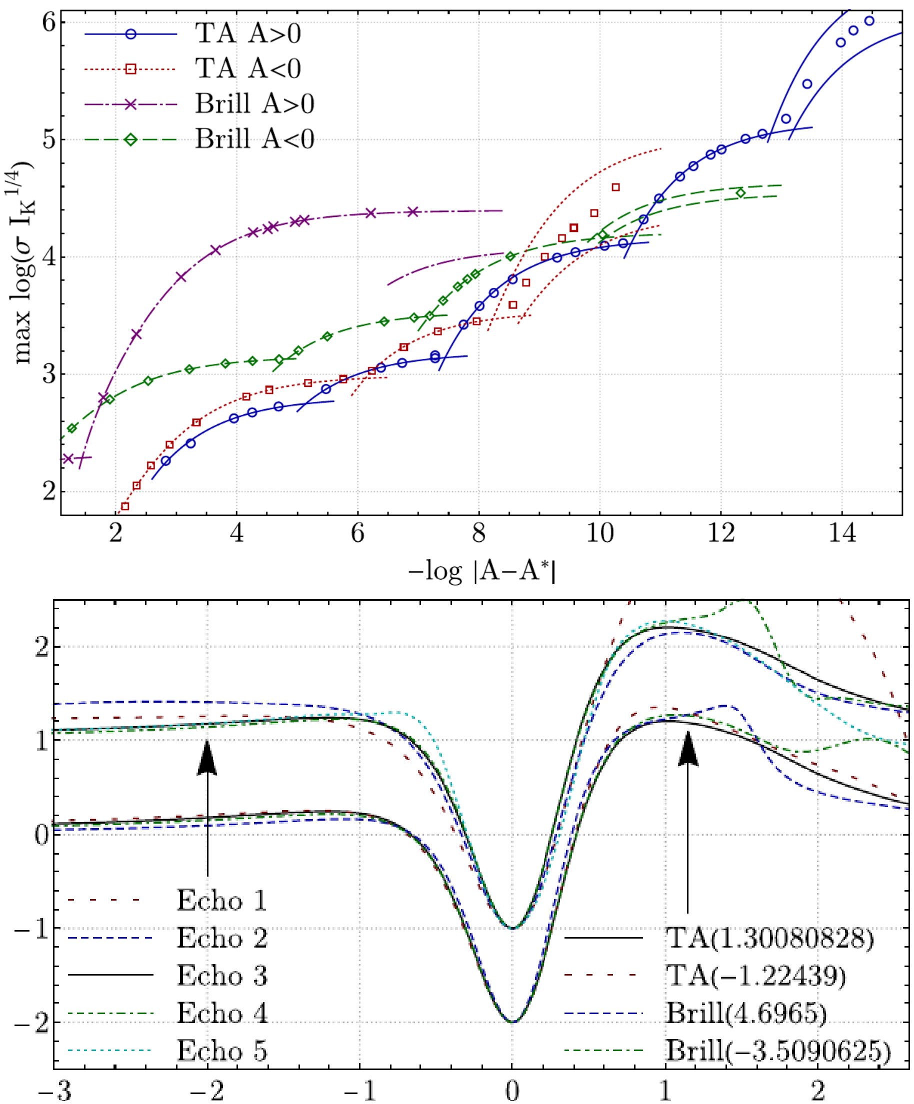
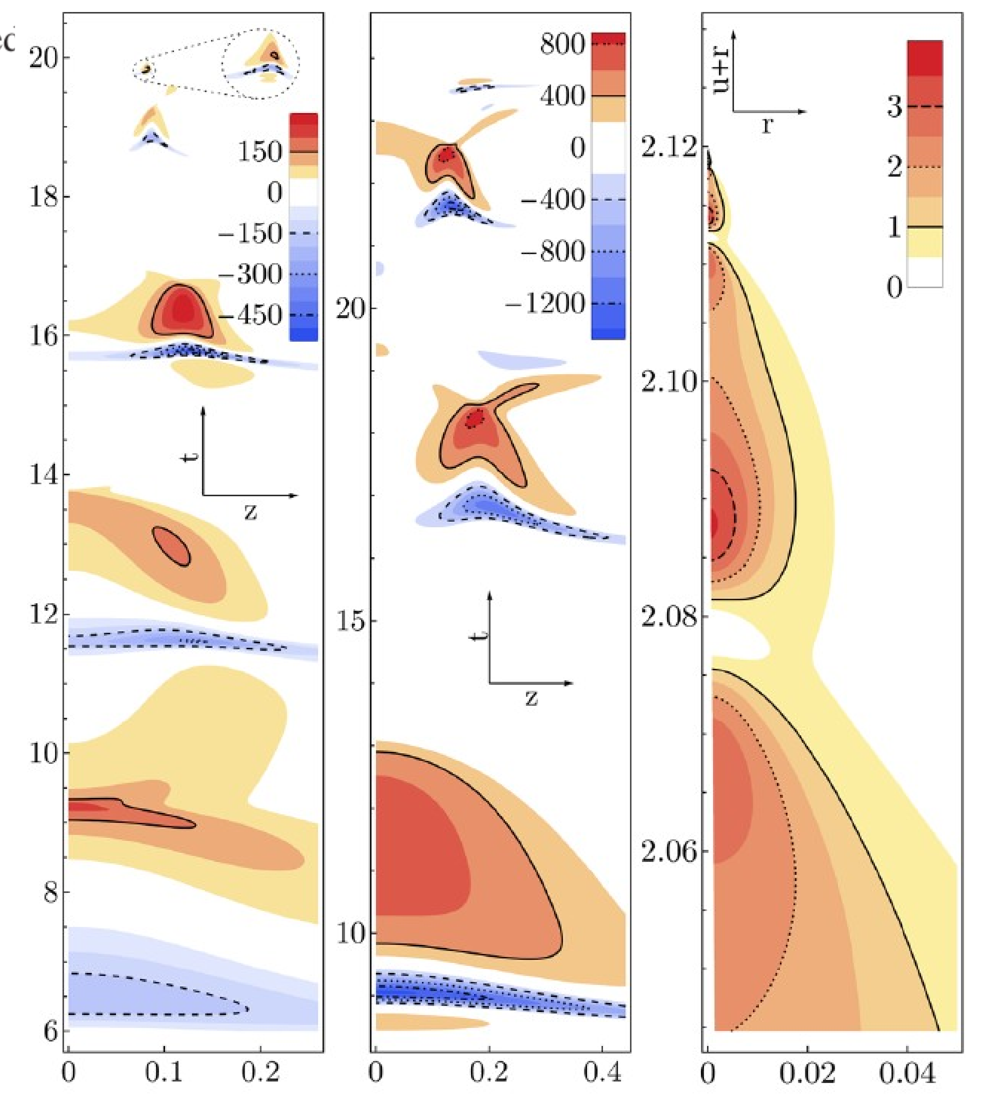
Universality of Curvature Invariants in Critical Vacuum Gravitational Collapse

