Signature change in loop quantum cosmology

Jakub Mielczarek

National Centre for Nuclear Research, Warsaw,
Jagiellonian University, Cracow

Prague, 25 June, 2012

- The signature change from Lorentzian to Euclidean

$$ds^2 = -dt^2 + dx^2 \to ds^2 = d\tau^2 + dx^2$$

is usually performed by the so-called Wick rotation ($t \to -i\tau$).

- The signature change from Lorentzian to Euclidean

$$ds^2 = -dt^2 + dx^2 \to ds^2 = d\tau^2 + dx^2$$

is usually performed by the so-called Wick rotation ($t \to -i\tau$).

- This computational trick relates path integral approach with statistical physics

$$e^{\frac{i}{\hbar}S} \to e^{-\frac{1}{\hbar}S_E},$$

improving convergence properties of some integrals.

- The signature change from Lorentzian to Euclidean

$$ds^2 = -dt^2 + dx^2 \to ds^2 = d\tau^2 + dx^2$$

is usually performed by the so-called Wick rotation ($t \to -i\tau$).

- This computational trick relates path integral approach with statistical physics

$$e^{\frac{i}{\hbar}S} \to e^{-\frac{1}{\hbar}S_E},$$

improving convergence properties of some integrals.

- In 1983, Hartle and Hawking suggested that Wick rotation may gain physical meaning at the Planck epoch introducing the so-called no-boundary proposal.

- The signature change from Lorentzian to Euclidean

$$ds^2 = -dt^2 + dx^2 \to ds^2 = d\tau^2 + dx^2$$

  is usually performed by the so-called Wick rotation ($t \to -i\tau$).
- This computational trick relates path integral approach with statistical physics

$$e^{\frac{i}{\hbar}S} \to e^{-\frac{1}{\hbar}S_E},$$

  improving convergence properties of some integrals.
- In 1983, Hartle and Hawking suggested that Wick rotation may gain physical meaning at the Planck epoch introducing the so-called no-boundary proposal.
- While such possibility is conceptually interesting, mechanism behind the signature change remains mysterious.

- The signature change from Lorentzian to Euclidean

$$ds^2 = -dt^2 + dx^2 \to ds^2 = d\tau^2 + dx^2$$

  is usually performed by the so-called Wick rotation ($t \to -i\tau$).
- This computational trick relates path integral approach with statistical physics

$$e^{\frac{i}{\hbar}S} \to e^{-\frac{1}{\hbar}S_E},$$

  improving convergence properties of some integrals.
- In 1983, Hartle and Hawking suggested that Wick rotation may gain physical meaning at the Planck epoch introducing the so-called no-boundary proposal.
- While such possibility is conceptually interesting, mechanism behind the signature change remains mysterious.
- What is the origin of the signature change?

- The signature change from Lorentzian to Euclidean

$$ds^2 = -dt^2 + dx^2 \rightarrow ds^2 = d\tau^2 + dx^2$$

  is usually performed by the so-called Wick rotation ($t \rightarrow -i\tau$).
- This computational trick relates path integral approach with statistical physics

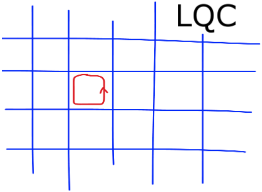
$$e^{\frac{i}{\hbar}S} \rightarrow e^{-\frac{1}{\hbar}S_E},$$

  improving convergence properties of some integrals.
- In 1983, Hartle and Hawking suggested that Wick rotation may gain physical meaning at the Planck epoch introducing the so-called no-boundary proposal.
- While such possibility is conceptually interesting, mechanism behind the signature change remains mysterious.
- What is the origin of the signature change? Can quantum gravity lead to the signature change?

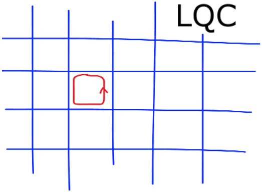
- The quantum gravity is usually related with some sort of discreteness of space at the Planck scale.

- The quantum gravity is usually related with some sort of discreteness of space at the Planck scale.
- In particular, in Loop Quantum Gravity (LQG), geometric operators (area, volume) have discrete spectra.

- The quantum gravity is usually related with some sort of discreteness of space at the Planck scale.
- In particular, in Loop Quantum Gravity (LQG), geometric operators (area, volume) have discrete spectra.
- Loop Quantum Cosmology (LQC) is a regular lattice model of LQG.



