Superradiance or total reflection?

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(recent work with András László)

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The stability of the Kerr black hole

- The stability problem for the Kerr family of black hole solutions to the vacuum Einstein equations is one of the most important unresolved issues in GR.
  - The ultimate goal is to understand the dynamical stability of Kerr, as a family of solutions, to the Cauchy problem for the system of nonlinear hyperbolic equations

$$\boxed{R_{ab}(g) = 0}$$

- Essentially all work in the black hole case has been confined to the linearized setting
  - The simplest problem: scalar perturbations on a fixed Kerr background

$$\boxed{\Box_g \Phi = 0}$$

  - which is a poor man's substitute for the more complicated problem of gravitational perturbations, obtained by linearizing $R_{ab}(g) = 0$ around a Kerr BH.
- A complete proof, covering the general subextremal case, of linear stability for scalar perturbations was given recently by M. Dafermos & I. Rodnianski [arXiv:1010.5137].

Superradiance

- The wave analog of the Penrose process: *allows energy to be extracted from black holes*.
  - "...if scalar, electromagnetic or gravitational wave is incident upon a black hole, part of the wave (the "transmitted wave") will be absorbed by the black hole and part of the wave (the "reflected wave") will escape to infinity."
- Superradiance, discovered at the early 70's as a new phenomenon, may be related to the names of Misner, Zel'dovich and Starobinskii
- By using the Teukolsky equation scalar, electromagnetic and gravitational perturbations can be investigated within the same setting.
- The conventional arguments ending up with superradiance, including the ones based on Teukolsky's equation, all refer to properties of individual modes.
- As it was shown first by Bekenstein whenever superradiance happens it can be seen to be completely consistent with the laws of BH thermodynamics.
- The aforementioned proof of linear stability by M. Dafermos & I. Rodnianski does not include a detailed investigation of superradiance.
  - Their main concern was to provide boundedness and decay statements for solutions of $\Box_g \Phi = 0$ arising from arbitrary finite-energy initial data.

Superradiance (mode analysis)

- It was realized first by Carter the d'Alembert operator separates for the t-Fourier transformed field.
  - the temporal Fourier transform, $\mathscr{F}\Phi$, of a solution to $\Box_g\Phi = 0$, in coordinates $t, r_*, \vartheta, \varphi$, may be decomposed as

$$\mathscr{F}\Phi(\omega, r_*, \vartheta, \varphi) = \frac{1}{\sqrt{r^2+a^2}} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} R^m_{\ell,\omega}(r_*) S^m_{\ell,a\omega}(\vartheta, \varphi)\,, \tag{1}$$

  - ω is the frequency in the time translation direction
  - $S^m_{\ell,a\omega}$ denotes the *oblate spheroidal harmonic functions* with oblateness parameter $a\omega$ and with angular momentum quantum numbers ℓ, m ($S^m_{\ell,a\omega}$ eigenfunctions of a self-adjoint op.)
  - for the radial functions $R^m_{\ell,\omega}$ a *one-dimensional Schrödinger equation* of the form

$$\frac{d^2 R^m_{\ell,\omega}}{dr_*^2} + \left[\left(\omega - \frac{ma}{r^2+a^2}\right)^2 + \Delta \cdot V^m_{\ell,\omega}(r_*)\right] R^m_{\ell,\omega} = 0\,, \tag{2}$$

  with suitable real potentials $V^m_{\ell,\omega}(r_*)$, can be derived from $\Box_g\Phi = 0$.

Superradiance (mode analysis)

- The "physical solutions" to (2) are supposed to possess the asymptotic behavior

$$R^{m}_{\ell,\omega} \sim \begin{cases} e^{-i\omega r_*} + \mathcal{R}\, e^{+i\omega r_*} & \text{as } \ r \to \infty \\ \mathcal{T}\, e^{-i(\omega - m\Omega_H) r_*} & \text{as } \ r \to r_+ \end{cases} \tag{3}$$

  - Ω_H: the angular velocity of the BH w.r.t the asymptotically stationary observers
  - with reflection and transmission coefficients, $\mathcal{R}$ and $\mathcal{T}$, respectively.

- **!** (3) presumes the existence of a transmitted wave submerging into the ergoregion.
- By evaluating the Wronskian of the corresponding fundamental solutions, "close" to infinity and "close" to the horizon, it can be shown that

$$(\omega - m\Omega_H)\,\mathcal{T} = (1 - \mathcal{R})\,\omega. \tag{4}$$

- Whenever $\mathcal{R} > 1$—or equivalently whenever the inequality

$$0 < \omega < m\,\Omega_H \tag{5}$$

holds—positive energy is supposed to be acquired by the backscattered scalar mode due to its interaction with the Kerr black hole.