Tomáš Ledvinka* and Anton Khirnov†
Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, CZ-180 00 Prague, Czech Republic



acceptec

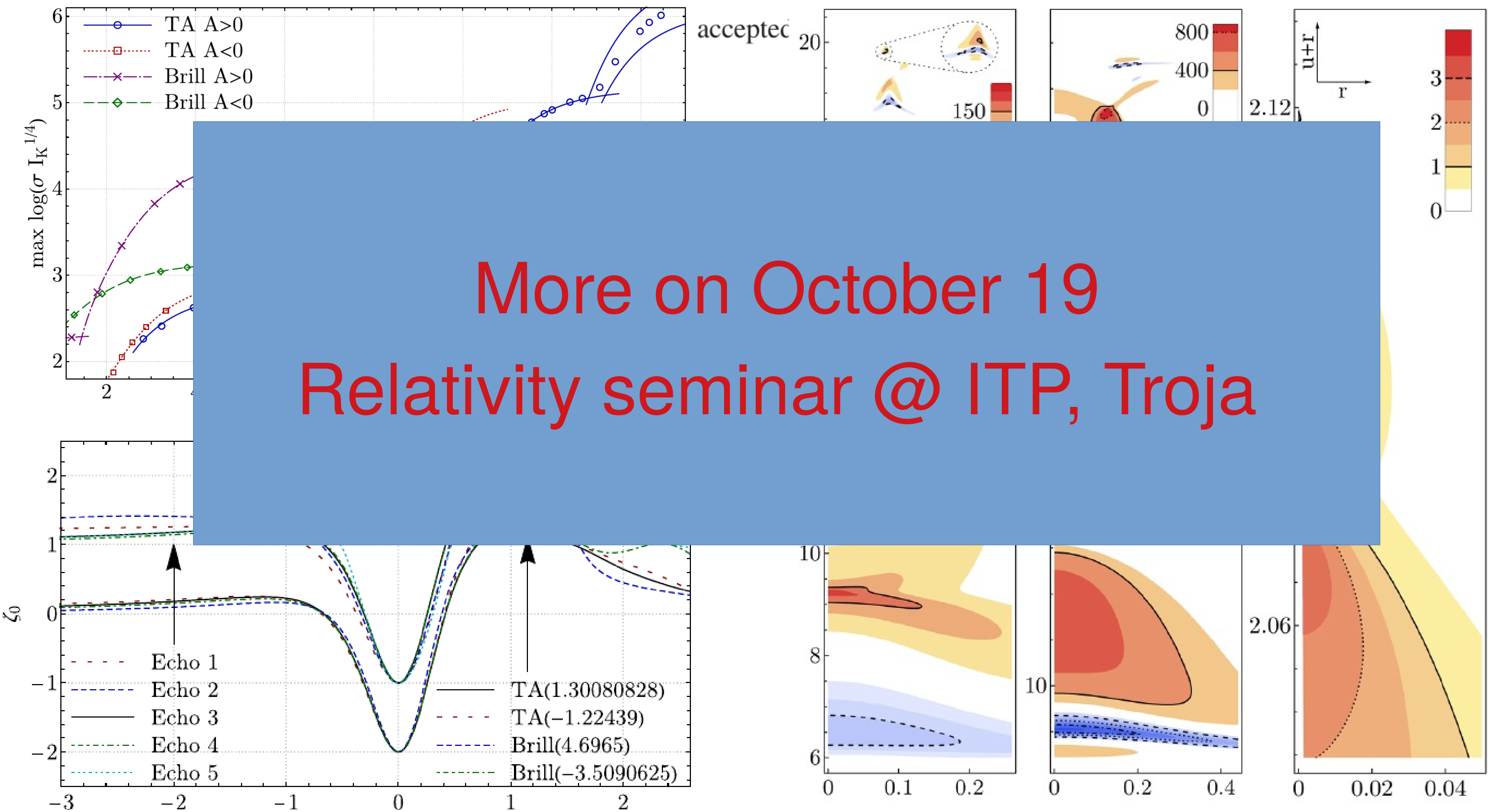


PHYSICAL REVIEW LETTERS **127**, 011104 (2021)

Universality of Curvature Invariants in Critical Vacuum Gravitational Collapse

Tomáš Ledvinka* and Anton Khirnov†

Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, CZ-180 00 Prague, Czech Republic

