Effect of magnetic fields on equatorial circular orbits around Kerr spacetimes

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Abstract

In this work we analyze the effects of an external magnetic field on charged particles following equatorial circular orbits around a Kerr spacetime, both in the black hole and the naked singularity cases. Understanding these phenomena is of great importance because equatorial circular orbits are a key ingredient of (simple) accretion disc models. In particular we study two important magnetic field configurations: a) a uniform magnetic field aligned with the angular momentum and b) a dipolar magnetic field. We center our attention on the effect of these external fields on the marginally bound and marginally stable equatorial circular orbits because they are potentially observable quantities that could be useful to determine the nature of the central object. Using a perturbative approach we are able to give analytic results and compare (in the black hole case) with previous results.

1 Introduction

Penrose's Cosmic Censorship Conjecture (CCC) [1] is among the most important open questions of Einstein's General Theory of Relativity, and although great efforts have been made over the past 40 years there is still no definitive answer to whether it is valid or not. In order to shed some light into this question and give indications to whether CCC is to be true or not but not conclusive anwers, many different lines of thought were used.

Even though the great efforts made, there are no conclusive observational evidence of the actual nature of the ultra compact objects, for instance up to date no direct evidence of an event horizon has been found. For this reason finding observable features that could help us to distinguish between black holes and naked singularities should be considered relevant. Any observation in this direction would enhance the current black hole paradigm.

A common feature observed in a wide variety of astrophysical enviorments, particulary near compact objects, is the formation of accretion discs.

The study of the differences between accretion discs formed around black holes and naked singularities is important because it may give us a different observational tool to determine the central object's nature. Circular equatorial orbits are a key ingredient to study more realistic disc models. In this direction the ecuatorial circular orbits in the field of a rotating naked singularity were studied in detail in [2].

The (main) astrophysical relevance of studying the accretion discs formed around ultracompact objects is that they are belived to be engines of the yet not completely understood astronomical phenomena of relativistic jet generation. This kind of extremely energetic phenomena appear on a wide range of object's scale: form AGN's (quasars) to stellar mass black holes or neutron stars (microquasars).

As a possible explanation of the jet formation a disc-jet coupling has been proposed by several authors [3], works trying to explain this unsolved question are to be considered of great importance. Two of the most accepted mechanisms that can explain the energetics involved in a relativistic jet (whose matter can have Lorentz factor grater that 100) are based on rotational energy extracted from the central rotating black hole to form the jet: the Blandford-Znajek process [4] and Penrose's mechanism [5, 6].

Associated with accretion discs and compact objects there are usually observed phenomena related with the precense of magnetic fields. The investigation of the differences between the effects of simple magnetic field configurations on the orbits formed around a black hole and naked singularity could give us mechanisms to distinguish the nature of the central compact object. The effects of magnetic fields on accretion discs around a rotating black hole where studied and the changes in the innermost stable orbit and in the marginally bound orbits is analized in [7], [8]. As we are only able to observe the effect of the presence of a black hole on particles, changes in these particular radii may give observable quantities that could allow us to distinguish between different available theoretical models for compact objects.

In this work were we are going to present some of the results of [9] in which we study the change in the position of the inner edge of an accretion disc in Kerr spacetime generalizing previous results by allowing the rotation parameter a to adopt values larger that 1. We present an analytical study using a perturbative (in the parameter λ that measures the coupling between matter and magnetic field strenght) approach that allow us not only to test previous numerical results but also to reinforce them. This approach to the problem limit our study to small values of λ. As we will see latter, this restriction is not strong as we are interested in studying only the plasma case which is a useful way to model a disc.

2 Stationary axisymmetric electromagnetic fields in Kerr spacetime

Before introducing the electromagnetic field configurations we are going to work with, we present some basic aspects of Kerr's spacetime.

Using Boyer-Lindquist coordinates and metric signature + - - -, the Kerr solution [10] to the vacuum Einstein field equations is expresed as:

$$ds^2 = \frac{\Delta - a^2\sin^2\theta}{\Sigma}dt^2 + 2a\sin^2\theta\frac{r^2+a^2-\Delta}{\Sigma}dtd\phi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 - \left(\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2,$$

where:

$$\Delta = r^2 - 2Mr + a^2 \quad \text{and} \quad \Sigma = r^2 + a^2\cos^2\theta.$$

2.1 Uniform Magnetic Field

The first exact solution for an external electromagnetic field in a Kerr background was found by Wald [11]. In this paper he derived the electromagnetic field of a rotating black hole placed in a magnetic field originally uniform and aligned with the rotation axis in order to preserve the Killing vector fields of the unperturbed background.

