} \ln\left|\frac{1 - \cos 2t}{\sin 2t}\right| & [\text{for } k = 0]. \end{cases}$$

- Ghost scalar field (negative energy density, only for k=-1)

$$\pm(\phi - \phi_0) = i\sqrt{\frac{2}{\kappa_4^2}} \arctan\left(\frac{-k\tan t + 2C_1}{\sqrt{k^2 - 4C_1^2}}\right)$$

Class-I solution for k=0

- Solution:

$$S(t) = C_1[\sin(n-2)t]^{2/(n-2)},$$

$$\pm(\phi-\phi_0) = \sqrt{\frac{n-3}{(n-2)\kappa_n^2}}\ln\left|\frac{1-\cos(n-2)t}{\sin(n-2)t}\right|.$$

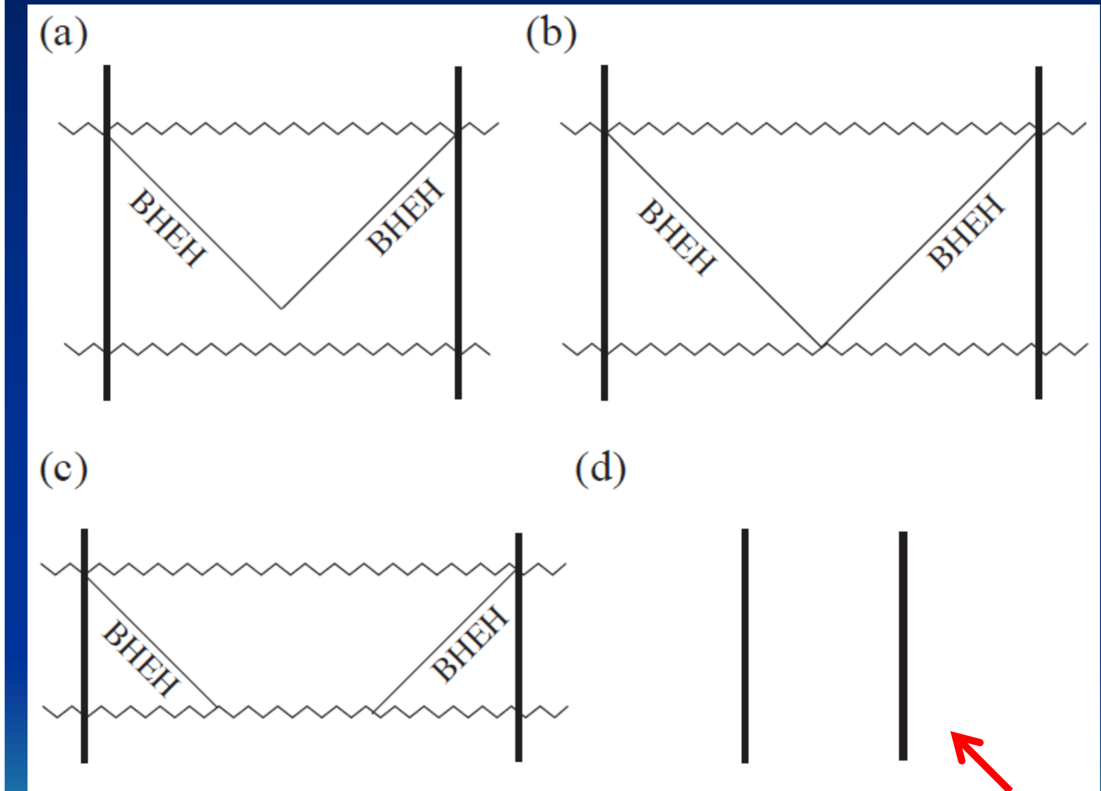
$$\mu = \frac{(n-2)(n-3)H^2}{2\kappa_n^2[\sin(n-2)t]^2},$$

$$m = \frac{(n-2)V_{n-2}^{(0)}C_1^{(n-1)/2}}{2\kappa_n^2(\varepsilon H)^{n-3}[\sin(n-2)t]^{(n-3)/(n-2)}}.$$

- Useful properties to show global structure
  - M^2 is conformally flat & there is a conformal Killing vector
  - Both t and ρ are periodic, and the ratio of these periods depends on k and n
    - See Sussman `91 for n=4, k=1