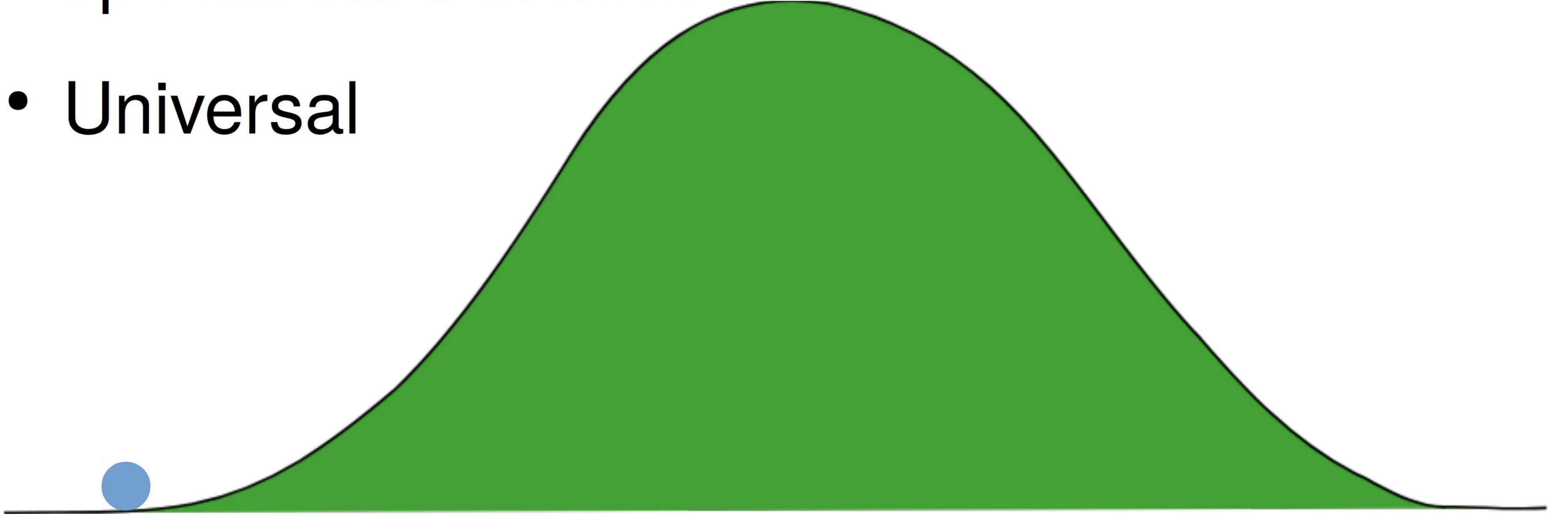


More on October 19

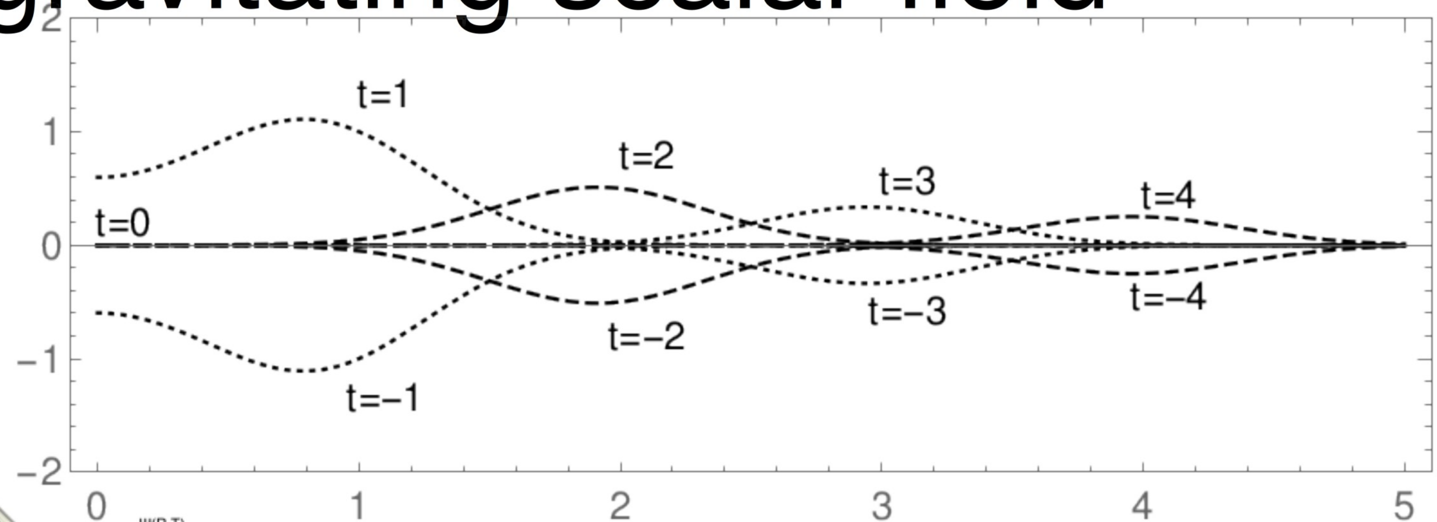
Relativity seminar @ ITP, Troja

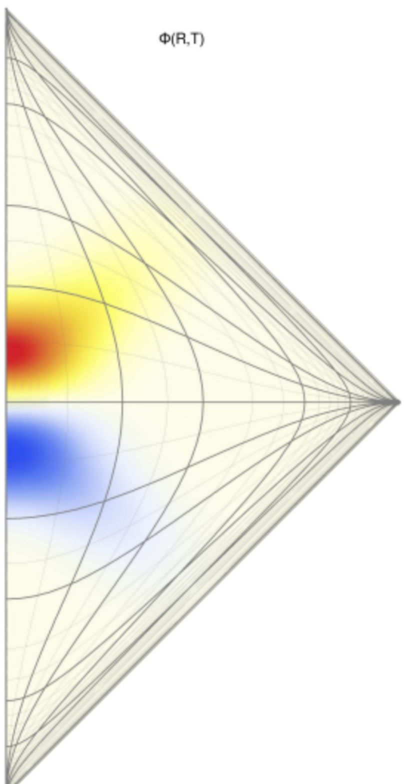
1D potential barrier

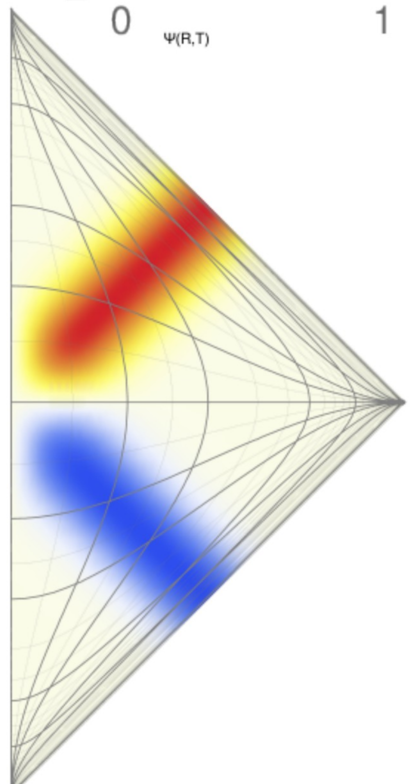
- reflects left
- continues right
- special case in between
- Universal



