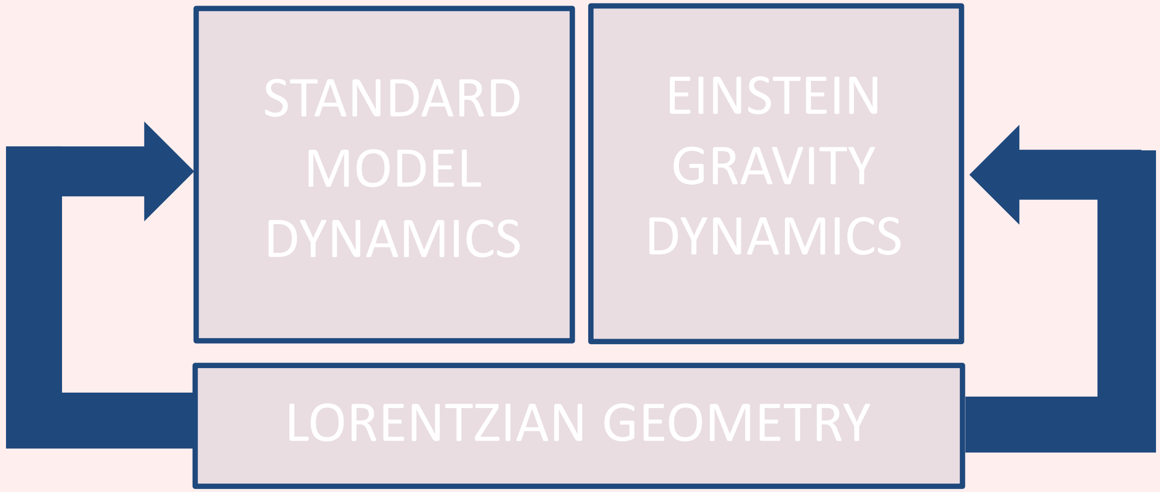
Geometrodynamics beyond Einstein



Frederic P. Schuller
Max Planck Institute for Gravitational Physics

fps@aei.mpg.de

Instead, consider geometries

$$(M, G)$$

smooth manifold → M

any tensor field → G

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smooth manifold → M ; any tensor field → G

Can such geometries provide spacetime kinematics?

Can one infer their gravitational dynamics?

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Can such geometries provide spacetime kinematics?

Yes, but not all.

Predictivity and quantizability of matter dynamics
impose powerful restrictions on geometry .

Can one infer their gravitational dynamics?

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Can such geometries provide spacetime kinematics?

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Predictivity and quantizability of matter dynamics impose powerful restrictions on geometry .

Can one infer their gravitational dynamics?

Yes.

Dynamics determined for all restricted geometries.

Matter dictates conditions on geometry

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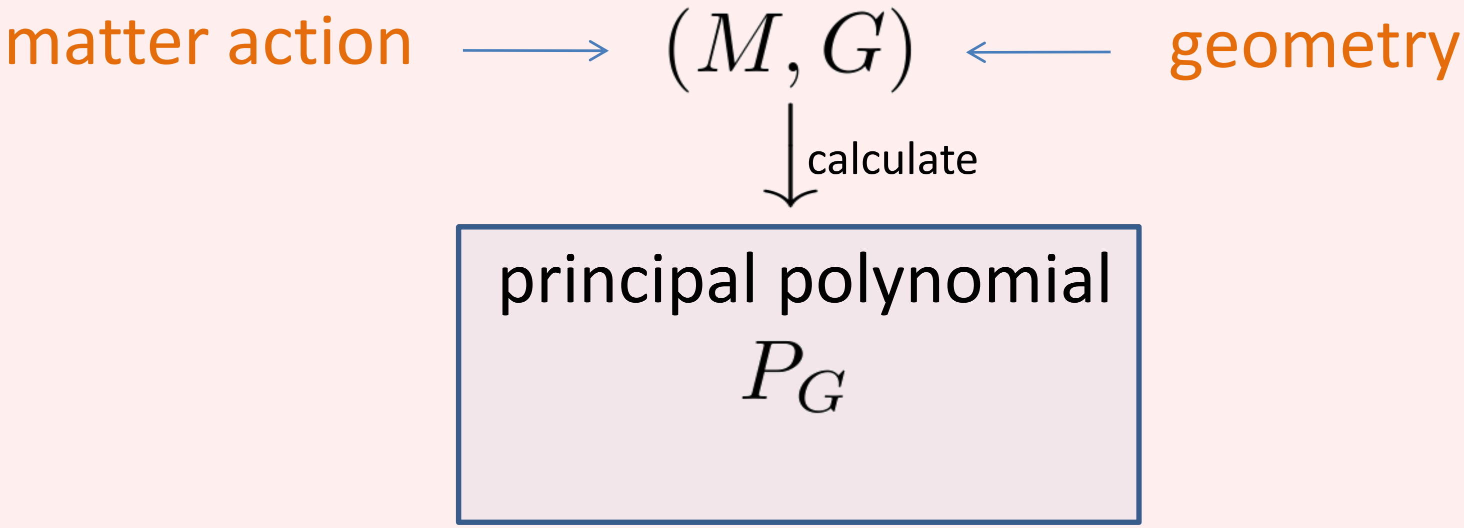
matter action $\longrightarrow$ (M, G) $\longleftarrow$ geometry

Matter dictates conditions on geometry



Matter dictates conditions on geometry

matter action → (M, G) ← geometry

↓ calculate

principal polynomial

P_G

predictive → hyperbolic

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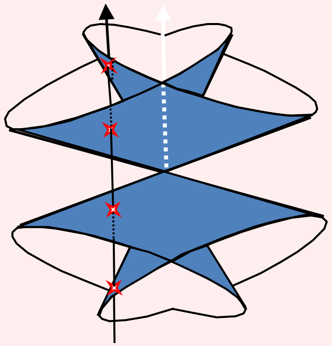

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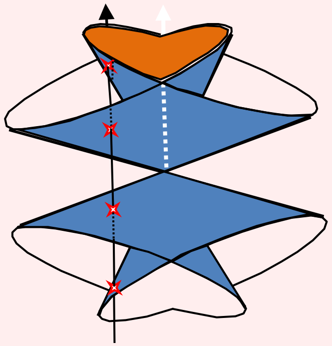

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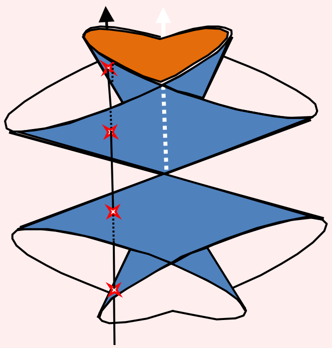

