On the Observability of the Granularity of Spatial Geometry

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States of Space:
The kinematics of loop quantum gravity (Completing dynamics is homework.)

$$\mathcal{H}_{kin} = \bigoplus_{\Gamma} L^2 \left(SU(2)^L / SU(2)^{\mathcal{N}} \right)$$

The spin network basis, $| \ \Gamma \, j_l \, i_n \rangle \in \mathcal{H}_{kin}$, are labeled by

- a graph Γ,
- **spins** j_l on the L links l, and
- **intertwiners** i_n for each of the $\mathcal{N}$ nodes n of the graph.

Intertwiners i_n have orthonormal basis labeled by a trivalent graph decomposition with spins labeling the virtual links.

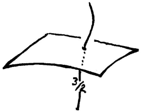
Granular Spatial Geometry

A hallmark of loop quantum gravity

With "geometric flux operator" L^i_I and "metric operator" $L_I \cdot L_{I'}$,

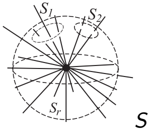
Area*: $\hat{A}_S \mid s\rangle = \sqrt{L_{I_S} \cdot L_{I_S}} \mid s\rangle = a \mid s\rangle$

$$a = \ell_P^2 \sum_{n=1}^{N} \sqrt{j_n(j_n+1)}$$



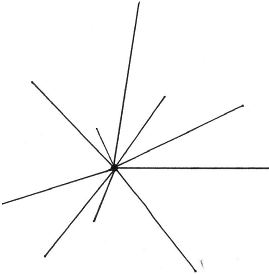
Angle†: $\hat{\theta}_{(12)} \mid s\rangle = \arccos \frac{L^i_1 L^i_2}{|L_1|\,|L_2|} \mid s\rangle = \theta \mid s\rangle$

$$\theta = \arccos\left(\frac{j_r(j_r+1) - j_1(j_1+1) - j_2(j_2+1)}{2\left[j_1(j_1+1)\, j_2(j_2+1)\right]^{1/2}}\right)$$



* Rovelli,Smolin Nuc. Phys. B **422** (1995) 593; Asktekar,Lewandowski Class. Quant. Grav. **14** (1997) A43

† SAM Class. Quant. Grav. **16** (1999) 3859 arXiv: gr-qc/9905019



Volume*:

$$V_n \sim \sqrt{|\epsilon^{ijk} L^i_{l_1} L^j_{l_2} L^k_{l_3}|} \sim_{j\to\infty} \ell_p^3 (\text{total flux})^{3/2},$$

with scaling for the mean eigenvalue for large spin,

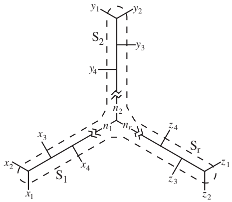
- Because of the angle and volume based at nodes, the node may be called an "atom of geometry".
- For large spin, an *effective length* $\ell = \ell_p \sqrt{\text{total spin}}$ and an *effective energy* $M = M_p / \sqrt{\text{total spin}}$
- **How might the granularity of spatial geometry be observed?** Observation would require a "lever arm".

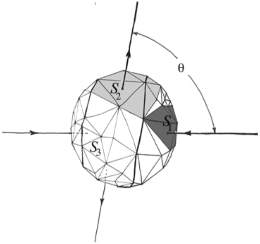
* Rovelli,Smolin Nuc. Phys. B **422** (1995) 593; Asktekar,Lewandowski Class. Quant. Grav. **14** (1997) A43

An Atom of Quantum Geometry

Idea: Use truncation of $\mathcal{H}_{kin}$ to a single N-valent spin network node

- Can have non-vanishing volume and angle
- Geometric atom have an orthonormal basis, the **intertwiners** i_n that label the ways to add spins
- Has dual picture of (fuzzy) convex polyhedra with N external faces*





* Bianchi et.al. Phys. Rev. D 83 (2011) 044035 arXiv: 1009.3402

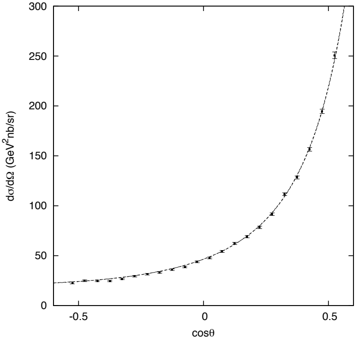
A Scattering Gedanken Experiment

How might discrete spatial geometry be observed?