Latter Petterson [12] derived explicit expresions for general stationary axisymmetric electromagnetic fields in a Kerr background. The explicit solution given in [12] for the 4-vector potential can be expresed as:

$$A_\mu = (A_t, 0, 0, A_\phi),$$

where:

$$A_\phi = \frac{B\sin^2\theta}{2\Sigma}\left[(r^2+a^2)^2 - \Delta a^2\sin^2\theta - 4Ma^2r\right]$$

where B is the magnetic field strenght.

The divergent behavior of the magnetic vector potential for large values of the radial coordinate r was expected from a physical point of view because the model assumes a constant magnetic field that fills all spacetime, so even without the presence of the massive body the energy at infinity is unbounded.

2.2 Dipolar Magnetic Field

As no magnetic monopole evidence has been found, the dipole magnetic field is the first approximation (in general a good one) to the more complex and realistic intrinsec magnetic field configuration of (astronomical) objects. This fact makes the study of the dipolar configuration a really important one.

Following the results obtained by Petterson in [12] for the particular case of a dipolar magnetic field with no electrostatic charge, we write the non zero components of the 4-vector potential as:

$$A_t = -\frac{3a\mu}{2(1-a^2)\Sigma}\left(\left[r(r-M)+(a^2-Mr)\cos^2\theta\right]\frac{1}{2\sqrt{1-a^2}}\ln\left(\frac{r-M+\sqrt{1-a^2}}{r-M-\sqrt{1-a^2}}\right) - (r-M\cos^2\theta)\right)$$

$$A_\phi = -\frac{3\mu\sin^2\theta}{4(1-a^2)\Sigma}\left((r-M)a^2\cos^2\theta + r(r^2+Mr+2a^2) - \left[r(r^3+a^2r-2Ma^2)+\Delta a^2\cos^2\theta\right]\frac{1}{2\sqrt{1-a^2}}\ln\left(\frac{r-M+\sqrt{1-a^2}}{r-M-\sqrt{1-a^2}}\right)\right)$$

where μ is the dipole moment, considered in our case, to be parallel to the rotation axis.

An important feature of the 4-vector potential is that is singular at the ring singularity. One can extend the analysis done in [7] for the extreme and super-extreme Kerr cases by studying the particular limiting case or the analytic extension of the logaritmic function respectively (see [9] for details).

3 Motion in the equatorial plane

The magnetic fields we are studying preserve the backgruond symmetries as they do not alter the Killing nature of ∂_ϕ and ∂_t.

With this fact we can do an analysis of the motion of charged particles in the equatorial plane using the same arguments used in the pioneering work of Carter [13] for uncharged particles.

The expresions involved are more complicated and the results a little more difficult to interpret. In the following we are going to study some aspects of this particular problem.

We are going to study both Kerr's black hole and Kerr's naked singularity, so we would allow the rotation parameter a to exceed unity. The analysis of the permited range for the λ parameter that messures the coupling between the charge of a volume element and the extenal magnetic field is presented in [8] and [14]. From the bounds they found we conclude that this perturbative analysis we are going to perform is completelly aceptable to study a fluid disc (wich we expect to be electrically neutral over large scales). In this poster we are going to focus our attention on the innermost stable and bound orbits for the co-rotating case.

As we know an exact solution for the non magnetized case we are going to treat perturbatively in the magnetic field using the following approach: expand in a Taylor polynomial the equations that govern the motion of charged particles and use a few steps of the Newton method for finding roots of a trascendental equation to get an exact (up to the perturbation order we are working with) solution.

3.1 Bound Orbits

We are going to analize the changes in the two solutions present in the unmagnetized case studied, for example, in [2]. The radii of these orbits (using the u-radial coordinate *i.e.* $u = 1/r$) are:

$$u_1^0 = \frac{1}{(\sqrt{1+a}-1)^2},$$

$$u_2^0 = \frac{1}{(1+\sqrt{1-a})^2}.$$

As mentioned previously one can reduce (even in the magnetized case) the circular orbit conditions to a quartic equation for x. Intead of numerically solving this equation we find "corrections" starting from the analytic result x^0 of the $\lambda = 0$ case and performing a few steps of a Newton method for findign roots of trascendental equations. After calculating it we have both $x(u; a, \lambda)$ and $E(u, x; a, \lambda)$. Then expanding in a Taylor series the condition for bound orbits, and explicitly using that $E(u^0_{a,b}, x^0; a, 0) = 1$ one gets the following expresion for the corrections in the radii of the two different marginally bound orbits:

$$E(u^0_{1,2}+\epsilon_{1,2}, x^0; a, \lambda=0) + \lambda\frac{dE}{d\lambda}(u^0_{1,2}, x^0; a, \lambda=0) = 1,$$