Superradiance (mode analysis)

- It is precisely in the frequency range $0 < \omega < m\,\Omega_H$ where for individual modes the sign of the energy flux through the event horizon is negative.
- An analogous conclusion can be drawn by looking at the "particle number current" in the scalar and electromagnetic wave cases.
- The linear stability problem solved first by Kay and Wald taught us important lessons:
  - **statements at the level of individual modes typically do not imply statements for the superposition of infinitely many modes**

Numerical studies

- We studied the evolution of complex scalar fields on Kerr background
  - **GridRipper (3+1)** is fully spectral in the angular directions while the dynamics in the complementary 1+1 Lorentzian spacetime is followed by making use of a fourth order finite differencing scheme with adaptive mesh refinement (AMR).

The initial data

- To investigate the way an incident scalar wave acquires extra energy by submerging into the ergoregion the solution, in the asymptotic region, was expected to posses the form

$$\Phi(t, r_*, \vartheta, \tilde{\varphi}) \approx e^{-\mathrm{i}\,\omega_0\,(r_* - r_{*0} + t)} f(r_* - r_{*0} + t)\, Y_\ell^m(\vartheta, \tilde{\varphi}). \tag{6}$$

  where $f : \mathbb{R} \to \mathbb{C}$ is a smooth function of compact support and ω_0, r_{*0} are real parameters.

- This, in a sufficiently small neighborhood of the initial data surface in the asymptotic region, may be generated by choosing the initial data as

$$\begin{aligned} \phi(r_*, \vartheta, \tilde{\varphi}) &= e^{-\mathrm{i}\,\omega_0\,(r_* - r_{*0})} f(r_* - r_{*0})\, Y_\ell^m(\vartheta, \tilde{\varphi})\,, \\ \phi_t(r_*, \vartheta, \tilde{\varphi}) &= -\mathrm{i}\,\omega_0\, \phi(r_*, \vartheta, \tilde{\varphi}) + e^{-\mathrm{i}\,\omega_0\,(r_* - r_{*0})}\, f'(r_* - r_{*0})\, Y_\ell^m(\vartheta, \tilde{\varphi})\,, \end{aligned}$$

  where f' denotes the first derivative of $f : \mathbb{R} \to \mathbb{C}$.

- the Fourier transform, $\mathscr{F}\Phi$, of the approximate solution (6) reads as

$$\mathscr{F}\Phi(\omega, r_*, \vartheta, \tilde{\varphi}) \approx e^{-\mathrm{i}\,\omega\,(r_* - r_{*0})} \mathscr{F}f(\omega - \omega_0)\, Y_\ell^m(\vartheta, \tilde{\varphi}), \tag{7}$$

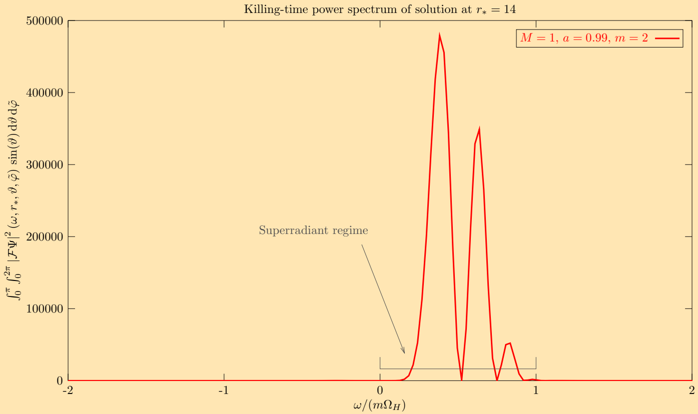
  - ω is the temporal frequency
  - $\mathscr{F}f$ stands for the Fourier-transform of f, playing the role of a frequency profile.