- The quantum gravity is usually related with some sort of discreteness of space at the Planck scale.
- In particular, in Loop Quantum Gravity (LQG), geometric operators (area, volume) have discrete spectra.
- Loop Quantum Cosmology (LQC) is a regular lattice model of LQG.



- Physical area of a loop $\mathrm{Ar}_{\square} = \bar{p}\bar{\mu}^2$, where $\bar{p} = a^2$ and a is a scale factor. In general $\bar{\mu} \propto \bar{p}^{\beta}$, where $-1/2 \leq \beta \leq 0$. For the so-called $\bar{\mu}-$scheme: $\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}}$, where $\Delta = 2\sqrt{3}\pi\gamma l_{\mathrm{Pl}}^2$ is the area gap derived from LQG.

- In the framework of Ashtekar variables, Hamiltonian of general relativity is a sum of three constraints:

$$0 \approx H_G[N, N^a, N^i]$$

- In the framework of Ashtekar variables, Hamiltonian of general relativity is a sum of three constraints:

$$0 \approx H_{\mathrm{G}}[N, N^a, N^i] \qquad = \frac{1}{16\pi G} \int_\Sigma d^3x \left(NC + NC_a + N^i C_i \right)$$

- In the framework of Ashtekar variables, Hamiltonian of general relativity is a sum of three constraints:

$$0 \approx H_{\mathrm{G}}[N, N^a, N^i] = \frac{1}{16\pi G} \int_\Sigma d^3x \left(NC + NC_a + N^i C_i \right)$$
$$= S[N] \quad \text{(scalar constraint)}$$

- In the framework of Ashtekar variables, Hamiltonian of general relativity is a sum of three constraints:

$$
\begin{aligned}
0 \approx H_{\mathrm{G}}[N, N^a, N^i] &= \frac{1}{16\pi G}\int_\Sigma d^3x \left(NC + NC_a + N^i C_i\right) \\
&= S[N] \quad \text{(scalar constraint)} \\
&+ D[N^a] \quad \text{(diffeomorphism constraint)}
\end{aligned}
$$

- In the framework of Ashtekar variables, Hamiltonian of general relativity is a sum of three constraints:

$$
\begin{aligned}
0 \approx H_{\mathrm{G}}[N, N^a, N^i] &= \frac{1}{16\pi G}\int_\Sigma d^3x \left(NC + NC_a + N^i C_i \right) \\
&= S[N] \quad \text{(scalar constraint)} \\
&+ D[N^a] \quad \text{(diffeomorphism constraint)} \\
&+ G[N^i] \quad \text{(Gauss constraint).}
\end{aligned}
$$

- In the framework of Ashtekar variables, Hamiltonian of general relativity is a sum of three constraints:

$$
\begin{aligned}
0 \approx H_{\mathrm{G}}[N, N^a, N^i] &= \frac{1}{16\pi G}\int_{\Sigma} d^3x \left(NC + NC_a + N^i C_i\right) \\
&= S[N] \quad \text{(scalar constraint)} \\
&+ D[N^a] \quad \text{(diffeomorphism constraint)} \\
&+ G[N^i] \quad \text{(Gauss constraint).}
\end{aligned}
$$

The constraints ($S \to \mathcal{C}_1$, $D \to \mathcal{C}_2$, $G \to \mathcal{C}_3$) form closed algebra

$$\{\mathcal{C}_I, \mathcal{C}_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}_K.$$

- In the framework of Ashtekar variables, Hamiltonian of general relativity is a sum of three constraints:

$$
\begin{aligned}
0 \approx H_{\mathrm{G}}[N, N^a, N^i] &= \frac{1}{16\pi G}\int_\Sigma d^3x\,(NC + NC_a + N^i C_i) \\
&= S[N] \quad \text{(scalar constraint)} \\
&+ D[N^a] \quad \text{(diffeomorphism constraint)} \\
&+ G[N^i] \quad \text{(Gauss constraint)}.
\end{aligned}
$$

The constraints ($S \to \mathcal{C}_1$, $D \to \mathcal{C}_2$, $G \to \mathcal{C}_3$) form closed algebra

$$\{\mathcal{C}_I, \mathcal{C}_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}_K.$$

- The effects of discreteness of space can be studied at the effective level by introducing appropriate corrections to the classical constraints.