Penrose diagrams

Thick lines: AdS infinity, zigzag lines: singularities



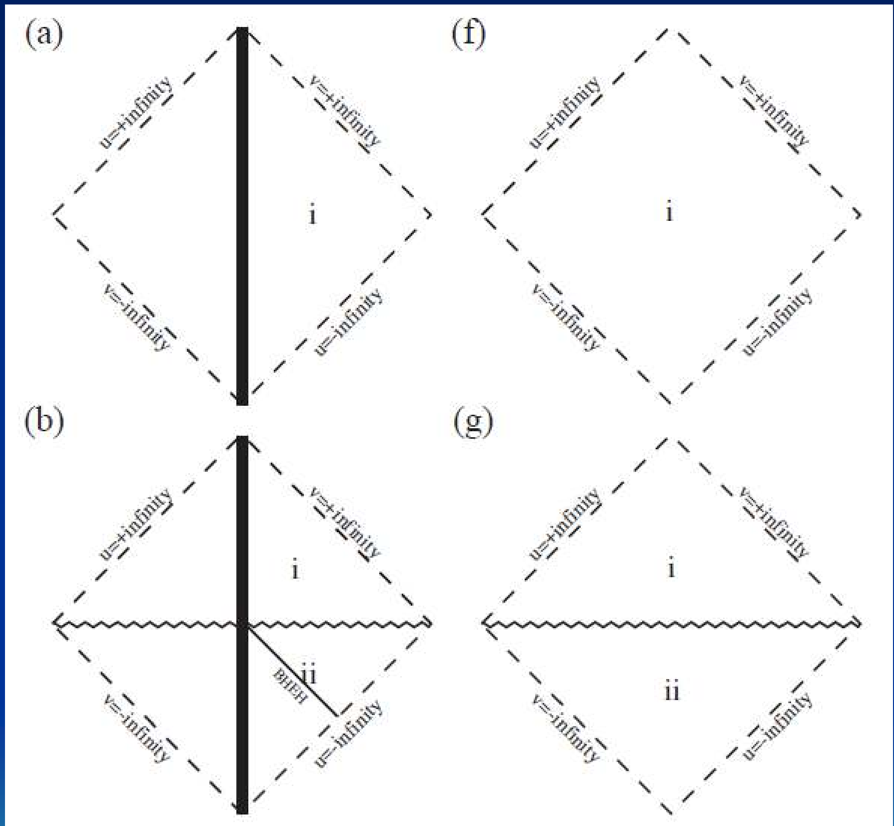
	$n=4$	$n \geq 5$
$k=0$	Fig. 1(b)	Fig. 1(c)
$k=1$	Fig. 1(c)	Fig. 1(a), (b), or (c)
$k=-1$	Fig. 1(a)	Fig. 1(a), (b), or (c)

(a) Dynamical BH (non-eternal)
(b,c) Dynamical BH (eternal)
(d) Dynamical wormhole