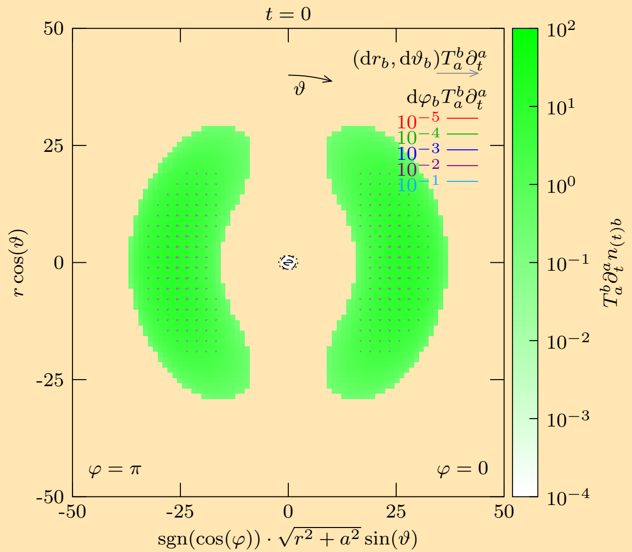
- Assuming that $\mathscr{F}f$ is sufficiently narrow the approximate solution (6) should be close to a monochromatic wave packet.

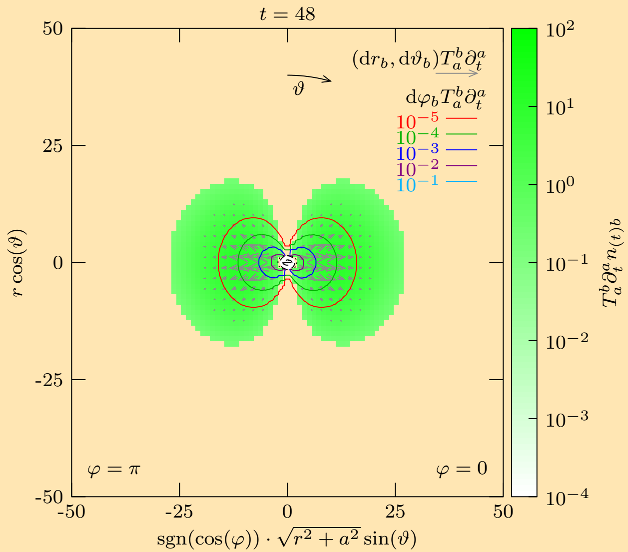
The chosen type of initial data is to be superradiant

- the frequency spectrum of a to be superradiant solution at $r_* = 14$ located towards the black hole with respect to the compact support of the initial data

$M = 1,\ a = 0.99,\ \ell = m = 2,$
$\omega_0 = \frac{1}{2} m \Omega_H,\ r_{*0} = 31.823$
(!pure quadrupole type initial data!)







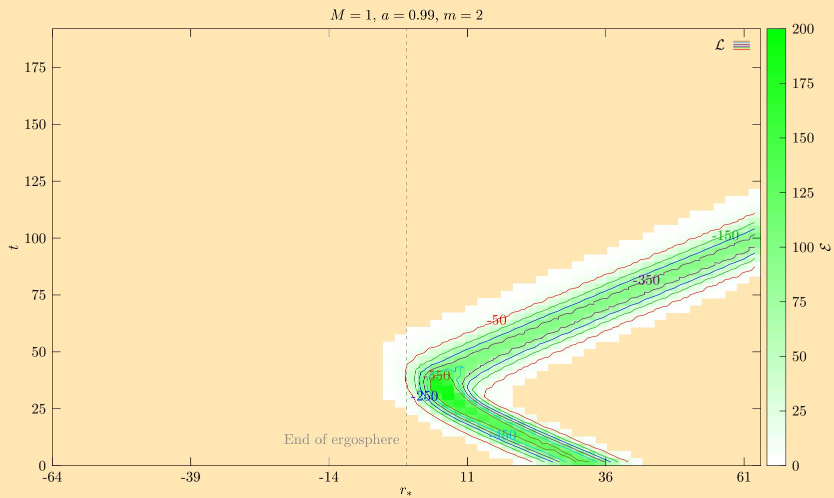
The time dependence of the radial energy and angular momentum distributions & and the power spectrum

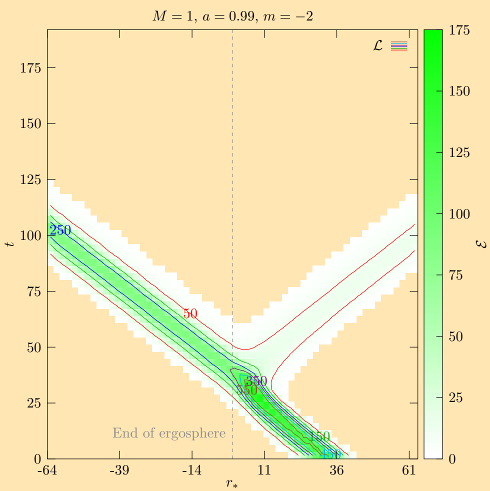
the energy and angular momentum, E and L, on a $t = const$ time level surface can be given as