In cosmology, Ashtekar variables can be decomposed for the backround (here, flat FRW) and perturbation parts:

$$
\begin{aligned}
E &= \bar{E} + \delta E, \\
A &= \bar{A} + \delta A.
\end{aligned}
$$

In cosmology, Ashtekar variables can be decomposed for the backround (here, flat FRW) and perturbation parts:

$$\begin{aligned} E &= \bar{E} + \delta E, \\ A &= \bar{A} + \delta A. \end{aligned}$$

Perturbations of Ashtekar variables can be related with the standard metric perturbations: scalar modes (Φ, Ψ, E, B), vector modes (S^a, F_a) and tensor modes (h_{ab}).

In cosmology, Ashtekar variables can be decomposed for the backround (here, flat FRW) and perturbation parts:

$$\begin{aligned} E &= \bar{E} + \delta E, \\ A &= \bar{A} + \delta A. \end{aligned}$$

Perturbations of Ashtekar variables can be related with the standard metric perturbations: scalar modes (Φ, Ψ, E, B), vector modes (S^a, F_a) and tensor modes (h_{ab}).
The scalar field φ and its canonically conjugated momenta π are also subject of decomposition:

$$\begin{aligned} \varphi &= \bar{\varphi} + \delta\varphi, \\ \pi &= \bar{\pi} + \delta\pi. \end{aligned}$$

In cosmology, Ashtekar variables can be decomposed for the backround (here, flat FRW) and perturbation parts:

$$
\begin{aligned}
E &= \bar{E} + \delta E, \\
A &= \bar{A} + \delta A.
\end{aligned}
$$

Perturbations of Ashtekar variables can be related with the standard metric perturbations: scalar modes (Φ, Ψ, E, B), vector modes (S^a, F_a) and tensor modes (h_{ab}).
The scalar field φ and its canonically conjugated momenta π are also subject of decomposition:

$$
\begin{aligned}
\varphi &= \bar{\varphi} + \delta\varphi, \\
\pi &= \bar{\pi} + \delta\pi.
\end{aligned}
$$

The analysis is performed up to the second order in the perturbative expansion:

$$
\mathcal{C}_I = \mathcal{C}_I^{(0)} + \mathcal{C}_I^{(1)} + \mathcal{C}_I^{(2)} + \ldots
$$

The quantum (holonomy) corrections are introduced at the level of total constraints $\mathcal{C}_{tot} = \mathcal{C}_G + \mathcal{C}_M$:

$$\mathcal{C}_{tot} \to \mathcal{C}^Q_{tot}.$$

The quantum (holonomy) corrections are introduced at the level of total constraints $\mathcal{C}_{tot} = \mathcal{C}_G + \mathcal{C}_M$:

$$\mathcal{C}_{tot} \to \mathcal{C}^{Q}_{tot}.$$

The procedure of introducing quantum corrections suffers from ambiguities.

The quantum (holonomy) corrections are introduced at the level of total constraints $\mathcal{C}_{tot} = \mathcal{C}_G + \mathcal{C}_M$:

$$\mathcal{C}_{tot} \to \mathcal{C}^Q_{tot}.$$

The procedure of introducing quantum corrections suffers from ambiguities. In general, the algebra of modified constraints is not closed:

$$\{\mathcal{C}^Q_I, \mathcal{C}^Q_J\} = g^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}^Q_K + \mathcal{A}_{IJ}.$$

The quantum (holonomy) corrections are introduced at the level of total constraints $\mathcal{C}_{tot} = \mathcal{C}_G + \mathcal{C}_M$:

$$\mathcal{C}_{tot} \to \mathcal{C}_{tot}^{Q}.$$

The procedure of introducing quantum corrections suffers from ambiguities. In general, the algebra of modified constraints is not closed:

$$\{\mathcal{C}_I^Q, \mathcal{C}_J^Q\} = g^K{}_{IJ}(A_b^j, E_i^a)\mathcal{C}_K^Q + \mathcal{A}_{IJ}.$$

Can we introduce quantum holonomy corrections in the anomaly-free manner? (i.e. such that $\mathcal{A}_{IJ} = 0$)?

The quantum (holonomy) corrections are introduced at the level of total constraints $\mathcal{C}_{tot} = \mathcal{C}_G + \mathcal{C}_M$:

$$\mathcal{C}_{tot} \to \mathcal{C}_{tot}^Q.$$

The procedure of introducing quantum corrections suffers from ambiguities. In general, the algebra of modified constraints is not closed:

$$\{\mathcal{C}_I^Q, \mathcal{C}_J^Q\} = g^K{}_{IJ}(A_b^j, E_i^a)\mathcal{C}_K^Q + \mathcal{A}_{IJ}.$$

Can we introduce quantum holonomy corrections in the anomaly-free manner? (i.e. such that $\mathcal{A}_{IJ} = 0$)?

