Quasi-normal modes, area spectra and multi-horizon spacetimes

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Abstract: There exists a long term debate about a possible link of the asymptotic quasi-normal (QNM) modes to the black hole thermodynamics. The first conjecture providing a link between the classical asymptotic black hole oscillation frequencies and black hole thermodynamics was formulated more than a decade ago by Hod. The conjecture was later modified by Maggiore. We analyse the behavior of the asymptotic frequencies of the spherically symmetric multi-horizon spacetimes (in particular Reissner-Nordström, Schwarzschild-deSitter, Reissner-Nordström-deSitter) and provide some suggestions for how to interpret the results following the spirit of the modified Hod's conjecture. The interpretation suggested is in some sense analogical to the Schwarzschild case, but has some new specific features.

This poster refers to work done over a longer period of time contained in papers [1, 2, 3, 4] and also to some extent in [5].

Introduction

Black holes, when perturbed, show certain characteristic damped oscillations which are called quasi-normal modes. Consider (scalar, electromagnetic, gravitational) perturbation of a spherically symmetric black hole spacetime given by a line element

$$-f(c_i,r)dt^2+f^{-1}(c_i,r)dr^2+r^2d\Omega^2. \qquad (1)$$

By c_i we mean black hole spacetime parameters, such as mass, charge, or eventually in the asymptotically non-flat spacetime the cosmological constant. After decomposing the perturbation into tensor spherical harmonics and furthermore applying the normal modes decomposition one can reduce the problem for the dynamics of the perturbations into a 1-dimensional Schrödinger-like equation:

$$\frac{d^2\psi(x)}{dx^2}+\left[\omega^2-V(c_i,l,x)\right]\psi(x)=0. \qquad (2)$$

By x we mean the tortoise coordinate defined as $dr/dx=f(r)$ and V is a Regge-Wheeler/Zerilli potential which depends on the black hole background parameters, the type of perturbation, the parity of the perturbation and the wave mode number l. Quasi-normal modes (QNMs) are solutions of the equation (2) such that correspond to purely outgoing wave boundary conditions. (There are some subtleties in this definition which are not going to be analysed here.) From the definition of the QNMs one can calculate for each perturbation the QNM frequencies ω. They are complex non-real frequencies, (for the $e^{i\omega t}$ normal mode convention they have positive imaginary parts). The frequencies depend on the wave-mode number and the black hole parameters. Furthermore, typically they form an infinite discrete set with unbounded imaginary part ω_I ($\omega=\omega_R+i\cdot\omega_I$). The physical meaning of QNM frequencies is that they are the poles of the Green's function and due to this fact they describe the characteristic oscillations of *arbitrary* compactly supported black hole perturbation. Thus QNMs can be described as the characteristic black hole oscillations (or the "sound" of black holes).

The highly damped modes and conjectured connection to quantum gravity

The highly damped (asymptotic) frequencies are in the known cases independent on the wave-mode number and depend only on the black hole parameters. For the Schwarzschild black hole they are given as (κ is the surface gravity of the black hole horizon):

$$\omega_n=(\text{offset})+i(\text{gap})\cdot n+O(n^{-1/2})=\kappa\cdot\ln(3)+i\kappa\cdot(n+1/2)+O(n^{-1/2}). \qquad (3)$$

Hod [6] used Bohr's correspondence principle to conjecture that the asymptotic modes might be linked to the quantum black hole state transition. He linked the mass of the emitted quantum to the real part of asymptotic frequencies as $\Delta M=\hbar\cdot\omega_{\infty R}$. This plugged into the first law of black hole mechanics gives an equispaced horizon area spectrum with the quantum $\Delta A=4\ln(3)\cdot l_p^2$. Such spectrum is of the form that was already previously suggested due to statistical reasons [8]. Moreover, in [9] a connection was made between the asymptotic real part of the frequencies and the Immirzi parameter in Loop Quantum Gravity. The same results as in [6] can be obtained also by a different line of thinking: by using Bohr-Sommerfeld quantization condition for the adiabatic invariants and by considering the frequencies asymptotic real part $\omega_{\infty\, R}$ to be black hole's characteristic frequency. However, Hod's conjecture led to many difficulties and as a result Maggiore suggested a modification which removed most of the complications [7]. According to Maggiore's modification of Hod's conjecture (or shorter Maggiore's conjecture) black hole perturbations have to be considered as a collection of damped harmonic oscillators and the mass quantum is connected to $n\to\infty$ limit of $\hbar\cdot\Delta_{n,n-1}\sqrt{\omega_{nR}^2+\omega_{nI}^2}$. Typically in the $n\to\infty$ limit it holds that: $\Delta_{n,n-1}\sqrt{\omega_{nR}^2+\omega_{nI}^2}=\Delta_{n,n-1}\omega_{nI}$. In Schwarzschild case this gives an equispaced area spectrum with a quantum of a form originally suggested by Bekenstein [10]: $\Delta A=8\pi l_p^2$. (The non-statistical form the of area quantum is thought *not* to be a problem, since the area spectrum is only a semi-classical result.)