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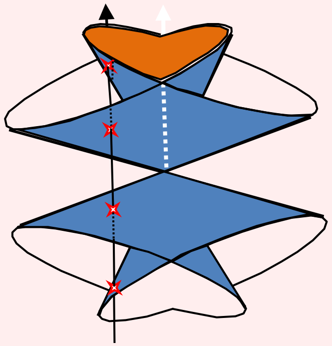

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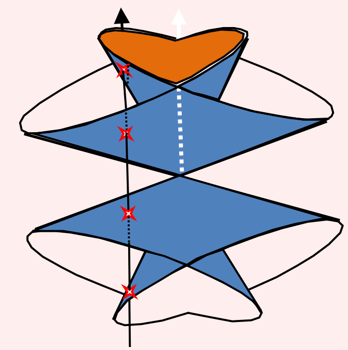

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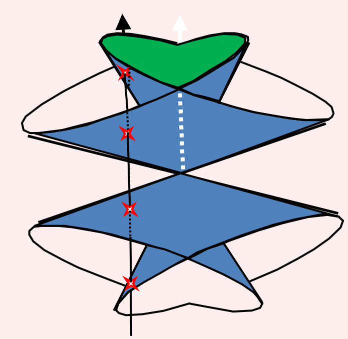

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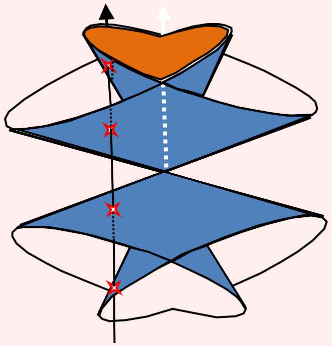

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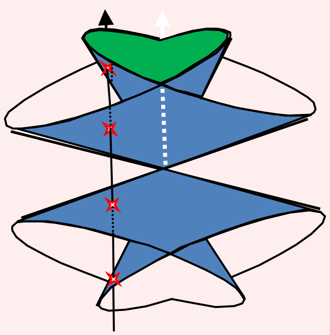

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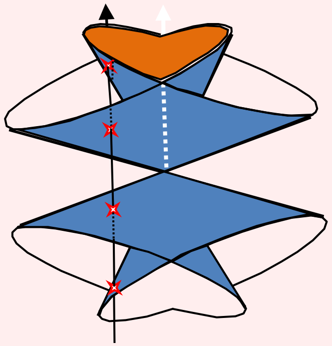

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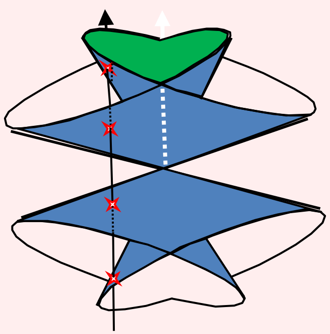

Geometry must be bi-hyperbolic to carry matter.

Example One

Maxwell theory $\longrightarrow$ $(M, G^{(\cdot\cdot)})$ $\longleftarrow$ metric geometry

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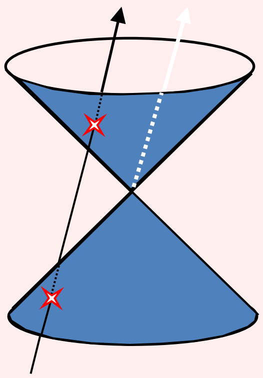
principal polynomial

$$P_G{}^{ab} = g^{ab}$$



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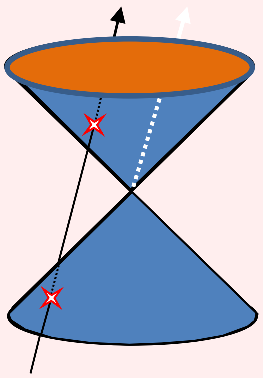

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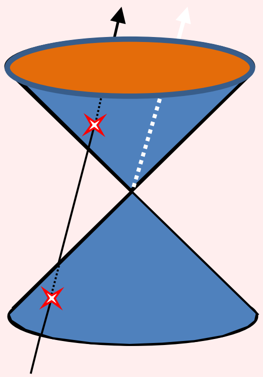

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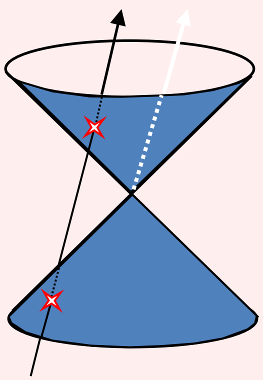

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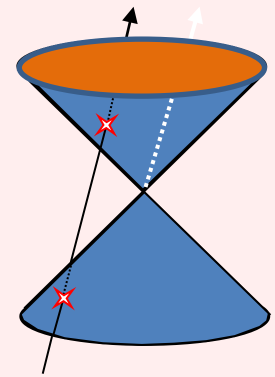

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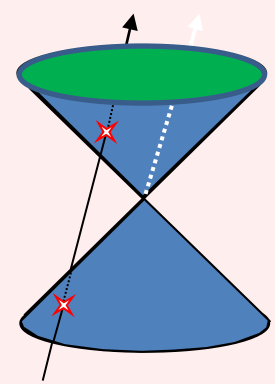

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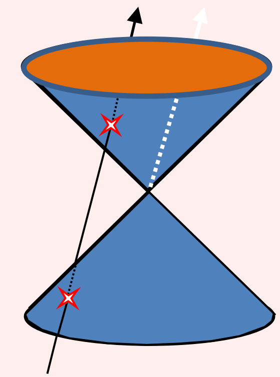

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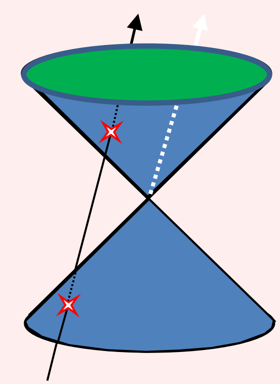

Metric must be of special algebraic class to carry Maxwell (Lorentzian)

Example Two

Maxwell theory $\longrightarrow (M, G^{[\cdot\cdot][\cdot\cdot]}) \longleftarrow$ area metric geometry

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$$P_G{}^{abcd} = \epsilon_{....}\epsilon_{....}G^{\cdots(a}G^{b|\cdot\cdot|c}G^{d)\cdots}$$

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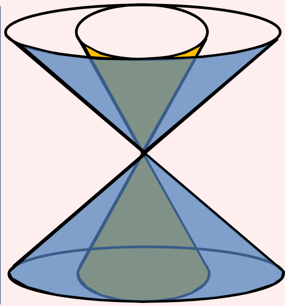

