Probability Distributions for Quantum Stress Tensors in 2D and 4D

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(PRD 81:121901, 2010; and PRD to be published)

- energy density in the vacuum state fluctuates
- both + and - energy fluctuations
- goal: to gain insight into the probability distributions from calculation of (65) moments

$$a_n = \int_{-x_0}^{\infty} x^n \, P(x) \, dx$$

Quantum Inequality

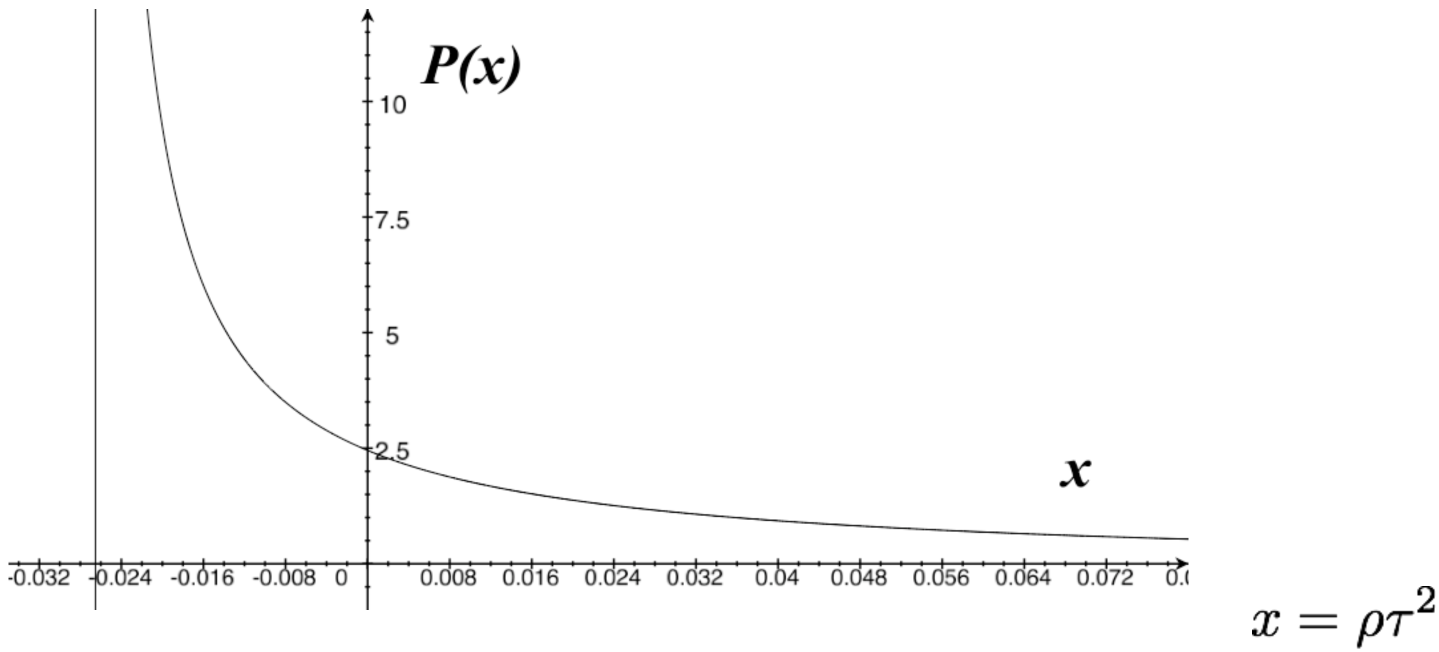
$$\int_{-\infty}^{\infty} f(t)\,\langle T_{\mu\nu} u^{\mu} u^{\nu} \rangle\, dt \geq -\frac{C}{\tau^{d}}$$

- d is the spacetime dimension
- $f(t)$ is a sampling function of width τ
- where $C \ll 1$
- holds for *any* (reasonable) quantum state and sampling function

2D Results

- Gaussian sampled energy density for massless scalar field
- found the exact analytic unique PD (shifted Gamma distribution)
- exact match between this distribution and 65 computed moments

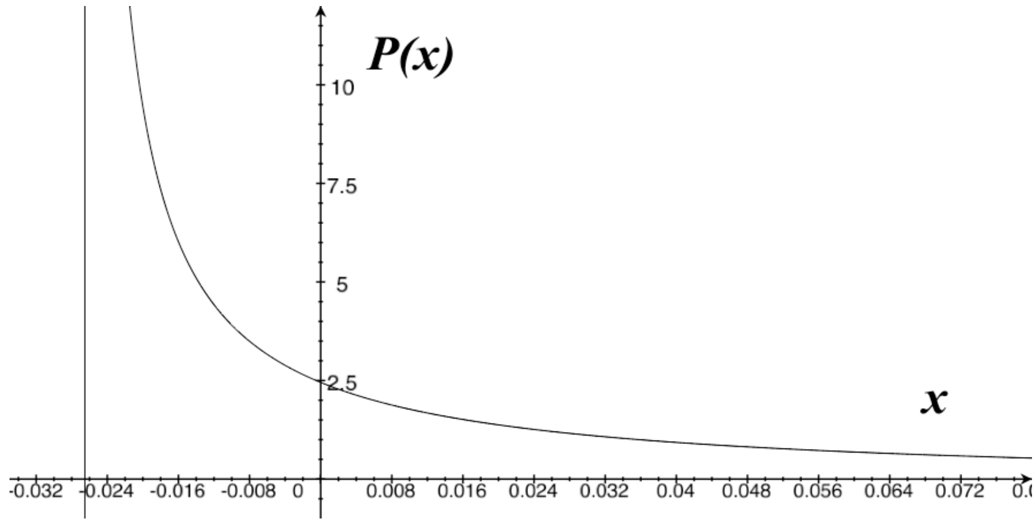
The 2D Probability distribution



-deep connection between QIs and probability distribution

- distribution spikes at Flanagan's optimal QI bound

- probability of getting a negative measurement is 84% !