- Bhabha scattering ($e^- e^+ \to e^- e^+$): The pure QED differential cross section for the process at lowest order is

$$s \left(\frac{d\sigma}{d\Omega} \right)_{\text{QED}} = \frac{\alpha^2}{4} \frac{(3 + \cos^2\theta)^2}{(1 - \cos\theta)^2}$$

  s is the square of the center of mass energy.



- In **quantum geometry**, scattering also measures cosine **angle**.

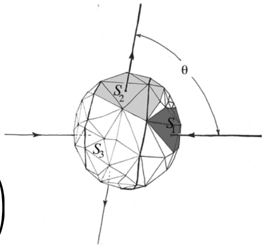
Center-of-mass energy has a corresponding length scale of ℓ. Effective geometry at this scale supports measured angles in cross section

What does a model of discrete geometry at this scale predict?

A Scattering Gedanken Experiment

- Angle operator

$$\hat{\theta}_{(12)} := \text{arccos}\, \frac{L_1^i L_2^i}{|L_1|\,|L_2|}$$



- Spectrum

$$\hat{\theta}_{(12)} \mid j_1\, j_2\, j_3\rangle = \theta_{(12)} \mid j_1\, j_2\, j_3\rangle \text{ with}$$

$$\theta_{(12)} = \arccos\left(\frac{j_3(j_3+1) - j_1(j_1+1) - j_2(j_2+1)}{2\left[j_1(j_1+1)\, j_2(j_2+1)\right]^{1/2}}\right)$$

will use $n = 2j$.

- Non-uniform spectrum so it is hard to model continuum angles.
This is the reason for the combinatorial lever arm that increases effective scale.

SAM Class. Quant. Grav. **16** (1999) 3859 arXiv: gr-qc/9905019

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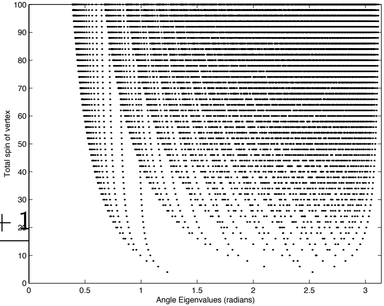

Seifert arXiv:gr-qc/0108047

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SAM Class. Quant. Grav. **16** (1999) 3859 arXiv: gr-qc/9905019

Model of atom of geometry assumptions:

- With "large" length scale we have large spins, external face areas, and mescopic volume.
- Large, **"semi-classical spins"** $1 \ll j_i \ll j_3$, $i = 1, 2$. Characterized by "shape parameter" $\epsilon = \sqrt{j_1 j_2 / j_3}$.
  With small ϵ can accurately model continuum angle
- **Simple and monochromatic**: $j = 1/2$ for all incident edge labels
- **Uniform probability distribution** on intertwiners i_v
- The 'shape' of the local geometry of space is altered by the combinatorics of the atom.

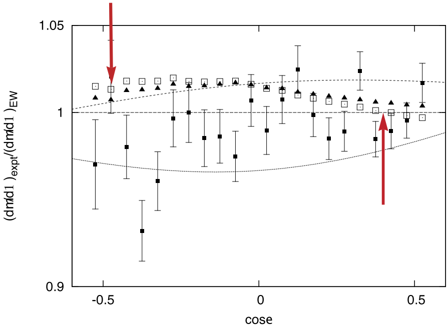
The model exhibits a systematic shift in the shape of geometry and angular measurements.

SAM arXiv:1005.5460, Major and Seifert arXiv: gr-qc/0109056

Bhabha Scattering: A Gedanken experiment

Shape corrected cross section has a correction of the form at lowest order

$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_{QED} = 1 \mp \left(\frac{3s}{\Lambda_\pm^2}\right)\left(\frac{\sin^2\theta}{3+\cos^2\theta}\right)\left(1 + \frac{8}{\pi}\cos(\theta)\epsilon + \dots\right)$$



corrections for $\epsilon = 5 \times 10^{-3}$

Derrick et. al. Phys. Rev. **D 34** *(1986) 3286; SAM 1005.5460*

Coherent states for an Atom of Geometry

Is this effect an artifact of the assumptions on the state of the atom?

- Coherent states. The semi-coherent states of Levine and Speziale are group averaged states $|\,\underline{j}\underline{\hat{n}}\rangle$ constructed from N $SU(2)$ coherent states (or face areas) and directions $\underline{\hat{n}}$.
- Angles between face normals are classical

$$\frac{\langle \underline{j}\underline{\hat{n}}|\cos\theta_{ij}\,|\,\underline{j}\underline{\hat{n}}\rangle}{\langle \underline{j}\underline{\hat{n}}\,|\,\underline{j}\underline{\hat{n}}\rangle} \simeq \hat{n}_i \cdot \hat{n}_j$$

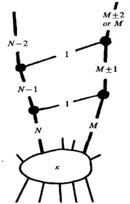
  at only moderate spins, e.g. $j \sim 100$.
- Form classical (oriented) convex polyhedra with N (fixed) faces at moderate spin.