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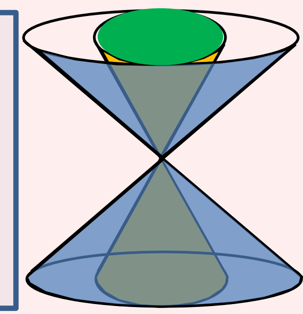

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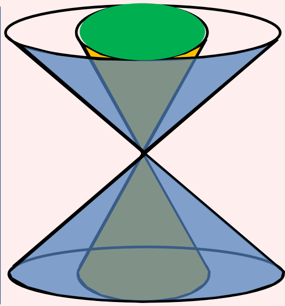

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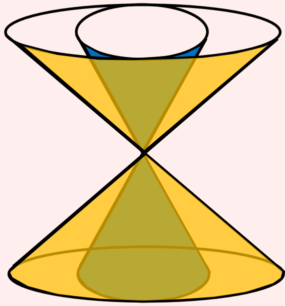

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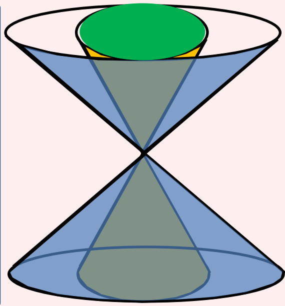

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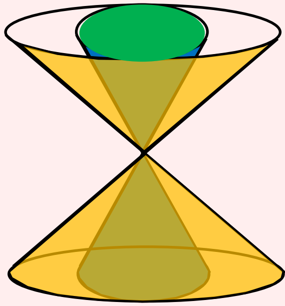

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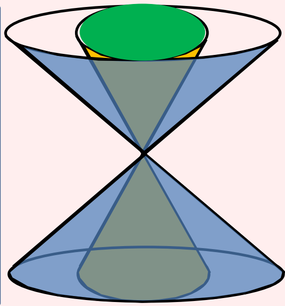

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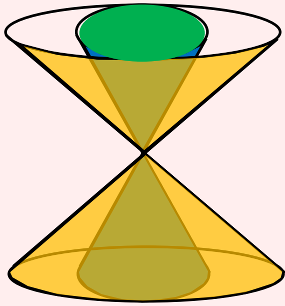


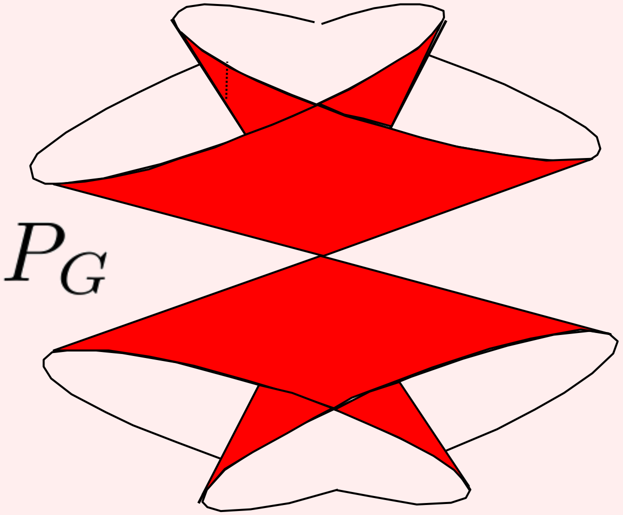
Area metric must be of special algebraic class to carry Maxwell
(6 out of 23)