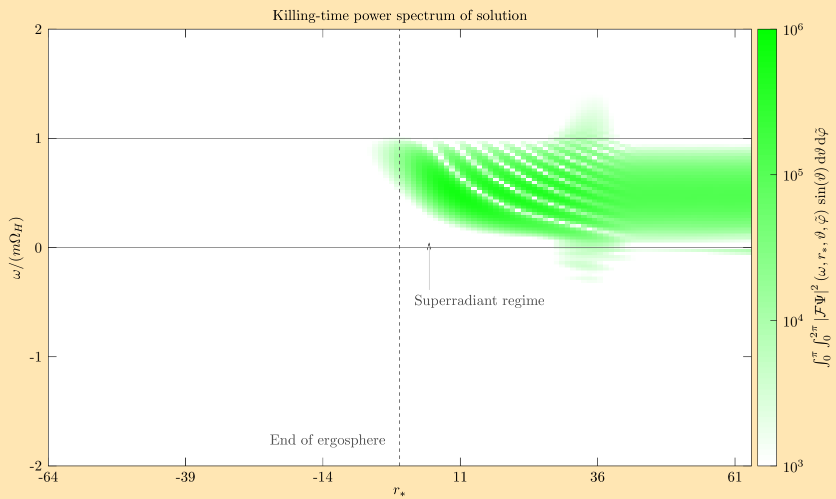
$$E = \int_{t=const} \mathscr{E}\, \mathrm{d}r_*$$

and

$$L = \int_{t=const} \mathscr{L}\, \mathrm{d}r_*$$

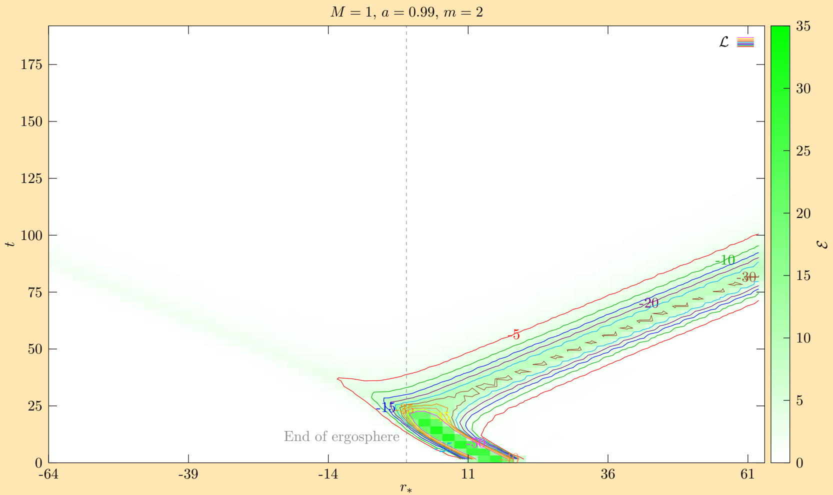


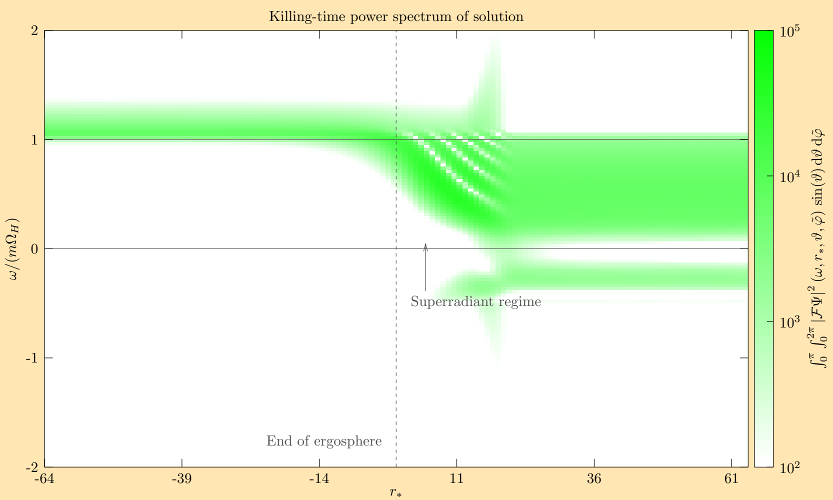




The time dependence of the radial energy and angular momentum distributions & and the power spectrum (2)