3+1 gravitating scalar field

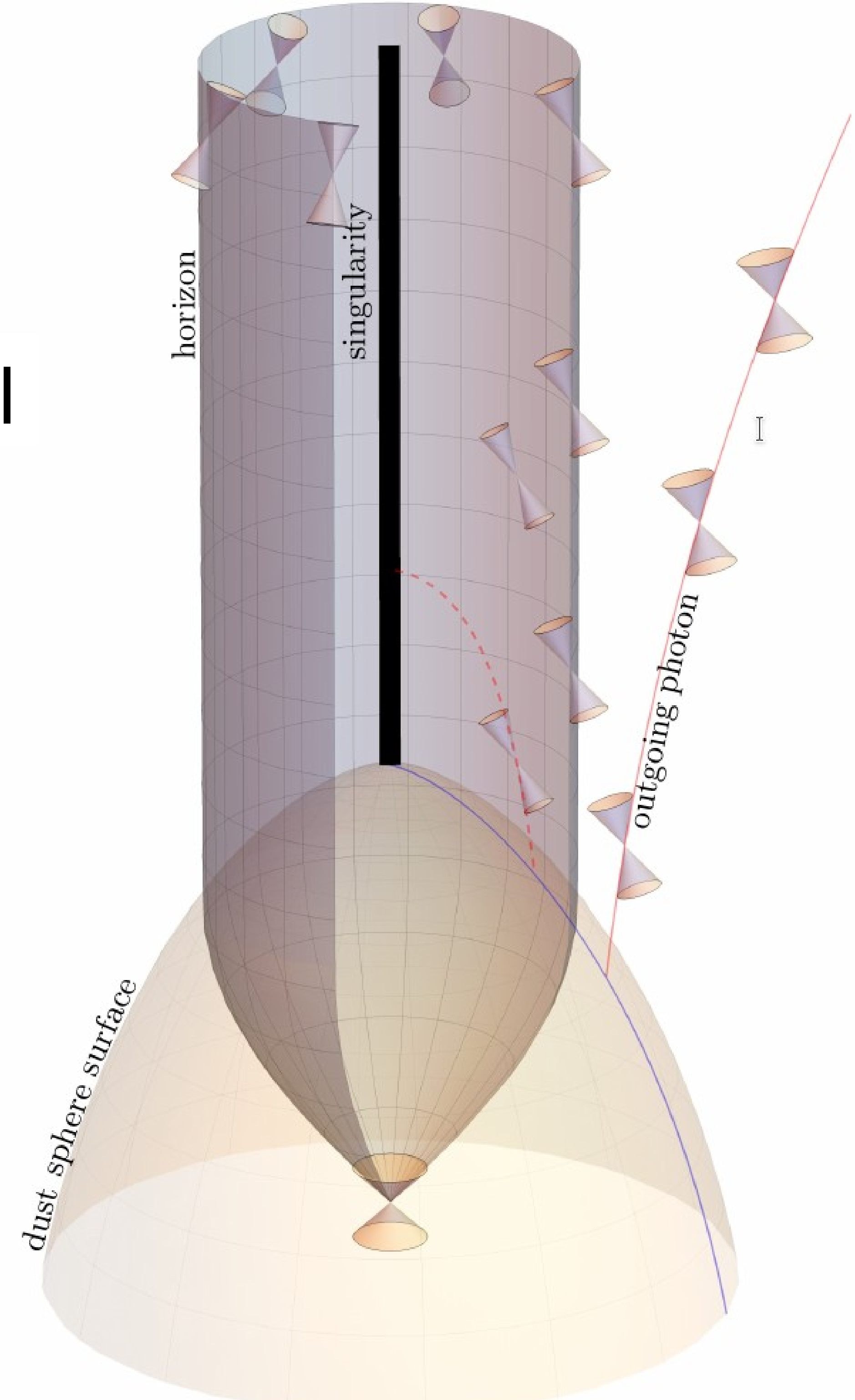






- Gravitational collapse

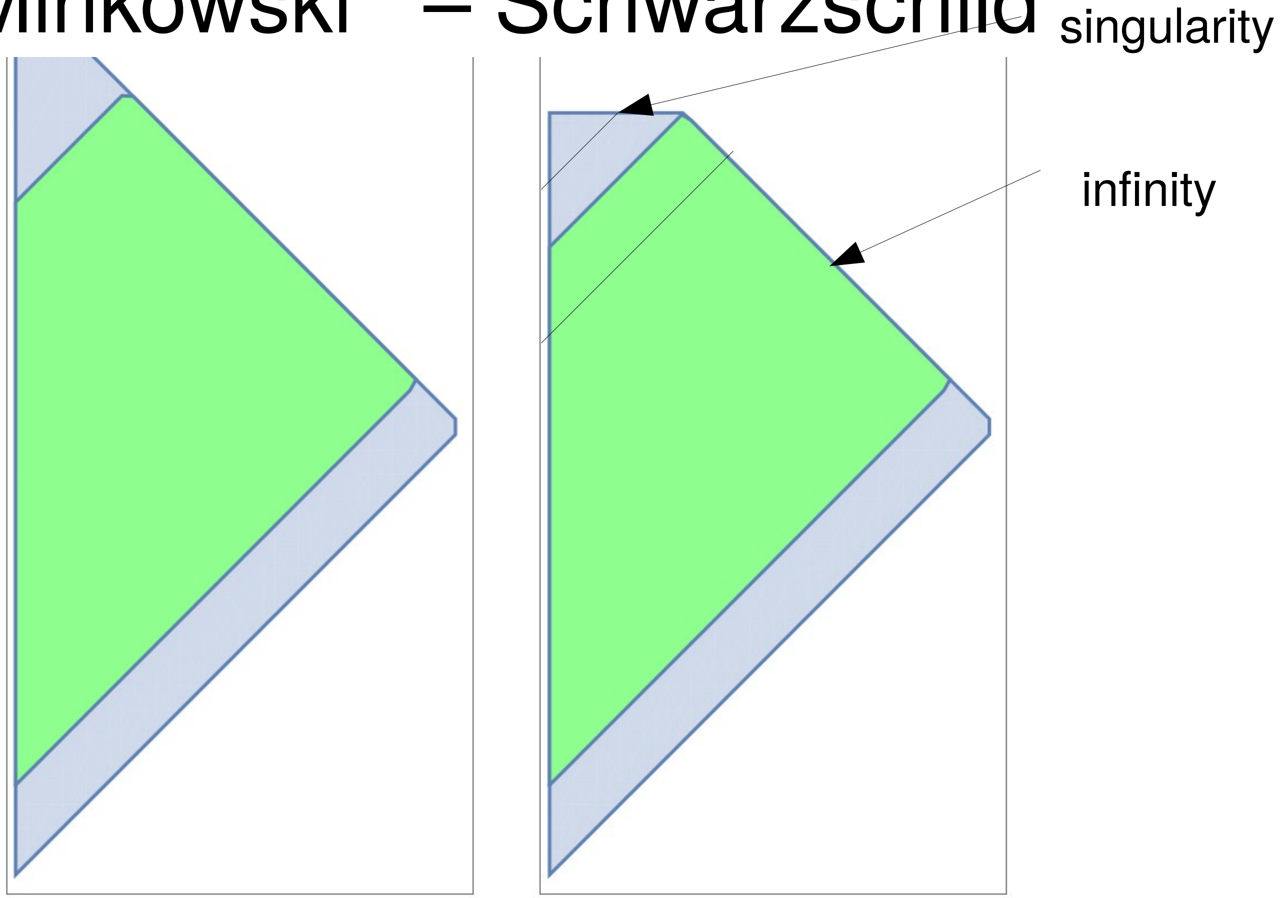