The answer turns out to be **yes**! There is a unique way of modifying constraints such that the algebra is closed.

The quantum (holonomy) corrections are introduced at the level of total constraints $\mathcal{C}_{tot} = \mathcal{C}_G + \mathcal{C}_M$:

$$\mathcal{C}_{tot} \to \mathcal{C}_{tot}^Q.$$

The procedure of introducing quantum corrections suffers from ambiguities. In general, the algebra of modified constraints is not closed:

$$\{\mathcal{C}_I^Q, \mathcal{C}_J^Q\} = g^K{}_{IJ}(A_b^j, E_i^a)\mathcal{C}_K^Q + \mathcal{A}_{IJ}.$$

Can we introduce quantum holonomy corrections in the anomaly-free manner? (i.e. such that $\mathcal{A}_{IJ} = 0$)?

The answer turns out to be **yes**! There is a unique way of modifying constraints such that the algebra is closed.

Additionally, the conditions of anomaly-freedom are fulfilled if and only if $\beta = -1/2$, which corresponds to "new quantization scheme".
[T. Cailleteau, J. Mielczarek, A. Barrau, J. Grain, Class. Quantum Grav. **29** (2012) 095010].

Algebra of constraints:

$$\begin{aligned}
\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} &= 0, \\
\{S_{tot}[N], D_{tot}[N^a]\} &= -S_{tot}[\delta N^a \partial_a \delta N], \\
\{S_{tot}[N_1], S_{tot}[N_2]\} &= \Omega D_{tot}\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right].
\end{aligned}$$

Algebra of constraints:

$$\begin{aligned}
\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} &= 0, \\
\{S_{tot}[N], D_{tot}[N^a]\} &= -S_{tot}[\delta N^a \partial_a \delta N], \\
\{S_{tot}[N_1], S_{tot}[N_2]\} &= \Omega D_{tot}\left[\frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right].
\end{aligned}$$

The algebra is closed but deformed with respect to the classical case due to presence of the factor

$$\Omega = \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c} \in [-1, 1] \quad \text{where} \quad \rho_c = \frac{3}{8\pi G\Delta} \sim \rho_{\text{Pl}}.$$

Algebra of constraints:

$$\begin{aligned}\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} &= 0, \\ \{S_{tot}[N], D_{tot}[N^a]\} &= -S_{tot}[\delta N^a \partial_a \delta N], \\ \{S_{tot}[N_1], S_{tot}[N_2]\} &= \Omega D_{tot}\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right].\end{aligned}$$

The algebra is closed but deformed with respect to the classical case due to presence of the factor

$$\Omega = \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c} \in [-1, 1] \quad \text{where} \quad \rho_c = \frac{3}{8\pi G\Delta} \sim \rho_{Pl}.$$

What is the interpretation? Classically, we have

$$\{S_{tot}[N_1], S_{tot}[N_2]\} = sD\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right],$$

where $s = 1$ corresponds to the Lorentzian signature and $s = -1$ to the Euclidean one.

- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.

- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.

- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.
- The similar effect was observed also for spherically symmetric models. [M. Bojowald and G. M. Paily, *Deformed General Relativity and Effective Actions from Loop Quantum Gravity*, arXiv:1112.1899 [gr-qc]].

- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.
- The similar effect was observed also for spherically symmetric models. [M. Bojowald and G. M. Paily, *Deformed General Relativity and Effective Actions from Loop Quantum Gravity*, arXiv:1112.1899 [gr-qc]].

A lot of questions arise, e.g. :

- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.
- The similar effect was observed also for spherically symmetric models. [M. Bojowald and G. M. Paily, *Deformed General Relativity and Effective Actions from Loop Quantum Gravity*, arXiv:1112.1899 [gr-qc]].

A lot of questions arise, e.g. :

- Is the sign change only a perturbative effect?

- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.
- The similar effect was observed also for spherically symmetric models. [M. Bojowald and G. M. Paily, *Deformed General Relativity and Effective Actions from Loop Quantum Gravity*, arXiv:1112.1899 [gr-qc]].

A lot of questions arise, e.g. :

- Is the sign change only a perturbative effect?
- What with the standard picture of the bouncing cosmology?

- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.
- The similar effect was observed also for spherically symmetric models. [M. Bojowald and G. M. Paily, *Deformed General Relativity and Effective Actions from Loop Quantum Gravity*, arXiv:1112.1899 [gr-qc]].