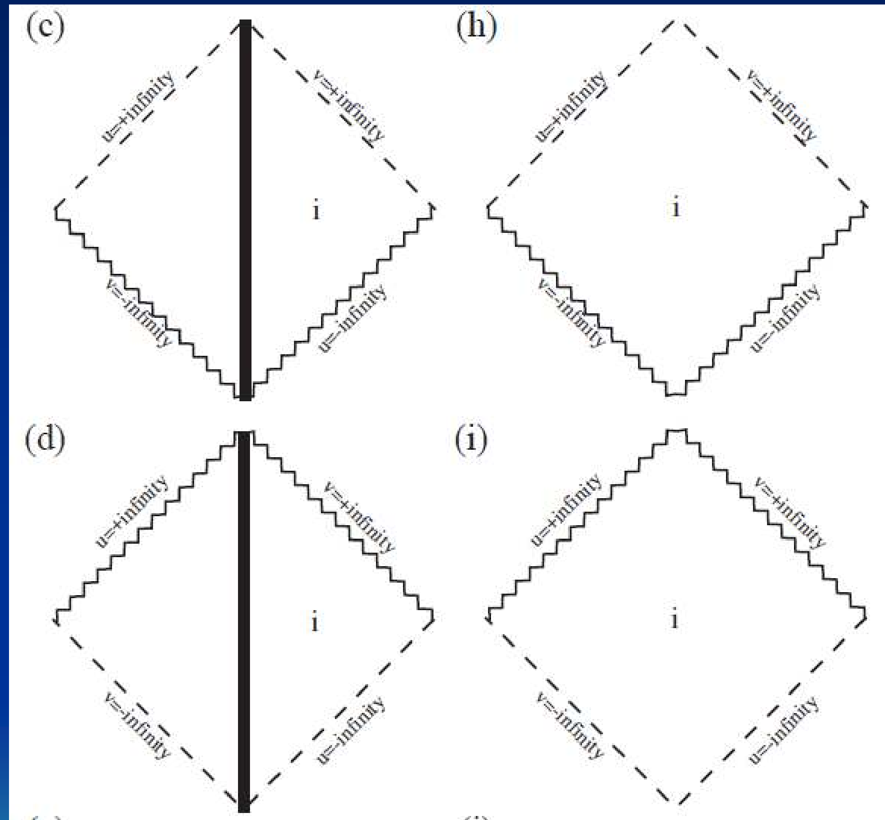
Negative energy density
for k=-1

Class-II & III solutions

Dashed lines: extendable boundaries



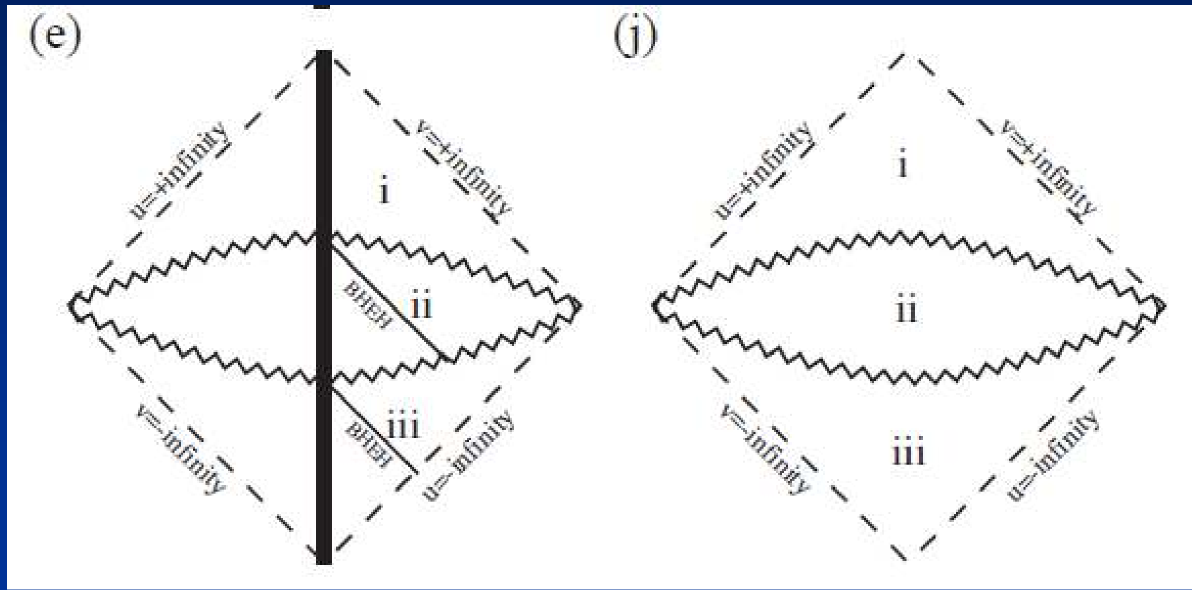
AdS dS



AdS dS

Region (ii) in (b) contains BH event horizon but spacetime non-maximally extended

Class-II & III solutions



AdS dS

Region (ii) in (e) contains BH event horizon but there is no regular Cauchy surface

Summary

- Generalized Lake solution obtained for arbitrary n,k
  - Einstein-Klein-Gordon system with Λ
- Class-I solution represents a dynamical AdS BH
  - Eternal BH for k=1 and n=4
    - Penrose diagram not completely identified for n>4 with k=1,-1
  - Asymptotically only locally AdS
    - ``Non-compact'' configuration because ϕ is homogeneous
    - Application to AdS/CFT or AdS instability seems difficult (?)
  - The spacetime is past incomplete (white hole horizon exists)
- Some of class-II & III solutions contain BH event horizon
  - But no regular Cauchy surface or spacetime not maximally extended
  - May be inhomogeneous cosmological models