Standard illustration



2 possible outcomes

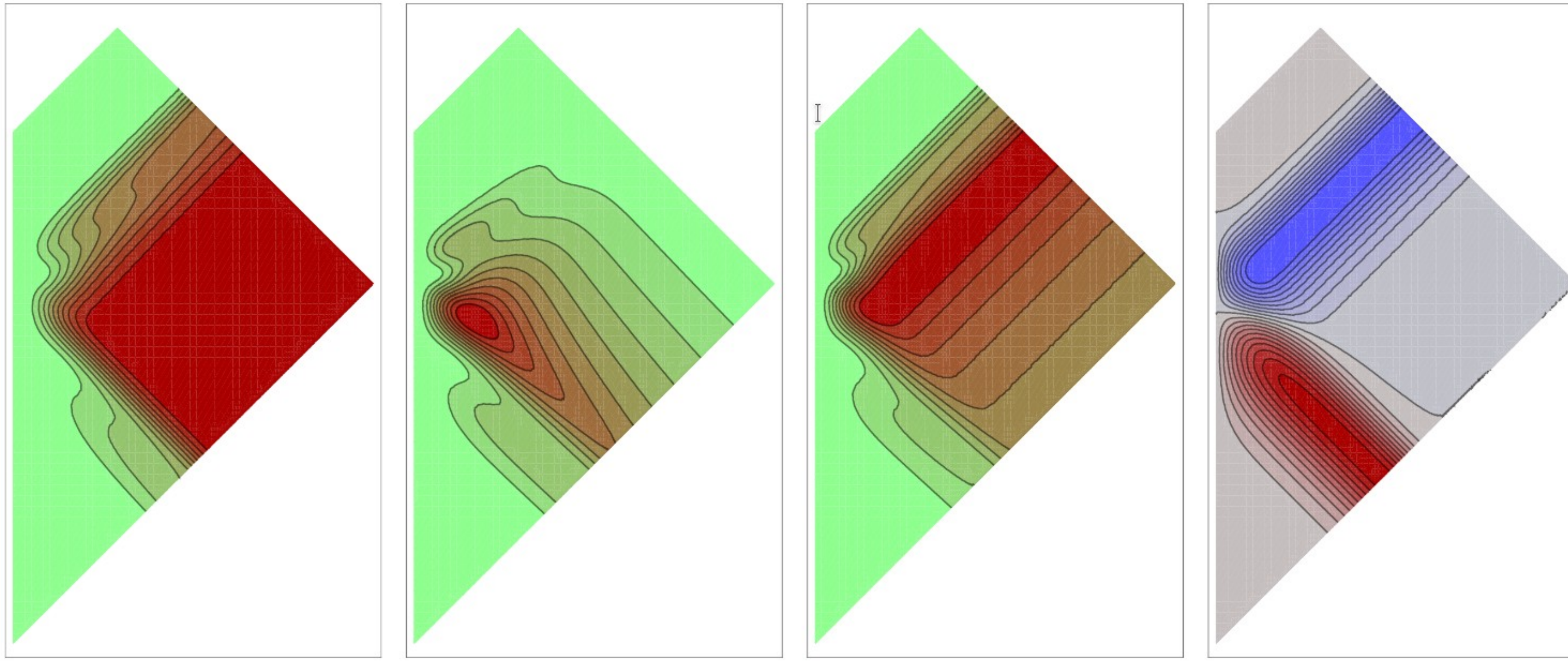
- (Assuming spherical symmetry)