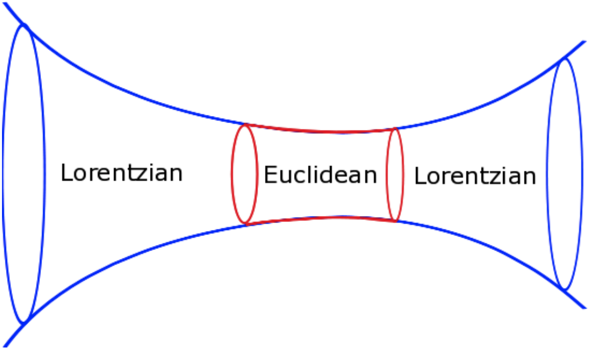
A lot of questions arise, e.g. :

- Is the sign change only a perturbative effect?
- What with the standard picture of the bouncing cosmology?
- Is this a hint that the quantum algebra in LQG is also modified? ($[\hat{H}, \hat{H}] = \Omega \hat{D}$?)

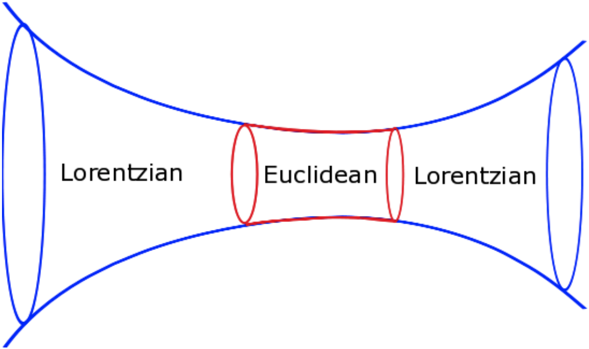
- The effective algebra of constraints shows that space is Euclidian for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.
- The similar effect was observed also for spherically symmetric models. [M. Bojowald and G. M. Paily, *Deformed General Relativity and Effective Actions from Loop Quantum Gravity*, arXiv:1112.1899 [gr-qc]].

A lot of questions arise, e.g. :

- Is the sign change only a perturbative effect?
- What with the standard picture of the bouncing cosmology?
- Is this a hint that the quantum algebra in LQG is also modified? ($[\hat{H}, \hat{H}] = \Omega \hat{D}$?)
- Is there relation to Hořava gravity? Flow from $z = 0$ to $z = 1$.



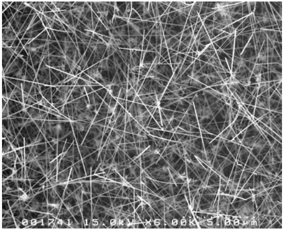
- Is there quantum tunneling through the Euclidean phase?



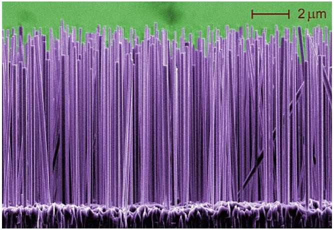
- Is there quantum tunneling through the Euclidean phase?
- Suppression of spatial derivatives while $\{H, H\} \to 0$. Possible support for the BKL conjecture.

Can physics of metamaterials help us? Signature change is observed e.g. in "wired" metamaterials as a result of negative dielectric permittivity[1].

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=\frac{1}{\epsilon_1}\frac{\partial^2\varphi}{\partial z^2}+\frac{1}{\epsilon_2}\frac{\partial^2\varphi}{\partial y^2}+\frac{1}{\epsilon_3}\frac{\partial^2\varphi}{\partial x^2}$$



$\epsilon_1=\epsilon_2=\epsilon_3$



$\epsilon_1<0$ and $\epsilon_2=\epsilon_3>0$

Spontaneous symmetry breaking? Emergence of time coordinate while passing to low temperatures?

[1] I. I. Smolyaninov, E. E. Narimanov, PRL **105**, 067402 (2010)

We find

$$\ddot{\phi} + 2\left[\mathcal{H} - \left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\dot{\phi} + 2\left[\dot{\mathcal{H}} - \mathcal{H}\left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\phi - c_s^2\nabla^2\phi = 0,$$

with the quantum correction

$$\epsilon = \frac{1}{2}\frac{\dot{\Omega}}{\Omega} = 3\mathbb{K}[2]\left(\frac{\rho + P}{\rho_c - 2\rho}\right),$$

and the squared velocity $c_s^2 = \Omega$.

