Dynamics of black holes in de Sitter spacetimes

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29th June 2012, 100 Years after Einstein in Prague

Contents

Outline

Why numerical relativity

Study of systems with strong and dynamical gravitational fields

- Gravitational radiation
  - Astrophysics, gravitational wave astronomy
- Mathematical and theoretical Physics:
  - Cosmic censorship
  - Instabilities (Black hole interior, Myers-Perry)
- High-energy particle systems:
  - AdS/CFT correspondence;
  - Black hole production at the LHC;

de Sitter

- Large-scale structure of our universe appears to be that of a de Sitter geometry:
  - How do inhomogeneities develop in time? Are they washed away by the cosmological expansion? (Shibata et. al 1994)
  - Cosmological dynamics should leave imprints in primordial black hole formation, which carry signatures of the cosmological acceleration; (Shibata & Sasaki, 1999)
- Two large BHs in de Sitter could, upon merger, give rise to too large a BH to fit in its cosmological horizon $\Rightarrow$ naked singularity.

Outline

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1. Motivation
2. Black holes in de Sitter
   - Formalism
   - Results
3. Final remarks
   - Conclusions

Formalism

Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 3H^2 g_{\mu\nu} = 0$$

Evolution equations

$$(\partial_t - \mathcal{L}_\beta)\, \gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\, K_{ij} = -D_i \partial_j \alpha + \alpha \left({}^{(3)}R_{ij} - 2K_i{}^k K_{jk} + K_{ij} K - 3H^2 \gamma_{ij} \right)$$

Constraints

$$\mathcal{H} \equiv {}^{(3)}R - K_{ij} K^{ij} + K^2 - 6H^2 = 0$$

$$\mathcal{M}_i \equiv D_i K - D_j K_i^j = 0$$

Initial data

Writing $K_{ij} \equiv A_{ij} + \frac{K}{3}\gamma_{ij}$

Constraints

$$ {}^{(3)}R - A_{ij}A^{ij} + \frac{2}{3}\left(K^2 - 9H^2\right) = 0 $$

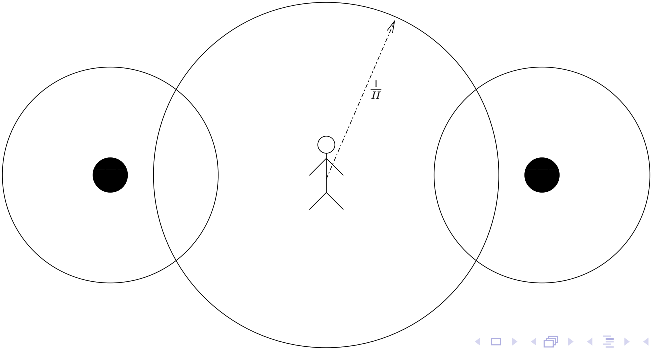
$$ D_j\left(A^j{}_i - \frac{2}{3}\delta^j_i K\right) = 0 $$

$\Rightarrow$ set $K = -3H$ to decouple equations. (Nakao et al 1993)

Initial data

Two BHs, asymptotically de Sitter

$$\gamma_{ij} dx^i dx^j = \left(1 + \frac{m_1}{2r_1} + \frac{m_2}{2r_2}\right)^4 (dx^2 + dy^2 + dz^2)$$
$$K^i_j = -H\delta^i_j,$$



Schwarzschild-de Sitter

S-dS, McVittie coordinates

$$ds^2 = -\left(\frac{1-\xi}{1+\xi}\right)^2 dt^2 + a(t)^2(1+\xi)^4(dr^2 + r^2 d\Omega_2)$$

$$K^i_j = -H\delta^i_j, \qquad a(t) = \exp(Ht), \quad \xi \equiv \frac{m}{2a(t)r}$$

with $m = 0$, cosmological horizon (centred at the origin) stands at

$$r_{\mathcal{H}_C} = 1/(He^{Ht})$$

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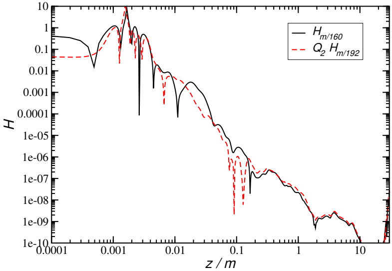
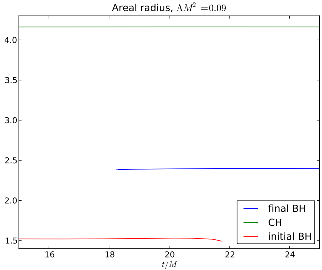
Head-on collision from “rest”

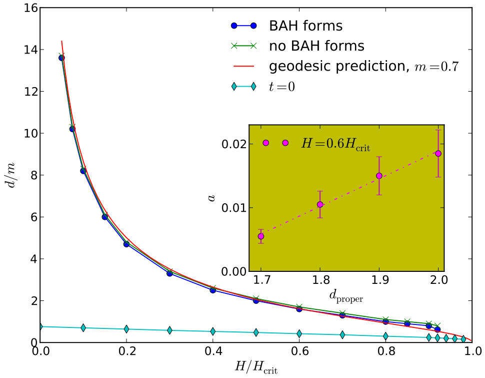




- Evolution is stable and the constraints are preserved;
- Successfully monitor the apparent horizons;

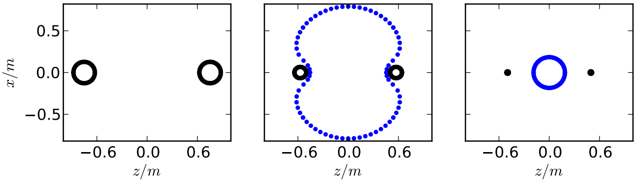
Black holes in de Sitter: "small" BHs

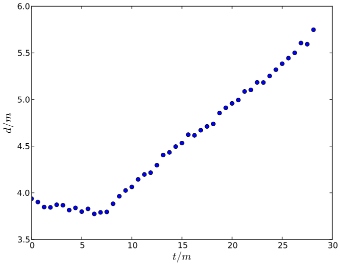
- Two parameters: H, d
- $d < d_{crit} \Rightarrow$ merger
  $d > d_{crit} \Rightarrow$ no common AH



- Critical coordinate distance for small mass binaries

Black holes in de Sitter: “large” BHs





- Proper distance between the black hole horizons as a function of time

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Conclusions

1. Numerically solving Einstein's equations in dynamical situations has the potential to answer important questions.
2. BHs in de Sitter:
   - Evolved head-on collision of BHs in asymptotically de Sitter spacetimes and monitored apparent horizons;
   - Studied formation of common apparent horizon as function of initial separation;
   - Results compatible with cosmic censorship.
3. To Do:
   - Perform numerical evolutions in collapsing universes.