Increase of black hole entropy in Lanczos Lovelock gravity

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General Relativity:

Einstein Hilbert action:

$$A = \frac{1}{16\pi} \int_M d^D X \sqrt{-g}\; R\left(g_{ab}, \partial_a g_{bc}, \partial_a \partial_b g_{cd}\right)$$

Eq. of motion are 2nd order partial hyperbolic diff. equations.

GR is perturbatively non-renormalizable, may make sense as an effective theory working perturbatively in the powers of a dimensionless small parameter *G (Energy)*$^{D-2}$

The presence of higher curvature terms from loop corrections is presumably inevitable.

The general form is:

$$A = \frac{1}{16\pi} \int_M d^D X \sqrt{-g} \left[R + \alpha \, O(R^2) + \beta \, O(R^3) + \ldots \right]$$

What kind of higher curvature terms you prefer?

EOM of a gravity theory:

$$A_{ab}\left(g, R, \nabla R, \ldots\right) = \kappa T_{ab}; \qquad \nabla^a A_{ab} = 0$$

Q. Whether all such eq. of motion are derivable from a Lagrangian?
Ans: Don’t know!!

Add a new condition, eq. of motion is second order just like Einstein’s equation, then answer is unique.
Lovelock Lagrangian.

D. Lovelock. J. Math. Phys. 1971

$$L_m^{(D)} = \frac{1}{16\pi} 2^{-m} \delta^{a_1 a_2 \ldots a_{2m}}_{b_1 b_2 \ldots b_{2m}} R^{b_1 b_2}_{a_1 a_2} \ldots R^{b_{2m-1} b_{2m}}_{a_{2m-1} a_{2m}}$$

m = 1 : $L_1^{(D)} = \frac{R}{16\pi}$ **m = 2 :** $L_2^{(D)} = \frac{1}{16\pi}\left(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\right)$

For m-th order Lovelock term to contribute: D ≥2m+1

Gauss Bonnet term is non trivial from D = 5

Lovelock eoms are the most general second rank, divergence free tensors, constructed from metric and curvatures, which contains not more than second derivative of the metric.

A natural generalization of Einstein tensor in higher dimensions.

Lovelock is a LOVELY theory……..

 # Lovelock gravity is unique (perturbative) ghost free theory of gravity . *Zwiebach 1986, Gross-Witten 1986.*

 # Eq. of motion is well-defined second order differential eqns. Initial value formalism is well-defined.

 # Explicit black hole solutions are known.

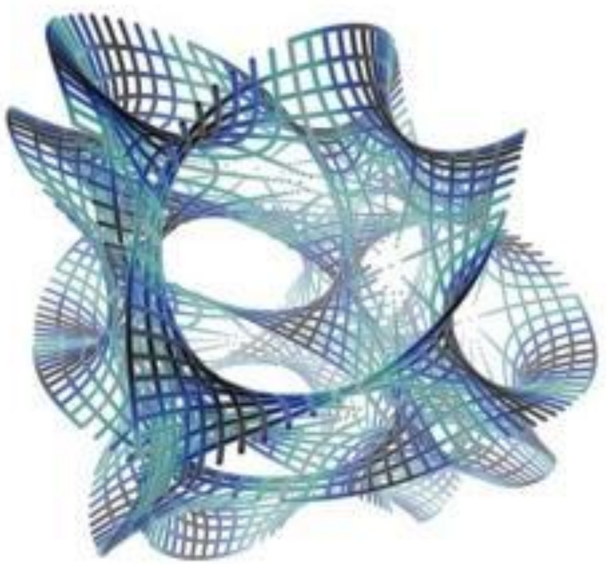
Boulware, Deser, PRL, 1986.

 # At least in the context of hetoretic string theory, GB term arises as a quantum correction to GR. The microscopic calculations produces correct Wald entropy even up to the right factors.

Cardoso et. al. PLB 1998, A Sen, JHEP, 2006

Extension of the laws of black hole mechanics for Lanczos Lovelock Gravity

An important question for people who live in the higher dimension!!



Zeroth Law:

Surface gravity is constant on a black hole horizon in GR provided dominant energy condition holds.
Bender et. al. (1974)

Surface gravity is constant on the horizon of a static black hole and stationary axisymmetric black hole with t-phi reflection Symmetry. *Racz & Wald, CQG (1995)*

Surface gravity is constant on a Killing horizon in Lovelock Gravity provided dominant energy condition holds.
***SS**, S Bhattacharya (2012),* arXiv:1205.2042.