Minkowski – Schwarzschild



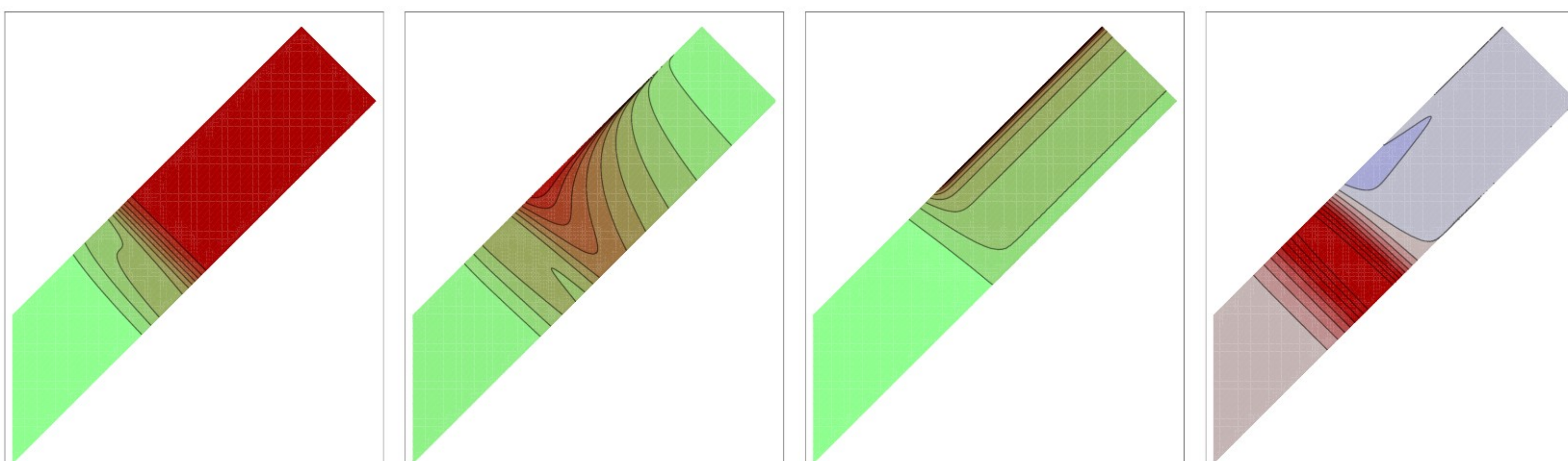
2 possible outcomes

- Minkowski



mass 2m/r redshift psi = r*phi

Schwarzschild



Choptuik (1993)