Bi-hyperbolic spacetime kinematics

cotangent space $\rightleftarrows$ tangent space

Bi-hyperbolic spacetime kinematics

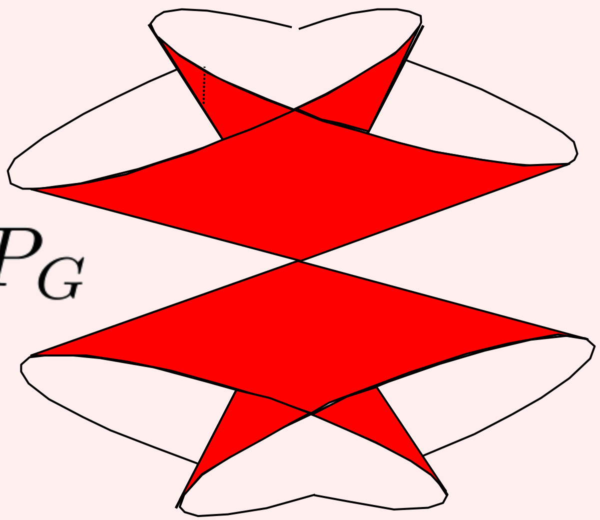
massless
momenta

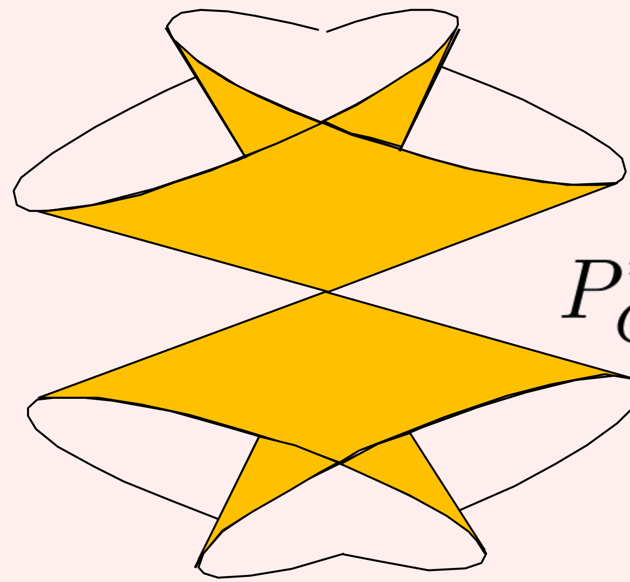


Bi-hyperbolic spacetime kinematics

massless momenta $\longmapsto$ light velocities

P_G





$P_G^{\#}$

Bi-hyperbolic spacetime kinematics

massless momenta ⟼ light velocities

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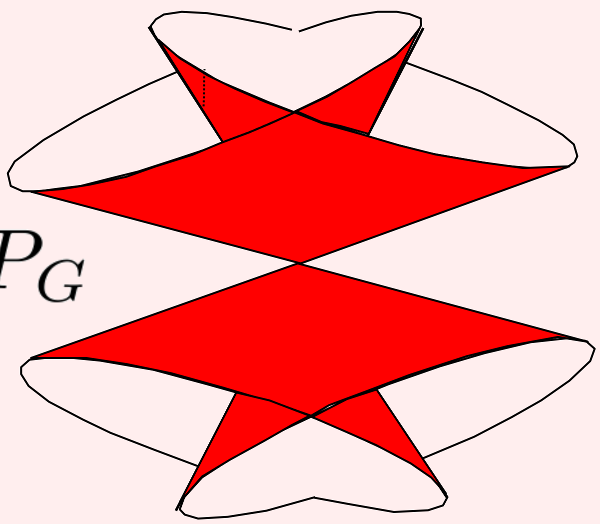
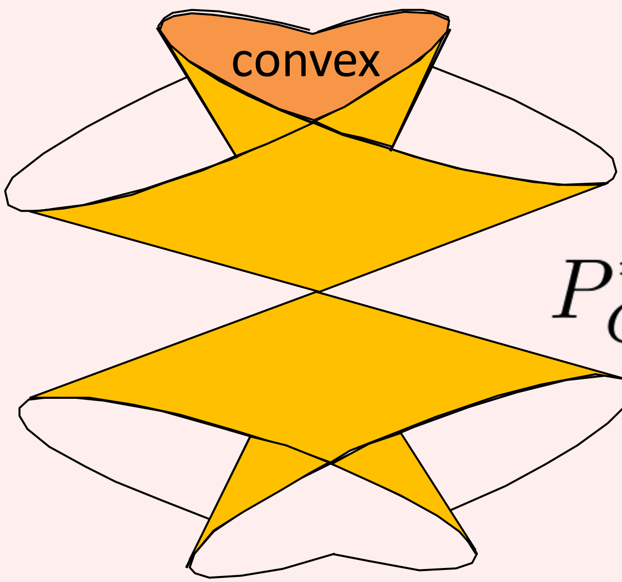


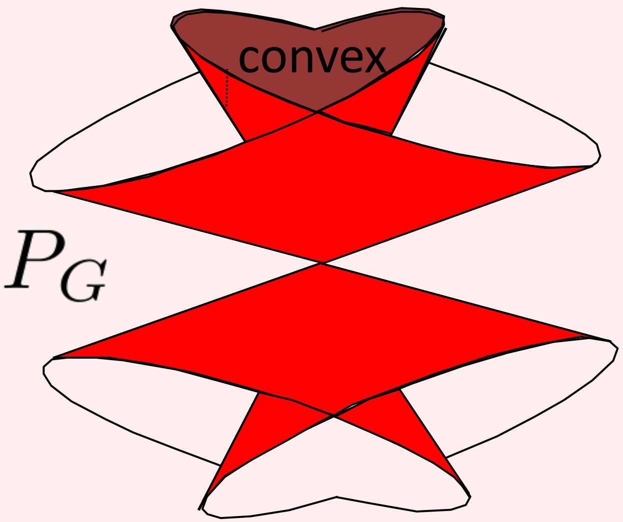


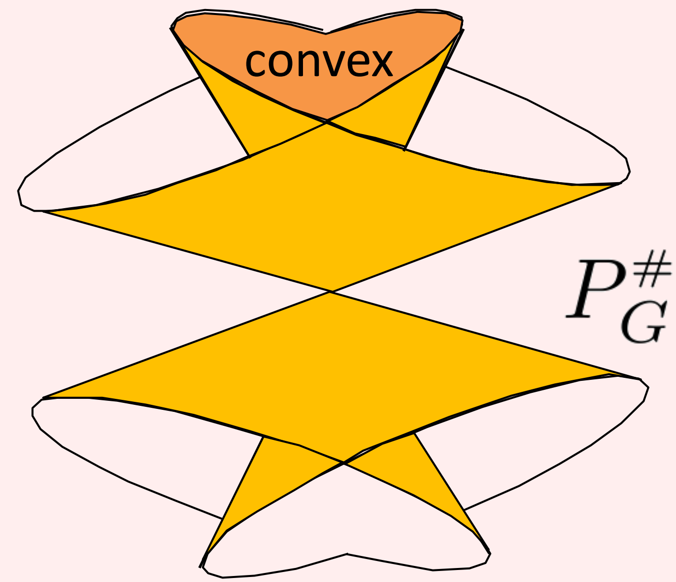
$P_G^{\#}$

observer velocities

Bi-hyperbolic spacetime kinematics

massless momenta ⟼ light velocities





stable massive momenta ⟻ observer velocities

Bi-hyperbolic spacetime kinematics

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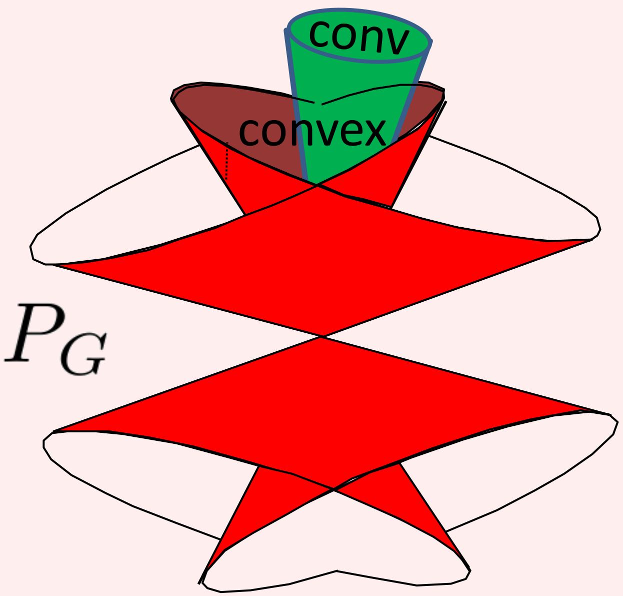