First Law: (Equilibrium state version)

For any diff. invariant Lagrangian, it is possible to show that for a stationary black hole: *Wald 1993, Wald and Iyer, 1994.*

$$\frac{\kappa}{2\pi}\delta S = \delta M - \Omega_H \delta J$$

$$S = 2\pi \int_C X^{abcd} \varepsilon_{ab}\varepsilon_{cd} \; ; X^{abcd} = \frac{\partial L}{\partial R_{abcd}}; \quad \varepsilon_{ab} = k_{[a} l_{b]}$$

In GR, $L = \frac{R}{16\pi}, \quad X^{abcd} = \frac{1}{32\pi}\left(g^{ac}g^{bd} - g^{ad}g^{bc}\right) \qquad S = \frac{1}{4}\int_C \sqrt{\sigma}\, d^{D-2}x = \frac{A}{4}$

In general, *entropy is no longer proportional to area*.

Physical process version:

How the area of a black hole changes when one throws matter in to the black hole. *Wald (1994), Jacobson et. al. (2001)*

$$\frac{\kappa}{2\pi}\delta\left(\frac{A}{4}\right)=\int_H T_{ab}\xi^a d\Sigma^b$$

; A local version of BH mechanics.

Generalization to Einstein Gauss Bonnet gravity:

*A Chatterjee, **SS** (PRL, 2011)*

$$\frac{\kappa}{2\pi}\delta S_W=\int_H T_{ab}\xi^a d\Sigma^b$$

The Wald entropy is an unambiguous notion of horizon entropy for stationary Killing horizons.

Does this entropy obeys a second law?

(Classical) **Second Law in GR** *(Hawking's area theorem)* **:**

Horizon cross-sectional area cannot decrease in any classical process, provided Einstein's equation together with the null energy condition hold.

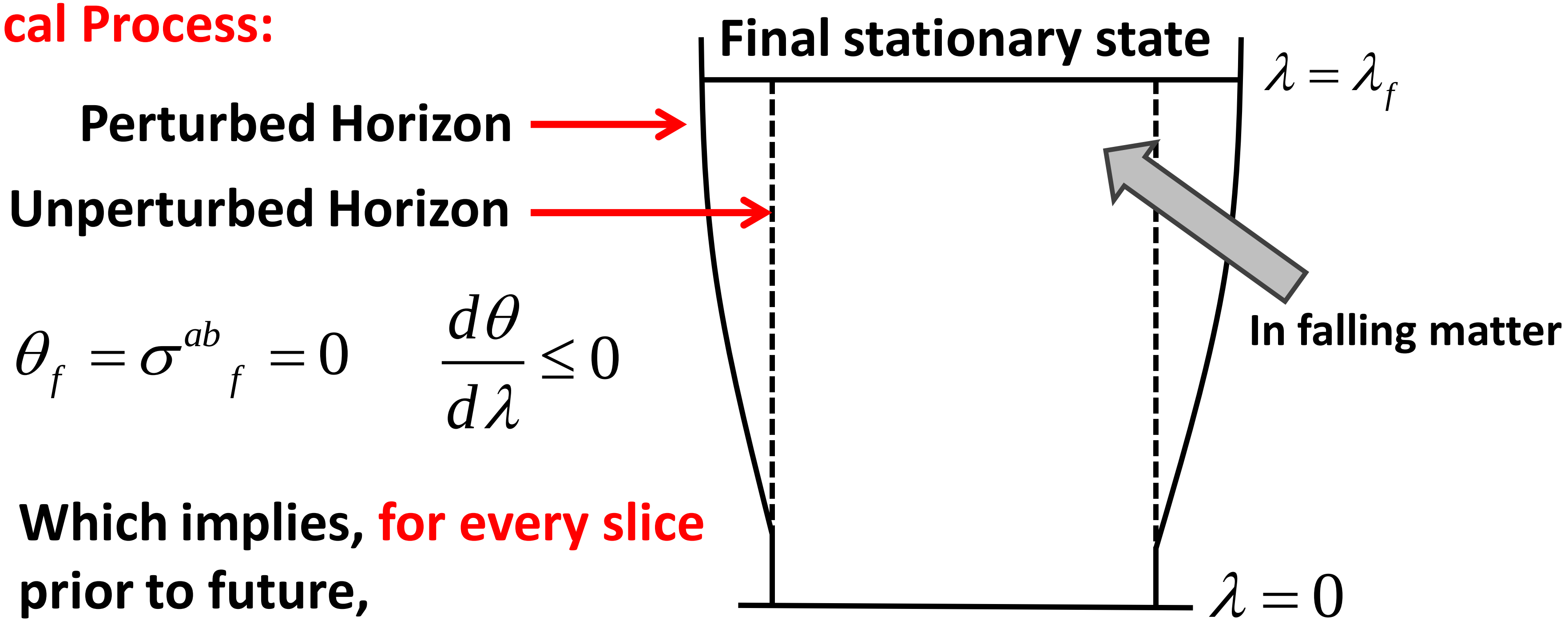
A linearized proof of area theorem:

Raychaudhuri eq. $$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{(D-2)} - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$$

We assume that the perturbation of the horizon are ``small"

$$\frac{d\theta}{d\lambda} \approx -R_{ab}k^a k^b = -T_{ab}k^a k^b \leq 0$$

Physical Process:



$$\theta_f = \sigma^{ab}{}_f = 0 \qquad \frac{d\theta}{d\lambda} \leq 0$$

Which implies, for every slice prior to future,

$$\theta(\lambda) \geq 0 \Rightarrow \frac{dA}{d\lambda} \geq 0$$

We like to proof this for Lovelock gravity, as an illustration

Let us concentrate first on m=2, Einstein Gauss Bonnet gravity:

Theory: $$A=\int d^D X\sqrt{-g}\left[\frac{1}{16\pi}\left(R+\alpha L_{GB}\right)\right];$$

EOM is: $$G_{ab}+\alpha\left(H_{ab}-\frac{1}{2}g_{ab}L_{GB}\right)=8\pi T_{ab}$$

$$H_{ab}=2\left(RR_{ab}-2R_{ac}R_b^c-2R^{cd}R_{acbd}+R_a^{cde}R_{bcde}\right)$$

Entropy candidate: $$S=\int_C \rho\, dA;\quad \rho=\frac{1}{4}\left(1+2\alpha\,{}^{D-2}R\right)$$

Prove that, this entropy always increases for small perturbations as long as NEC holds.

$$\Delta S=\int_C\left(\rho\theta+\frac{d\rho}{d\lambda}\right)dA$$

Define: $$\Theta=\frac{d\rho}{d\lambda}+\theta\,\rho$$

Note: $$\Theta_f=0$$

$$\frac{d\Theta}{d\lambda} = -T_{ab}k^a k^b + \Re_{ab}k^a k^b + \mathrm{O}(\varepsilon^2)$$

$$\Re_{ab}k^a k^b = \left(H_{ab} - 2\,{}^{D-2}R\,R_{ab} + 2\nabla_a \nabla_b\,{}^{D-2}R\right)k^a k^b$$

Note that, we only need to evaluate first order departures from the background space time.

Perturbation Scheme: $XY \approx X^{(B)}Y^{(B)} + X^{(B)}Y^{(P)} + X^{(P)}Y^{(B)}$

Let us concentrate on the remaining terms:

$$H_{ab} - 2\,{}^{D-2}R\,R_{ab} = 2\,{}^{D-2}R^{(B)ab}R^{(P)}_{acbd}k^c k^d$$

$$k^a \nabla_a\,{}^{D-2}R = Lie_k\left({}^{D-2}R\right) = {}^{D-2}R_{ab}Lie_k\left(\gamma^{ab}\right) + D_a\left(\delta v^a\right)$$

$$k^a k^b \nabla_a \nabla_b\,{}^{D-2}R = {}^{D-2}R^{(B)ab}R^{(P)}_{acbd}k^c k^d$$

$$\Re_{ab}k^a k^b = \left(H_{ab} - 2\,{}^{D-2}R\,R_{ab} + 2\nabla_a\nabla_b\,{}^{D-2}R\right)k^a k^b = \mathrm{O}\left(\varepsilon^2\right)$$

Hence, for small perturbations:

$$\frac{d\Theta}{d\lambda} = -T_{ab}k^a k^b \leq 0 \text{ and } \Theta_f = 0$$

Which implies, on every slice prior to future,

$$\Theta(\lambda) \geq 0 \Rightarrow \frac{dS}{d\lambda} \geq 0$$

The proof can be extended to all Lovelock terms in any dimensions.

arXiv:1201.2947, PRD, Rapid Communications.

In a physical process, the entropy for Lovelock black holes Increases as long as matter obeys null energy condition.

Going beyond small perturbation assumption?

It depends on the signs of the higher order terms.

$$\frac{d\Theta}{d\lambda} = -T_{ab}k^a k^b + O\left(\varepsilon^2\right)$$

GR result: $$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{(D-2)} - \sigma_{ab}\sigma^{ab} - T_{ab}k^a k^b \leq 0$$

We need to calculate higher order terms: A thermodynamic generalization of Raychaudhuri equation.

(SS, Sanved Kolekar, in progress)

Open problems:

1. Extend the result beyond linear perturbations.

2. Study uniqueness theorems for black holes in Lovelock gravity (at least for static case).

3. Possible topologies of black holes. Are they constrained as in the case of GR .

4. (Most important) Establish positive mass theorem.

Thanks