[2]E. Wilson-Ewing, Class. Quant. Grav. **29** (2012) 085005

Equations of motion - Longitudinal gauge ($E = 0 = B$)

We find

$$\ddot{\phi} + 2\left[\mathcal{H} - \left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\dot{\phi} + 2\left[\dot{\mathcal{H}} - \mathcal{H}\left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\phi - c_s^2\nabla^2\phi = 0,$$

with the quantum correction

$$\epsilon = \frac{1}{2}\frac{\dot{\Omega}}{\Omega} = 3\mathbb{K}[2]\left(\frac{\rho + P}{\rho_c - 2\rho}\right),$$

and the squared velocity $c_s^2 = \Omega$. The derived equation is the same as this found by E. Wilson-Ewing[2] in his approach. This non-trivial equivalence of both approaches may suggests uniqueness in defining theory of scalar perturbations with holonomy corrections in anomaly-free manner.

[2] E. Wilson-Ewing, Class. Quant. Grav. **29** (2012) 085005

Equations of motion - Gauge-invariant variables

Gauge-invariant variables (modified Bardeen's potentials):

$$
\begin{aligned}
\Phi &= \phi + \frac{1}{\Omega}(\dot{B} - \ddot{E}) + \left(\frac{\mathbb{K}[2]}{\Omega} - \frac{\dot{\Omega}}{\Omega}\right)(B - \dot{E}), \\
\Psi &= \psi - \frac{\mathbb{K}[2]}{\Omega}(B - \dot{E}), \\
\delta\varphi^{GI} &= \delta\varphi + \frac{\dot{\bar{\varphi}}}{\Omega}(B - \dot{E}).
\end{aligned}
$$

Equations of motion - Gauge-invariant variables

Gauge-invariant variables (modified Bardeen's potentials):

$$\begin{aligned}
\Phi &= \phi + \frac{1}{\Omega}(\dot{B} - \ddot{E}) + \left(\frac{\mathbb{K}[2]}{\Omega} - \frac{\dot{\Omega}}{\Omega}\right)(B - \dot{E}), \\
\Psi &= \psi - \frac{\mathbb{K}[2]}{\Omega}(B - \dot{E}), \\
\delta\varphi^{GI} &= \delta\varphi + \frac{\dot{\bar{\varphi}}}{\Omega}(B - \dot{E}).
\end{aligned}$$

The gauge invariant variables are modified since the very structure of spacetime is deformed.

Equations of motion - Gauge-invariant variables

Gauge-invariant variables (modified Bardeen's potentials):

$$\begin{aligned}\Phi &= \phi + \frac{1}{\Omega}(\dot{B} - \ddot{E}) + \left(\frac{\mathbb{K}[2]}{\Omega} - \frac{\dot{\Omega}}{\Omega}\right)(B - \dot{E}),\\ \Psi &= \psi - \frac{\mathbb{K}[2]}{\Omega}(B - \dot{E}),\\ \delta\varphi^{GI} &= \delta\varphi + \frac{\dot{\bar{\varphi}}}{\Omega}(B - \dot{E}).\end{aligned}$$

The gauge invariant variables are modified since the very structure of spacetime is deformed.

The equations of motion for Φ and Ψ are the same as this found for the longitudinal gauge. Moreover

$$\delta\ddot{\varphi}^{GI} + 2\mathbb{K}[2]\delta\dot{\varphi}^{GI} - \Omega\nabla^2\delta\varphi^{GI} + \bar{p}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^{GI} + 2\bar{p}V_{,\varphi}(\bar{\varphi})\Psi - 4\dot{\bar{\varphi}}^{GI}\dot{\Psi} = 0.$$

Equations of motion:

Equations of motion:

Scalar pertubations. One can derive modified Mukhanov equation:

$$\frac{d^2}{d\eta^2}v - \Omega\nabla^2 v - \frac{z''}{z}v = 0,$$

where $z := \sqrt{\bar{p}}\frac{\dot{\varphi}}{H}$. Spatial curvature $\mathcal{R} = v/z$.

Equations of motion:

Scalar pertubations. One can derive modified Mukhanov equation:

$$\frac{d^2}{d\eta^2}v - \Omega\nabla^2 v - \frac{z^{''}}{z}v = 0,$$

where $z := \sqrt{\bar{p}}\frac{\dot{\varphi}}{H}$. Spatial curvature $\mathcal{R} = v/z$.

Vector perturbations. For the considered model with a scalar field vector modes are pure gauge.