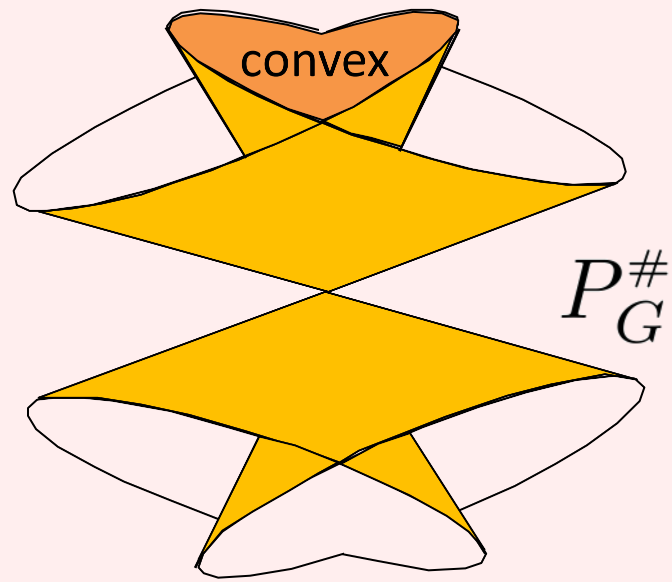

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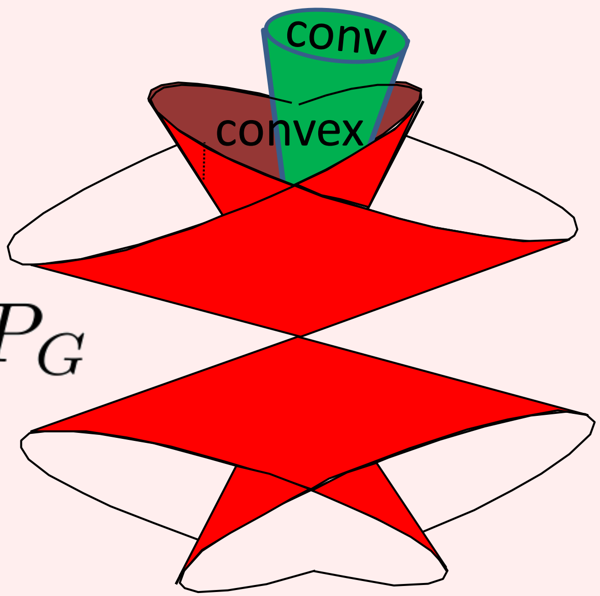
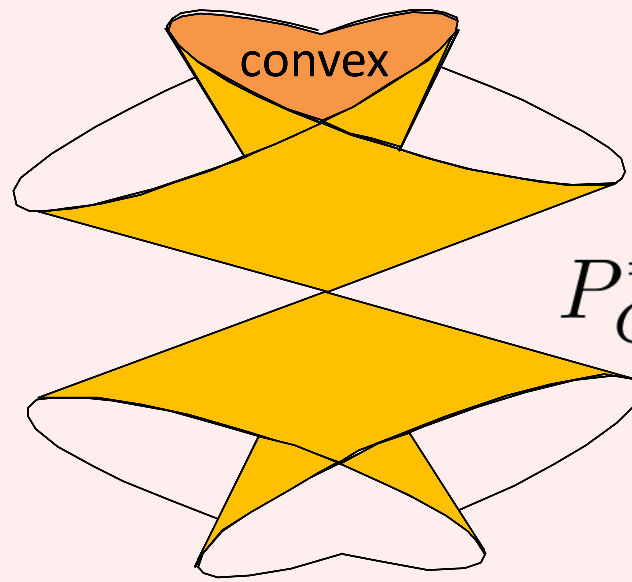




$P_G^{\#}$

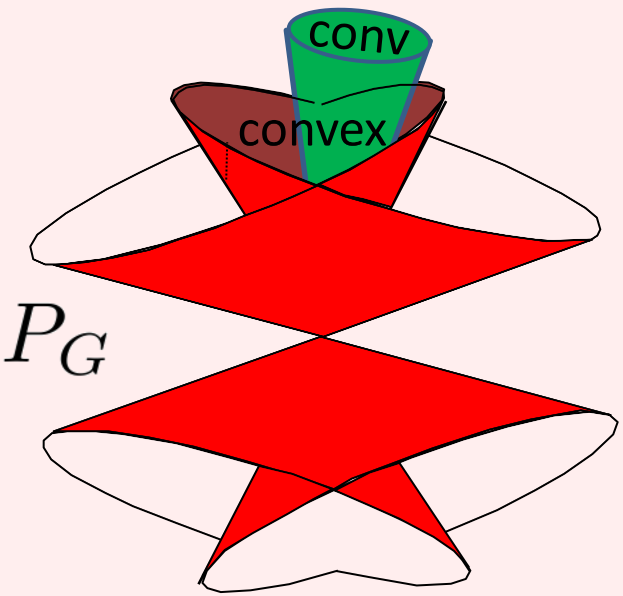
stable massive momenta

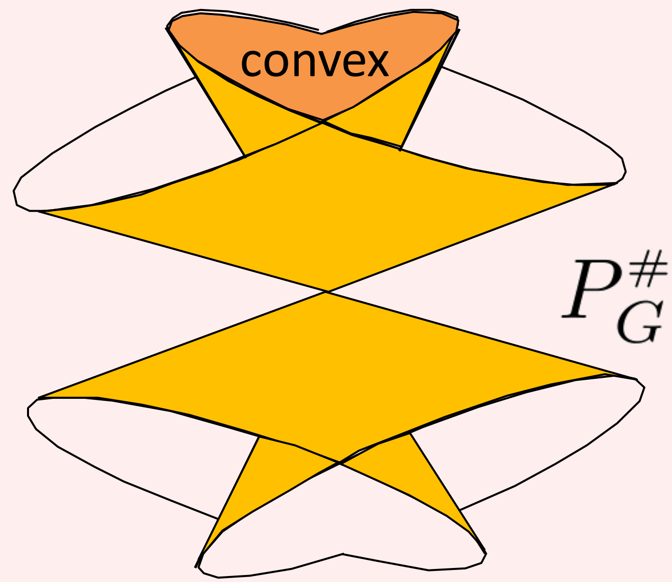
$$k_a \underset{\longleftarrow\!\!\!\dashv}{\overset{\text{Legendre}}{\longmapsto}} \frac{\partial \ln P(k)}{\partial k_a}$$

observer velocities

Bi-hyperbolic spacetime kinematics

massless momenta $\quad k_a \quad \underset{\longleftarrow\!\!\dashv}{\overset{\text{Gauss}}{\longmapsto}} \quad \dfrac{\partial P(k)}{\partial k_a} \quad$ light velocities