For an almost to be superradiant solution





“statements at the level of individual modes typically do not imply statements for the superposition of infinitely many modes”

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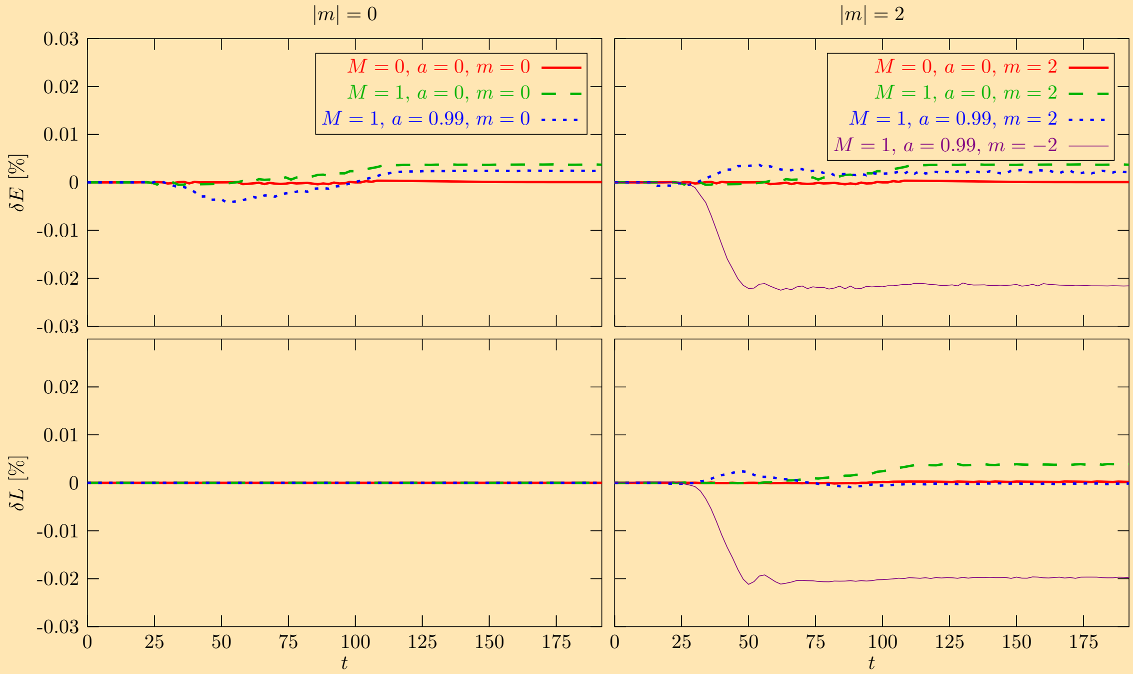

- Time dependence of the relative variation of the energy and angular momentum balances

$$\delta E = \frac{[E(t)+E_{rad}(t)]-E_0}{E_0} \quad \text{and} \quad \delta L = \frac{[L(t)+L_{rad}(t)]-L_0}{L_0}$$

  – E_0 and L_0 are the initial energy and angular momentum, respectively.

Summary

- The evolution of massless scalar field on Kerr background, arising from initial data with compact support in the distant region, was considered.
- The incident wave packet was tuned to maximize the effect of superradiance.
- For perfectly tuned data instead of the occurrence of energy extraction from black hole the inward sent radiation fail to reach the ergoregion rather it suffers total reflection.
- By examining the energy to angular momentum content of the to be superradiant wave packets it is clear that far too much angular momentum is stored by them, $E < \Omega_H L$, which does not allow them to reach the horizon of the black hole.
- This new phenomenon may be considered as the field theoretical analog of the one in Wald's thought experiments demonstrating, in the early 70', that a Kerr black hole does not capture a particle that would cause a violation of the relation $m^2 \geq a^2 + e^2$.
- Our findings do also have implications related to the concept of BH bomb. If superradiance does not occur the solutions to the massive Klein-Gordon equation

$$\boxed{\Box_g \Phi = \mu^2 \Phi}$$

on Kerr background, as opposed to some fashionable speculations, should remain bounded and, in turn, our results support the stability of Kerr black holes.