So these coherent states states exhibit **no lever arm**.
But the **directions are classical, arbitrary inputs**.

Livine and Speziale Phys. Rev. **D 76** *(2007) 084028 arXiv:0705.0674*

Geometric atoms from Combinatorics

Penrose's Goal: "build up physical theory from discreteness ... from combinatorial principles "



Spin Geometry Theorem (Penrose Moussouris) The scalar products of spins acting on a correlated state of spins (an atom of geometry) gives vectors in 3D Euclidean space, as long as the (relative) uncertainty of the scalar products is small.

- Can we obtain classical directions from only the quantum state of the atom? Yes! The spin geometry theorem is constructive - by minimizing the relative uncertainties of $\cos\theta$ we can derive normals in 3D from pure combinatorics.
- Do these states exhibit a lever arm?

Penrose, "Angular momentum: An approach to combinatorial spacetime" in *Quantum Theory and Beyond* T. Bastin

Moussouris, "Quantum Models as Space-time Based on Recoupling Theory" D.Phil. Thesis, Oxford University, Oxford, 1983.

Geometric atoms from Combinatorics

Apparently, Yes!

Numerical work on simplest "hydrogen atom of geometry", the tetrahedron.

- For equal area faces we used the spin geometry theorem algorithm to determine normals.
- Linear combination of states with amplitudes given by a complex Gaussian $f(j)$
- Minimizing the relative uncertainties

$$\frac{\langle \Delta(\vec{J}_2 \cdot \vec{J}_3) \rangle}{j_2 j_3} < \delta$$

based on peak, width and phase using

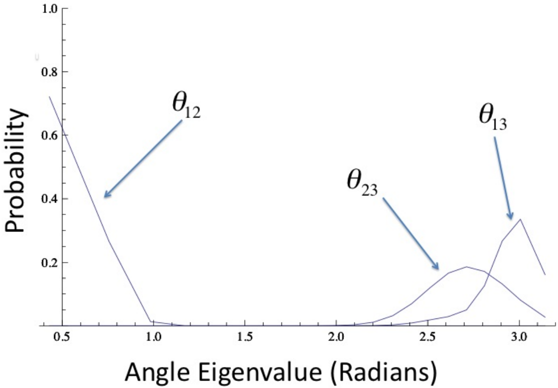
$$\delta^2 > \frac{1}{j_r^4}\Big(\sum_{j_{23}}^{2j_r}\sum_{j_{12}}^{2j_r}\sum_{j'_{12}}^{2j_r}(j_{23}(j_{23}+1)-2j_r(j_r+1))^2 f^*(j_{12})\mathrm{dim}j_{12}\mathrm{dim}j_{23}\left\{\begin{array}{ccc} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{23}\end{array}\right\}\left\{\begin{array}{ccc} j_1 & j_2 & j_{23} \\ j_3 & j_4 & j'_{12}\end{array}\right\} f(j'_{12}) -$$

$$\Big(\sum_{j_{23}}^{2j_r}\sum_{j_{12}}^{2j_r}\sum_{j'_{12}}^{2j_r}(j_{23}(j_{23}+1) - 2j_r(j_r+1)) f^*(j_{12})\mathrm{dim}j_{12}\mathrm{dim}j_{23}\left\{\begin{array}{ccc} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{23}\end{array}\right\}\left\{\begin{array}{ccc} j_1 & j_2 & j_{23} \\ j_3 & j_4 & j'_{12}\end{array}\right\} f(j'_{12})\Big)^2\Big)$$

we computed the distribution of possible angles.

Geometric atoms from Combinatorics

Minimizing relative uncertainties favors squashed geometries with $\theta \sim 0, \pi$, indicating a the presence of a lever arm.



Summary

- Observing Planck scale effects requires lever arm
- An atom of geometry used as model for physical effects of LQG kinematics. Corrections of continuum spatial geometry could arise
- Combinatorics can play a role in quantum geometry phenomenology
- Spin geometry theorem and operational point of view suggests how to build up coherent states from only spins
- Apparently there is a lever arm in this approach.

Further work:

- Define these states for general N
- Determine extent of lever arm
- Develop the effective-geometry field theory