$$E(u^0_{1,2}, x^0; a, \lambda=0) + \epsilon_{1,2}\frac{dE}{d\epsilon_{1,2}}\bigg|_{\epsilon_{1,2}=0}(u^0_{1,2}, x^0; a, \lambda=0) + \lambda\frac{dE}{d\lambda}(u^0_{1,2}, x^0; a, \lambda=0) = 1$$

from where we can obtain that:

$$\epsilon_{1,2} = -\lambda\frac{dE}{d\lambda}(u^0_{1,2}, x^0; a, \lambda=0)\left(\frac{dE}{d\epsilon_{1,2}}\bigg|_{\epsilon_{1,2}=0}(u^0_{1,2}, x^0; a, \lambda=0)\right)^{-1}. \quad (1)$$

3.2 Innermost Stable Orbit

The analysis is virtually the same for this case, using the exact radius for the innermost stable orbit available for the unmagnetized case. The expresion for $r^0_{st} = 1/u^0_{st}$ is given by the following expresion:

$$r^0_{st} = 3 + Z_2 - \sqrt{(3-Z_1)(3+Z_1+2Z_2)}$$

where

$$Z_1 = 1 + (1-a^2)^{1/3}\left[(1+a)^{1/3} + (1-a)^{1/3}\right], \quad Z_2 = \sqrt{3a^2 + Z_1^2}.$$

The main difference is that the linear (in λ) correction is null, because $\frac{dE}{d\lambda}(u^0_{st}, x^0; a, \lambda = 0) = 0$, so the first non zero correction to the position of the innermost stable orbit due to the presence of an uniform magnetic field is cuadratic in λ and its expresion is given by:

$$\epsilon_{st} = -\frac{1}{2}\lambda^2\frac{d^2E}{d\lambda^2}(u^0_{st}, x^0; a, \lambda=0)\left(\frac{dE}{d\epsilon_{st}}\bigg|_{\epsilon_{st}=0}(u^0_{st}, x^0; a, \lambda=0)\right)^{-1}. \quad (2)$$

4 Results

Using expresions (1) and (2) we can write down analytic expresions for the important radii we are studying. We present our results in the following Figures.

4.1 Uniform Magnetic Field

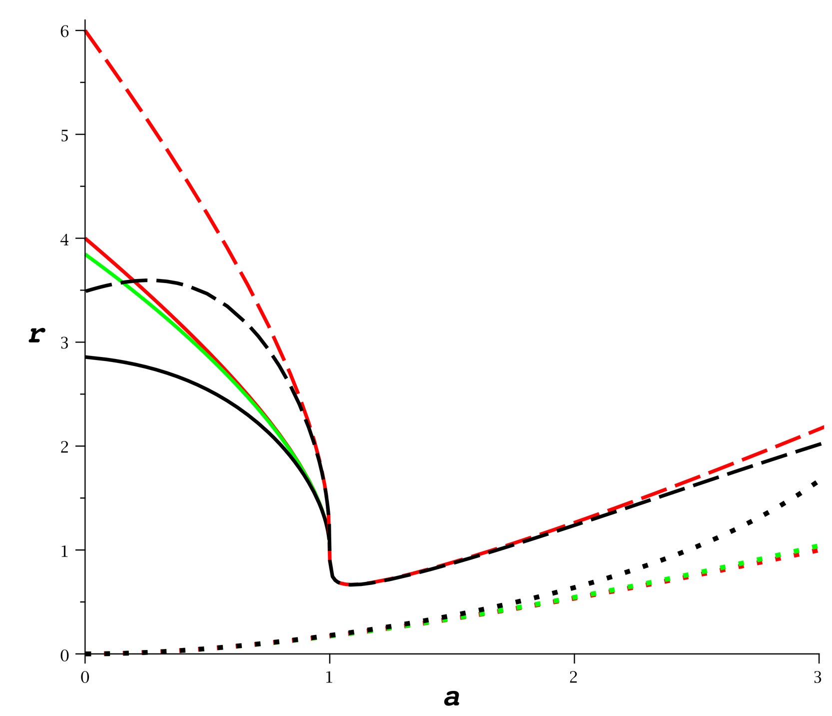

Figure 1: We present the radii of the marginally bound orbit and of the innermost stable ones. Equal colors mean equal values of the parameter λ: red for $\lambda = 0$, green for $\lambda = 0.01$ and black for $\lambda = 0.1$. The solid and dotted lines correspond to the marginally bound orbits and the dashed ones to the innermost stable one. Our results are in complete agreement with those of [2] and [8] for black holes.