Equations of motion:

Scalar pertubations. One can derive modified Mukhanov equation:

$$\frac{d^2}{d\eta^2}v - \Omega\nabla^2 v - \frac{z''}{z}v = 0,$$

where $z := \sqrt{\bar{p}}\frac{\dot{\varphi}}{H}$. Spatial curvature $\mathcal{R} = v/z$.

Vector perturbations. For the considered model with a scalar field vector modes are pure gauge.

Tensor perturbations. Equation of motion for the gravitational waves is the following:

$$\frac{d^2}{d\eta^2}h_{ab} + 2\left(aH - \frac{1}{2\Omega}\frac{d\Omega}{d\eta}\right)\frac{d}{d\eta}h_{ab} - \Omega\nabla^2 h_{ab} = 0.$$

Slow-roll inflation with holonomy corrections

Modified Friedmann equation:

$$H^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}\rho\left(1-\frac{\rho}{\rho_c}\right).$$

Slow-roll inflation with holonomy corrections

Modified Friedmann equation:

$$H^2 = \frac{8\pi}{3m_{\rm Pl}^2}\rho\left(1-\frac{\rho}{\rho_c}\right).$$

Slow-roll parameters:

$$\begin{aligned}
\epsilon &:= \frac{m_{\rm Pl}^2}{16\pi}\left(\frac{V_{,\varphi}}{V}\right)^2\frac{1}{(1-V/\rho_c)},\\
\eta &:= \frac{m_{\rm Pl}^2}{8\pi}\left(\frac{V_{,\varphi\varphi}}{V}\right)\frac{1}{(1-V/\rho_c)},\\
\delta &:= \eta-\epsilon\left(1-\frac{V}{\rho_c}\right).
\end{aligned}$$

Based on the derived equations of motion one can determine inflationary scalar and tensor power spectra ($\rho > \rho_c/2$).
For this purpose we perform quantization of the v and h_{ab} fields.

Based on the derived equations of motion one can determine inflationary scalar and tensor power spectra ($\rho > \rho_c/2$).
For this purpose we perform quantization of the v and h_{ab} fields.

Scalar power spectrum:

$$\mathcal{P}_S(k) = A_S \left(\frac{k}{aH}\right)^{n_S - 1},$$

$$A_S = \frac{1}{\pi\epsilon}\left(\frac{H}{m_{Pl}}\right)^2 \left(1 + 2\frac{V}{\rho_c}\right),$$

$$n_S = 1 + 2\eta - 6\epsilon(1 - V/\rho_c).$$

Based on the derived equations of motion one can determine inflationary scalar and tensor power spectra ($\rho > \rho_c/2$).
For this purpose we perform quantization of the v and h_{ab} fields.

Scalar power spectrum:

$$\mathcal{P}_{\rm S}(k) = A_S \left(\frac{k}{aH}\right)^{n_{\rm S}-1},$$

$$A_S = \frac{1}{\pi\epsilon}\left(\frac{H}{m_{\rm Pl}}\right)^2 \left(1+2\frac{V}{\rho_c}\right),$$

$$n_{\rm S} = 1 + 2\eta - 6\epsilon(1 - V/\rho_c).$$

Tensor power spectrum:

$$\mathcal{P}_{\rm T}(k) = A_T \left(\frac{k}{aH}\right)^{n_{\rm T}},$$

$$A_T = \frac{16}{\pi}\left(\frac{H}{m_{\rm Pl}}\right)^2 \left(1+3\frac{V}{\rho_c}\right),$$

$$n_{\rm T} = -2\epsilon(1 - 3V/\rho_c).$$

Based on the derived equations of motion one can determine inflationary scalar and tensor power spectra ($\rho > \rho_c/2$).
For this purpose we perform quantization of the v and h_{ab} fields.

Scalar power spectrum:

$$\mathcal{P}_S(k) = A_S \left(\frac{k}{aH}\right)^{n_S-1},$$

$$A_S = \frac{1}{\pi\epsilon}\left(\frac{H}{m_{Pl}}\right)^2 \left(1 + 2\frac{V}{\rho_c}\right),$$

$$n_S = 1 + 2\eta - 6\epsilon(1 - V/\rho_c).$$

Tensor power spectrum:

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{aH}\right)^{n_T},$$

$$A_T = \frac{16}{\pi}\left(\frac{H}{m_{Pl}}\right)^2 \left(1 + 3\frac{V}{\rho_c}\right),$$

$$n_T = -2\epsilon(1 - 3V/\rho_c).$$

Consistency relation $r := \frac{A_T}{A_S} \simeq 16\epsilon\left(1 + \frac{V}{\rho_c}\right)$.