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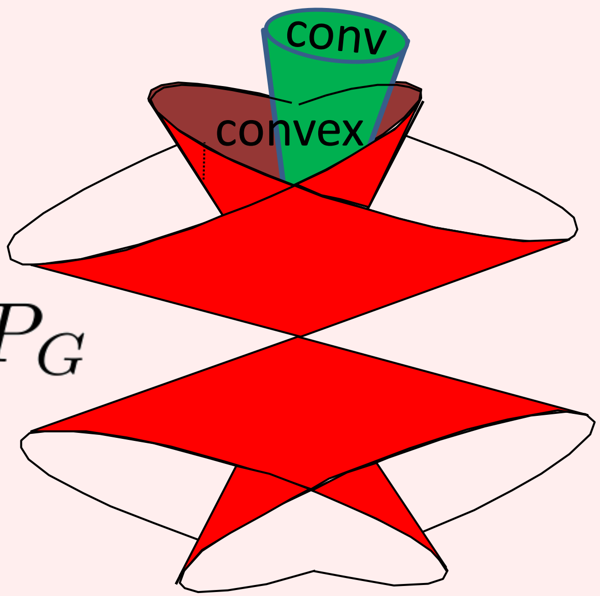
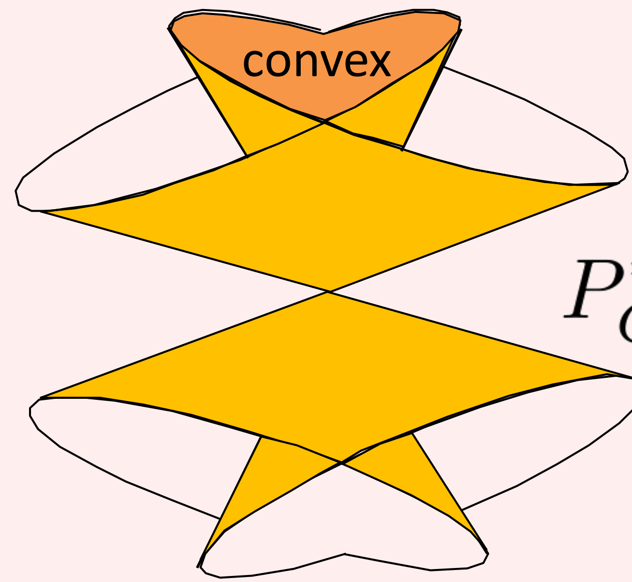


contain all kinematical information



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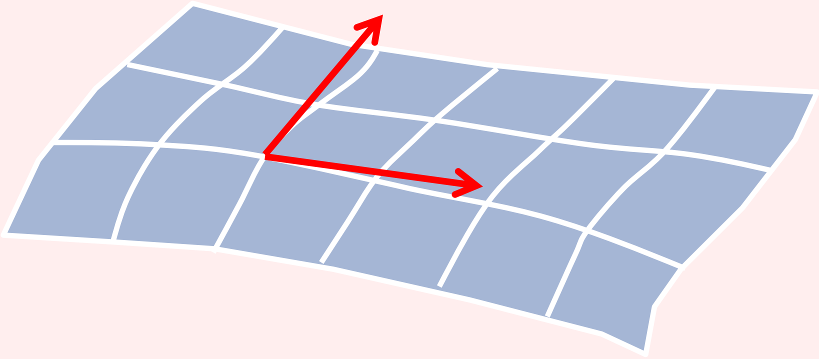
Evolving geometric data



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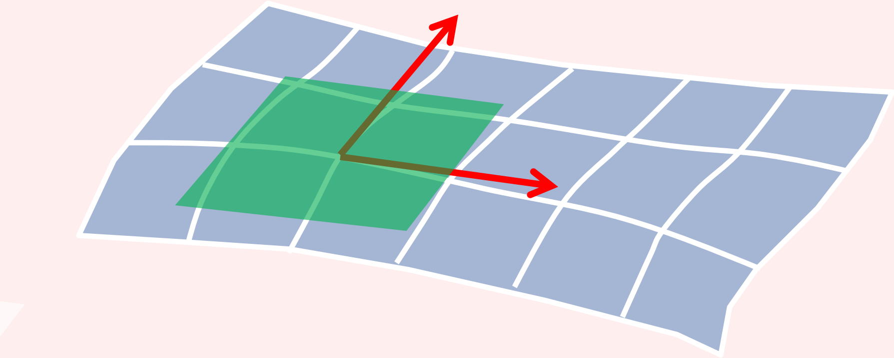

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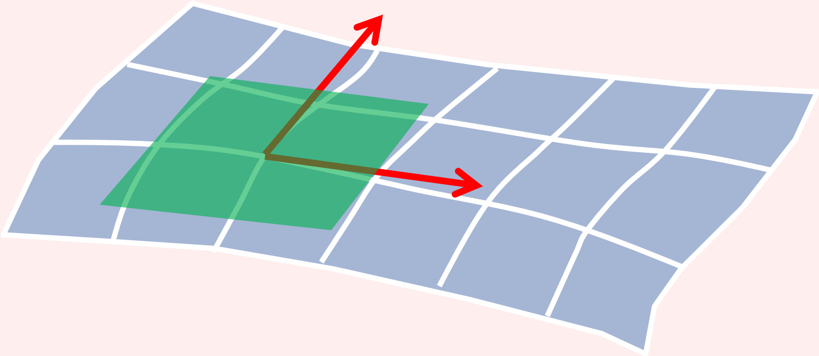
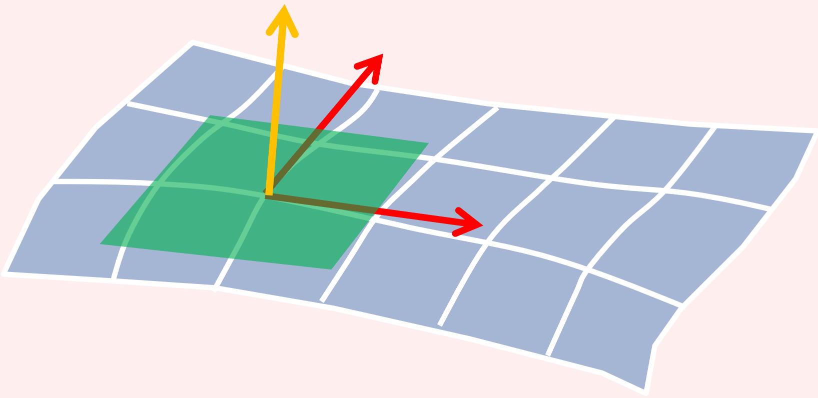


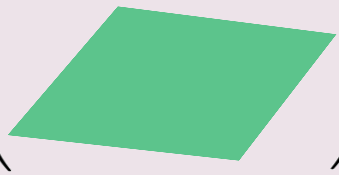
vital injection from physics:

"encodes info about matter"

Evolving geometric data



vital injection from physics:

 $= \text{Legendre}\Big($  $\Big)$

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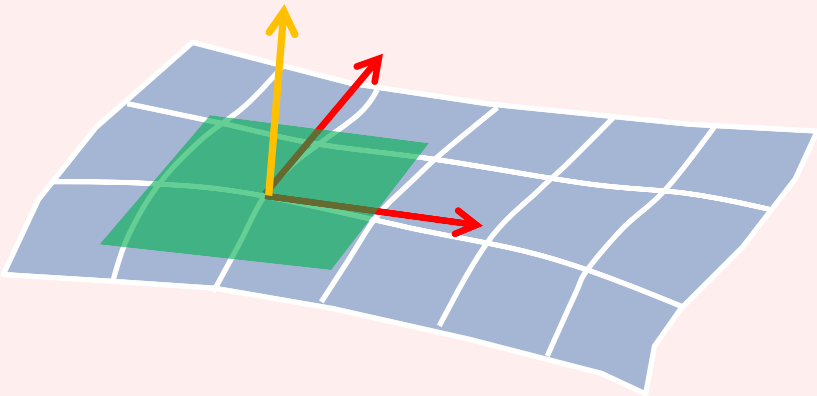

vital injection from physics:

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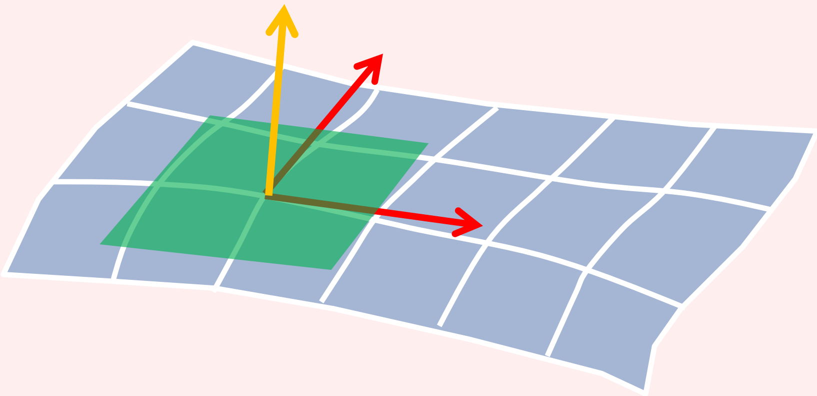
Divine view

Human view

Evolving geometric data



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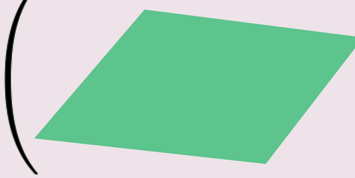
 $= \mathrm{Legendre}\Big($  $\Big)$

"encodes info about matter"

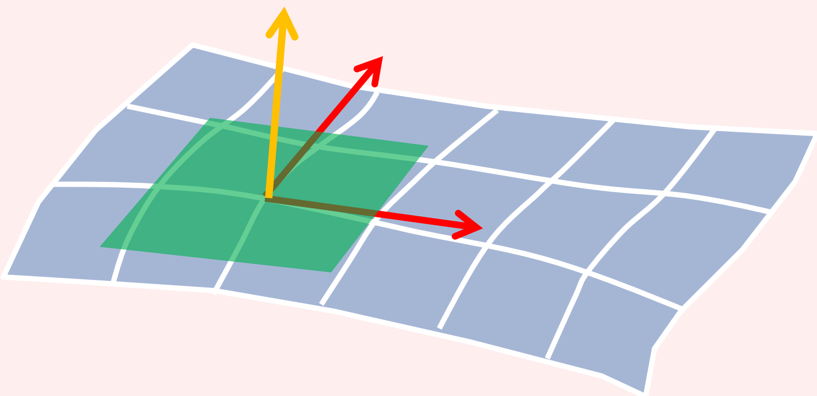
Divine view

- 4-geometry is known everywhere

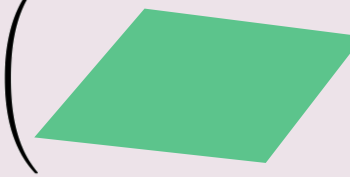
Human view

- only 3-geometry on one surface known

Evolving geometric data



vital injection from physics:

 $= \mathrm{Legendre}\Big($  $\Big)$

“encodes info about matter“

Divine view

- 4-geometry is known everywhere
- 3-geometry pushed by linear operators

normal $\mathcal{H}(N)$

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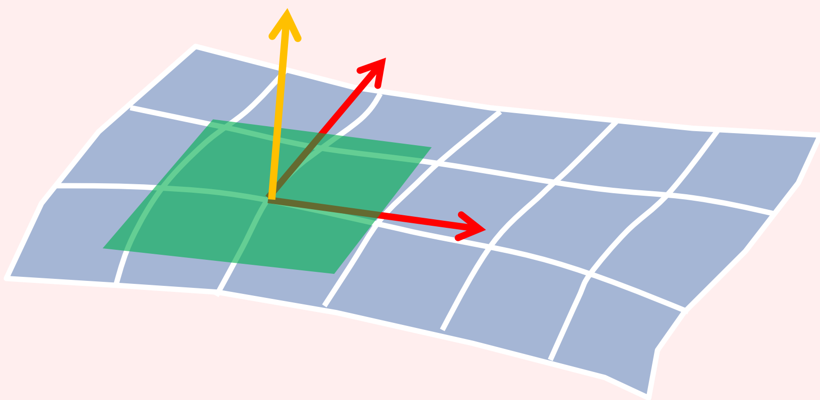
Human view

- only 3-geometry on one surface known

Evolving geometric data



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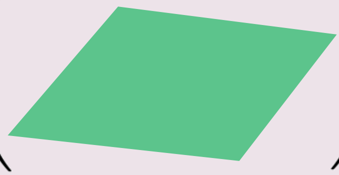
 $= \text{Legendre}\Big($  $\Big)$

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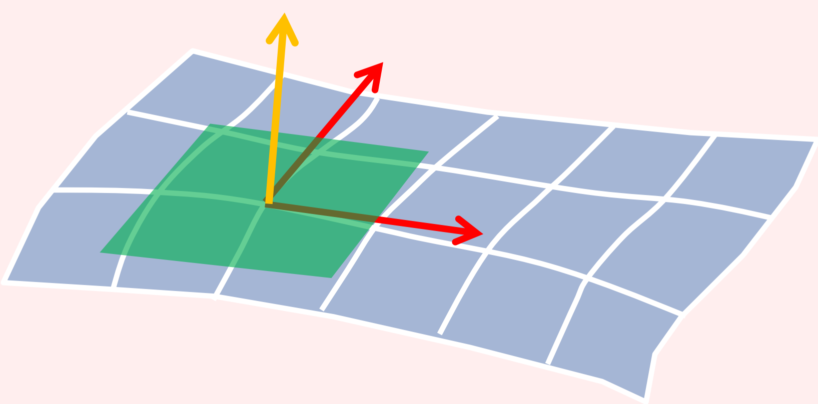

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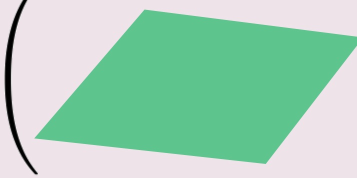
 $= \text{Legendre}\Big($  $\Big)$