Spherically symmetric multi-horizon black hole spacetimes

For the multi-horizon spherically symmetric black hole spacetimes (Reissner-Nordström, Schwarzschild-deSitter, Reissner-Nordström-deSitter) and the scalar, vector, tensor perturbations the asymptotic QNM frequencies obey the following equation:

$$\sum_{A=1}^{M}C_A\exp\left(\sum_{i=1}^{N}Z_{Ai}\frac{2\pi\omega}{|\kappa_i|}\right)=0. \qquad (4)$$

Z_{Ai} takes one of the values $Z_{Ai}=0,1,2$, furthermore N is the number of horizons and κ_i are the surface gravities of the different horizons. The solutions of this formula have much more complicated behaviour as the QNM frequencies of the Schwarzschild single horizon case (3). In [3, 4] we analysed the behaviour of the solutions of (4) with the following results: If the ratio of all of the surface gravities is a rational number then all the frequencies split in a *finite* number of equispaced families (labeled by a) of the form:

$$\omega_{an}=(\text{offset})_a+in\cdot\text{lcm}(|\kappa_1|,|\kappa_2|,...,|\kappa_N|). \qquad (5)$$

Here lcm is the least common multiple of the numbers in the bracket, hence

$$\text{lcm}(|\kappa_1|,...,|\kappa_N|)=p_1|\kappa_1|=...=p_N|\kappa_N|,$$

where $\{p_1,...,p_N\}$ is a set of relatively prime integers. If the ratio of arbitrary two of the surface gravities is irrational, then there does *not* exist an equispaced subsequence in the sequence of asymptotic QNM frequencies. Moreover one can prove [2] for the Reissner-Nordström (R-N) black hole (but one expects it to hold for all the three cases) that, in case the ratio of the surface gravities is irrational, the $n\to\infty$ limit for $\Delta_{n-1}\omega_{nI}$ does *not* exist. Also for the rational ratio of the surface gravities and the R-N black hole the only case in which the limit $n\to\infty\ \Delta_{n,n-1}\omega_{nI}$ exists is if *all* the frequencies are given by families of the form (5) with the same $(\text{offset})_I$. But this cannot be the case when the ratio of the surface gravities is given by two relatively prime integers whose product is an odd number [2].

The previous considerations seem to suggest, that the modified Hod's conjecture has very little chance to survive the multi-horizon case. However the significantly different behaviour for the cases of rational / irrational ratios of surface gravities and the general splitting of frequencies into families (5) seem to indicate something important. Moreover, it was already observed that surface gravities rational ratios have significant consequences for the multi-horizon spacetime thermodynamics [11]. Based on this observations let us pick the R-N case where the thermodynamical interpretation is straightforward (but keep in mind that all the calculations can be repeated in an exact analogy for the other S-dS, R-N-dS cases) and consider the following: Let us presuppose that both of the horizons in the R-N spacetime, the outer horizon with the area A_+ and the inner Cauchy horizon with the area A_- have equispaced area spectra given as[1] $A_\pm=8\pi l_p^2\gamma\cdot n_\pm$.

The perturbations are supposed to carry *no* charge, so one expects that only the ADM mass of the black hole will be changed. Thus one can write the change of the areas of the black hole horizons as (from now on we use Planck units):

$$\Delta A_\pm=\frac{8\pi\Delta M}{\kappa_\pm}. \qquad (6)$$

But $\Delta A_\pm$ can be given only as $\Delta A_\pm=8\pi\gamma m_\pm$, which implies $\Delta M=\gamma\kappa_\pm m_\pm$. Furthermore, this implies

$$m_+\kappa_+=m_-\kappa_-\to\frac{\kappa_+}{\kappa_-}=\frac{m_-}{m_+}. \qquad (7)$$

This means that if the single ADM mass transitions have to be allowed the surface gravities ratio must be rational. Furthermore if one wants the emitted mass quantum to be as small as possible, such that it is still compatible with the quantization of the two horizon's areas one obtains:

$$\Delta M=\gamma\cdot\text{lcm}(\kappa_+,|\kappa_-|). \qquad (8)$$

Then modified Hod's conjecture suggests that

$$\lim_{n\to\infty}\Delta_{n,n-1}\omega_{nI}=\gamma\cdot\text{lcm}(\kappa_+,|\kappa_-|). \qquad (9)$$

This is indeed true if one takes the following interpretation of the frequencies (slight modification of Maggiore's conjecture): the straightforward extension of Maggiore's conjecture to the multi-horizon case is misleading, in fact only the equispaced families carry information about the quantum black hole mass transitions. (Every frequency belongs to one of the families.) Thus one has to first identify the equispaced families and then take the limit in the spacing in the imaginary part of the frequencies within each of the families. Such interpretation then fixes together with the formula (5) the parameter γ to be $\gamma=1$. This means the area spectra of both of the horizons are given as $8\pi n$. Let us remind here, that the same analysis can be repeated for both S-dS and R-N-dS spacetimes: Assuming that all the horizons have the same equispaced area spectra, the single M parameter transitions lead to the surface gravities rational ratio condition and the QNM frequencies given by the formula (5) fix the spectra of all the horizons to be $8\pi n$, (after one considers our generalization/modification of Maggiore's conjecture).

Conclusions

To summarize: We suppose that also in the multi-horizon case the modified Hod's conjecture provides information about the spacetime horizons spectra, only the way the information is encoded is more tricky than in the single horizon case. (This is hardly anything surprising as the quantization of more than one horizon might play a role in the game.) The QNM frequencies are consistent with each of the horizons being quantized with spectra given as $8\pi n$ (in Planck units).

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[1] This type of spectrum with $\gamma=1$ is the one originally suggested by Bekenstein for the black hole horizon area and represents the far most popular choice in the current literature. Let us also mention that in [12] it was already suggested that both of the horizons in the R-N spacetime have the same area spectra of the form $A=8\pi l_p^2\cdot n$.