4.2 Dipolar Magnetic Field

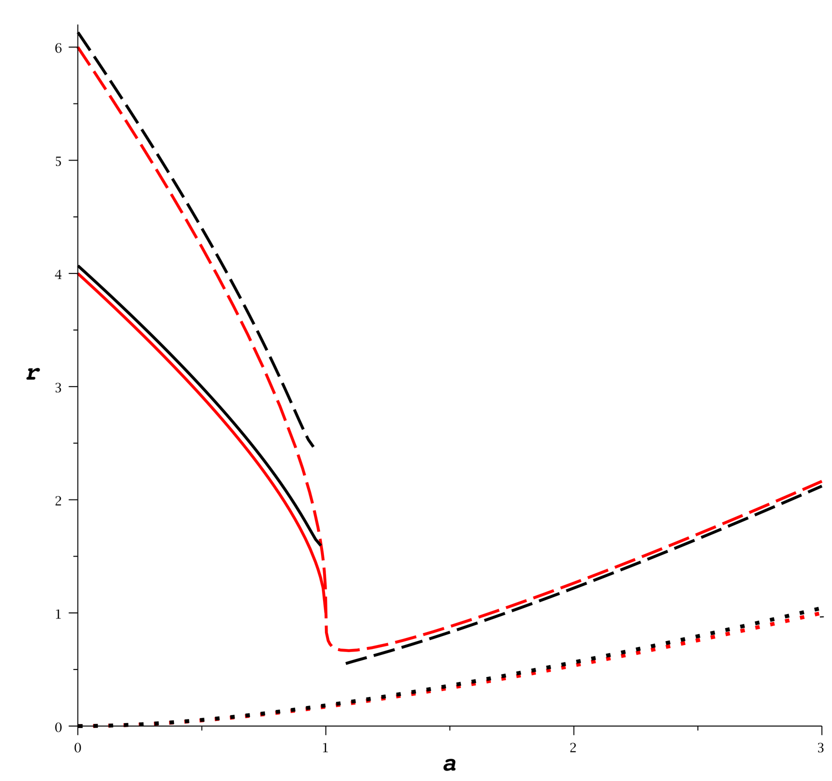

Figure 2: We present the radii of the marginally bound orbit and of the innermost stable ones. Our results are in agreement with the obtained for a black hole in [14]. The line styles and colours correspond to the ones explained in Figure 1.

The effect of the two magnetic field configurations is cualitatively different as the uniform magnetic field decreases the values of the important raddi for a given a whearas the dipolar one produces the oposite effect.

We conclude that observations of the disc structure around compact objects might be used to determine the nature of the central compact object as our results suggest that external magnetic field affect these discs differently.

References

[1] R. Penrose, "The Question of Cosmic Censorship", in R.M. Wald (ed), "Black Holes and Relativistic Stars", pp. 103-122. Chicago, IL: University of Chicago Press, 1998.

[2] Z. Stuchlík, Bull. Astron. Inst. Czechosl. **31**, 129, (1980); for a recent review see D. Pugliese, H. Quevedo and R. Ruffini, Physical Review D, vol. 84, Issue 4, id. 044030, (2011).

[3] I.F. Mirabel and L.F. Rodriguez, Nature, **371**, 46, (1994); R.D. Blandford and D.G. Payne, Mon, Not. R. astr. Soc., **199**, 883, (1982); H. Falcke and P.L. Biermann, Astron. Astrophys. **342**, 49, (1999); R. Fender and T. Belloni, Annu. Rev. Astron. Astrophys., **42**, 317, (2004) and references therein.

[4] R.D. Blandford, and R.L. Znajek, Monthly Notices of the Royal Astronomical Society, 179, 433, (1977).

[5] R. Penrose, Nuovo Cimento Rivista, Numero Speciale 1, 252-276, (1969).

[6] R.K. Williams, Physical Review 51 (10): 5387âĂ§5427, (1995); R.K. Williams, The Astrophysical Journal, 611, 952-963, (2004).

[7] A.R. Prasanna and C.V. Vishveshwara, Pramana, **11**, 359, (1978).

[8] P.J Wiita, C.V. Vishveshwara, M.J. Siah and B.R. Iyer, J.Phys.A: Math. Gen., **16**, 2077, (1983).

[9] I.F. Ranea-Sandoval and H. Vucetich (in preparation).

[10] Kerr, R. P., *Phys. Rev. Lett* **11**, 237 (1963).

[11] R.M. Wald, Phys. Rev. D, **10**, 1680, (1974).

[12] J.A. Petterson, Phys. Rev. D, **12**, 2218, (1975).

[13] B. Carter, Phys. Rev. **175**, 1559, (1968).

[14] B.R. Iyer, C.V. Vishveshwara, P.J. Wiita and J.J Goldstein, Pramana-J.Phys., Vol. 25, No. 2, pp. 135-148, (1985).