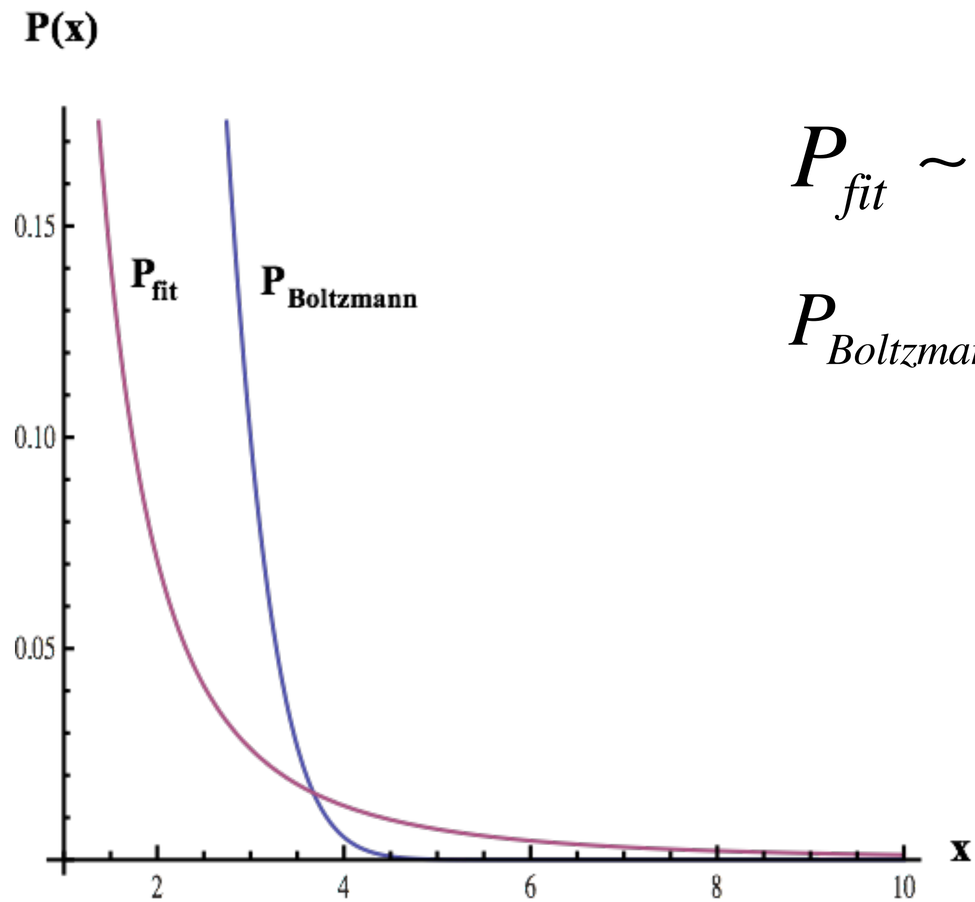


- lower bound but no upper bound
- no large negative energy fluctuations
- the mean value (first moment) of the vacuum distribution is zero
- small frequent negative fluctuations cancelled by large but rarer positive fluctuations

4D Results

- Lorentzian sampled energy density for massless scalar and EM fields
- not clear whether it is possible to *uniquely* determine the distribution from moments alone (we suspect that one cannot)
- lower bound but no upper bound (as in 2D)
- possible to give approx lower bounds and asymptotic tails
- no distribution can have a tail which decreases faster than ours
- rough fit to intermediate distribution

vacuum fluctuations outweigh thermal fluctuations at high energies



$$P_{fit} \sim e^{-ax^{1/3}}$$

$$P_{Boltzmann} \sim e^{-\beta x}$$

Applications – nucleation of black holes, Boltzmann brains
(these depend only on the asymptotic form of the tail)

Summary - 2D

- PD for energy density fluctuations in 2D is unique
- lower bound gives *optimal* QI bound
- PD has lower bound but no upper bound
- frequent but small negative fluctuations; large but rare positive fluctuations

Summary - 4D

- similar behavior, i.e., lower bound but no upper bound
- numerical estimates of lower bound and asymptotic form of the tail
- vacuum fluctuations dominate thermal fluctuations at high energies
- not clear whether it is possible to *uniquely* determine the distribution from moments alone (we suspect that one cannot)

Further Work

- settle the issue of whether the moments determine the probability distribution (we conjecture that they don't)
- understand better what information about the distribution can be obtained nonetheless
- determine what the optimal QI bound actually is in 4D
- more general sampling functions?