Obtained quantum gravitational corrections are of the order $\frac{V}{\rho_c} \sim 10^{-12}$.

Further comments

- The considerations apply to the modes with $\lambda > l_{\text{Pl}}$. The issue of trans-Planckian modes cannot be addressed.

Further comments

- The considerations apply to the modes with $\lambda > l_{\mathrm{Pl}}$. The issue of trans-Planckian modes cannot be addressed.
- Euclidean phase is crucial in *Causal Dynamical Triangulation*. The issue of conformal divergence of the classical Einstein action. Is this related with our sign change?

Further comments

- The considerations apply to the modes with $\lambda > l_{\text{Pl}}$. The issue of trans-Planckian modes cannot be addressed.
- Euclidean phase is crucial in *Causal Dynamical Triangulation*. The issue of conformal divergence of the classical Einstein action. Is this related with our sign change?
- At $\rho = \rho_c/2$ where $\{H, H\} = 0$ the *ultralocal gravity* (Isham - 1976) is recovered.

Further comments

- The considerations apply to the modes with $\lambda > l_{\text{Pl}}$. The issue of trans-Planckian modes cannot be addressed.
- Euclidean phase is crucial in *Causal Dynamical Triangulation*. The issue of conformal divergence of the classical Einstein action. Is this related with our sign change?
- At $\rho = \rho_{\text{c}}/2$ where $\{H, H\} = 0$ the *ultralocal gravity* (Isham - 1976) is recovered.
- The general covariance is modified. What is the physical meaning of this modification?

Further comments

- The considerations apply to the modes with $\lambda > l_{\text{Pl}}$. The issue of trans-Planckian modes cannot be addressed.
- Euclidean phase is crucial in *Causal Dynamical Triangulation*. The issue of conformal divergence of the classical Einstein action. Is this related with our sign change?
- At $\rho = \rho_c/2$ where $\{H, H\} = 0$ the *ultralocal gravity* (Isham - 1976) is recovered.
- The general covariance is modified. What is the physical meaning of this modification?
- How to pass from the algebra of constraints to the Lagrangian formulation? (Kuchař -1974; Bojowald, Paily - 2011)

Further comments

- The considerations apply to the modes with $\lambda > l_{\text{Pl}}$. The issue of trans-Planckian modes cannot be addressed.
- Euclidean phase is crucial in *Causal Dynamical Triangulation*. The issue of conformal divergence of the classical Einstein action. Is this related with our sign change?
- At $\rho = \rho_c/2$ where $\{H, H\} = 0$ the *ultralocal gravity* (Isham - 1976) is recovered.
- The general covariance is modified. What is the physical meaning of this modification?
- How to pass from the algebra of constraints to the Lagrangian formulation? (Kuchař -1974; Bojowald, Paily - 2011)
- D'Alambert operator changes smoothly its type from hyperbolic to elliptic one: $\Box = \frac{\partial^2}{\partial t^2} - \Omega(t)\nabla^2$.

Outlook

Outlook

- Better understanding of the transition between Lorentzian and Euclidian domains.

Outlook

- Better understanding of the transition between Lorentzian and Euclidian domains.
- Relation with the Hartle-Hawking proposal.

Outlook

- Better understanding of the transition between Lorentzian and Euclidian domains.
- Relation with the Hartle-Hawking proposal.
- The modified Mukhanov equation can be directly applied to compute power spectrum of the scalar perturbations with the holonomy corrections.

Outlook

- Better understanding of the transition between Lorentzian and Euclidian domains.
- Relation with the Hartle-Hawking proposal.
- The modified Mukhanov equation can be directly applied to compute power spectrum of the scalar perturbations with the holonomy corrections.
- The issue of initial conditions (matching conditions) for the perturbations at $\rho = \rho_c/2$ $(\Omega = 0)$. Maybe scale-invariant spectrum without inflation?

Outlook

- Better understanding of the transition between Lorentzian and Euclidian domains.
- Relation with the Hartle-Hawking proposal.
- The modified Mukhanov equation can be directly applied to compute power spectrum of the scalar perturbations with the holonomy corrections.
- The issue of initial conditions (matching conditions) for the perturbations at $\rho = \rho_c/2$ $(\Omega = 0)$. Maybe scale-invariant spectrum without inflation?
- Comparison with the CMB data (TT, TE, EE and BB spectra).

Thank you!