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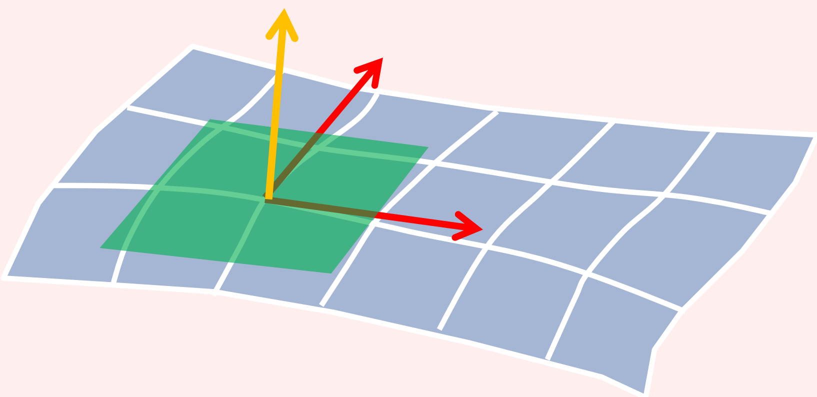

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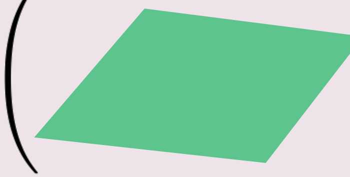
 = Legendre ()

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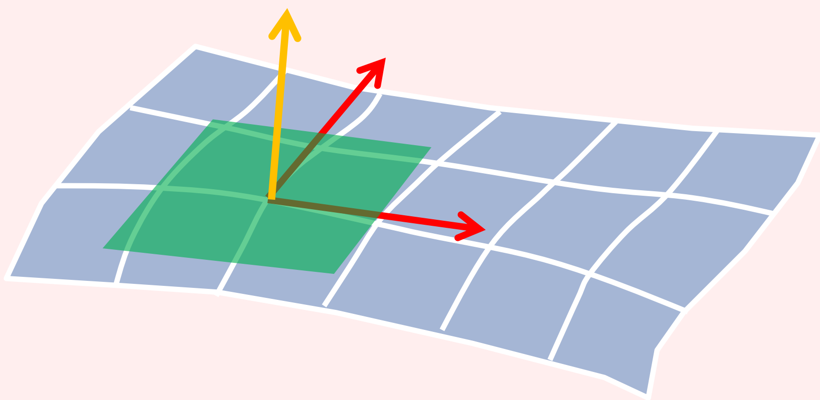

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Main Theorem

Devising consistent

gravitational dynamics for bi-hyperbolic spacetimes*

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for the coefficients
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piece of cake

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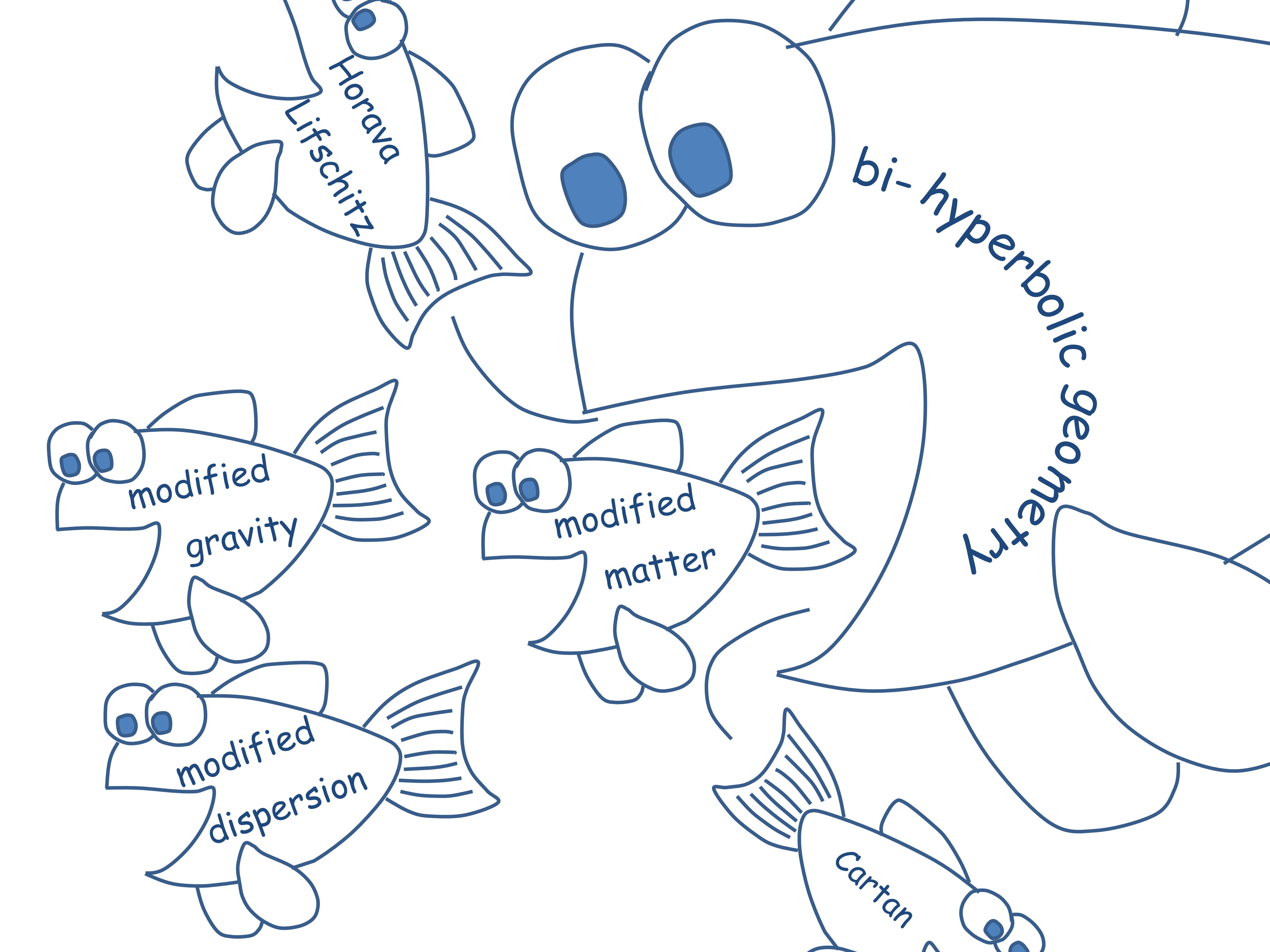
„<u>All</u> info about modified gravity is encoded in <u>one</u> linear system of PDE.“



Horava Lifschitz
bi-hyperbolic geometry
modified gravity
modified matter
modified dispersion
Cartan