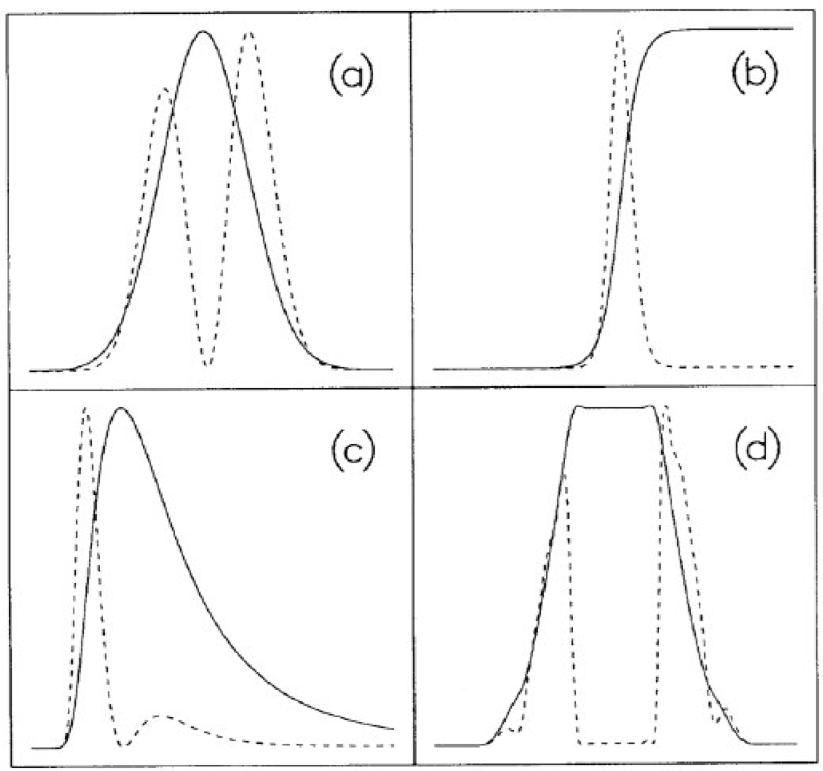


FIG. 1. Typical initial profiles of the scalar field ϕ (solid lines) and the radial mass-energy density dm/dr (dotted lines) for the four families defined in Table I.

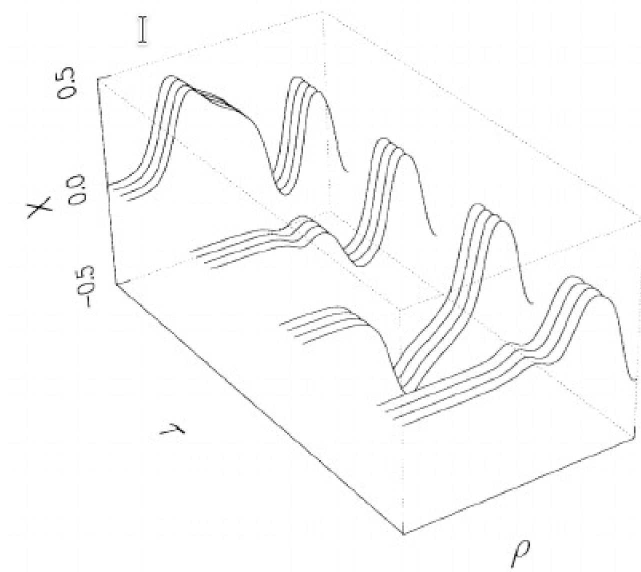
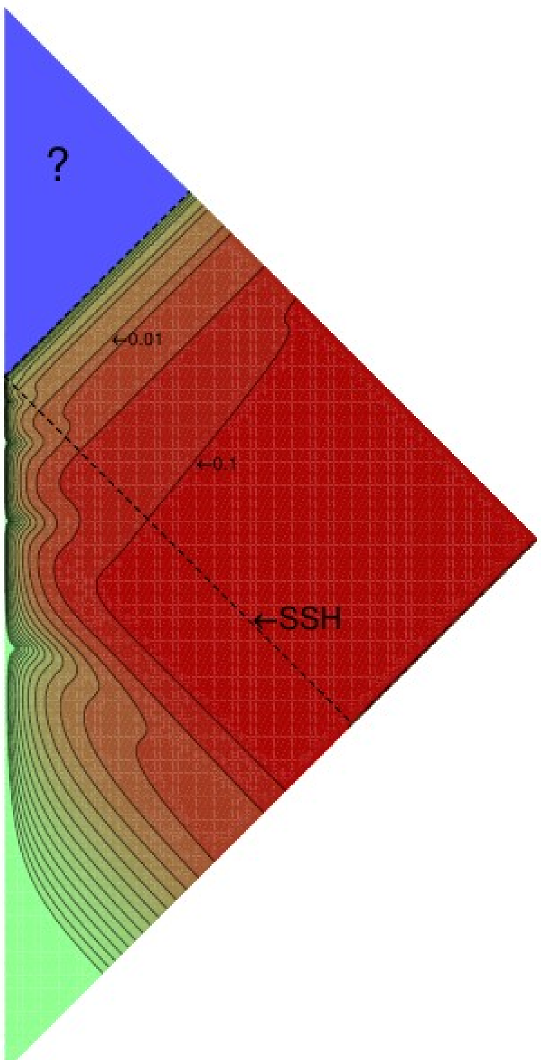
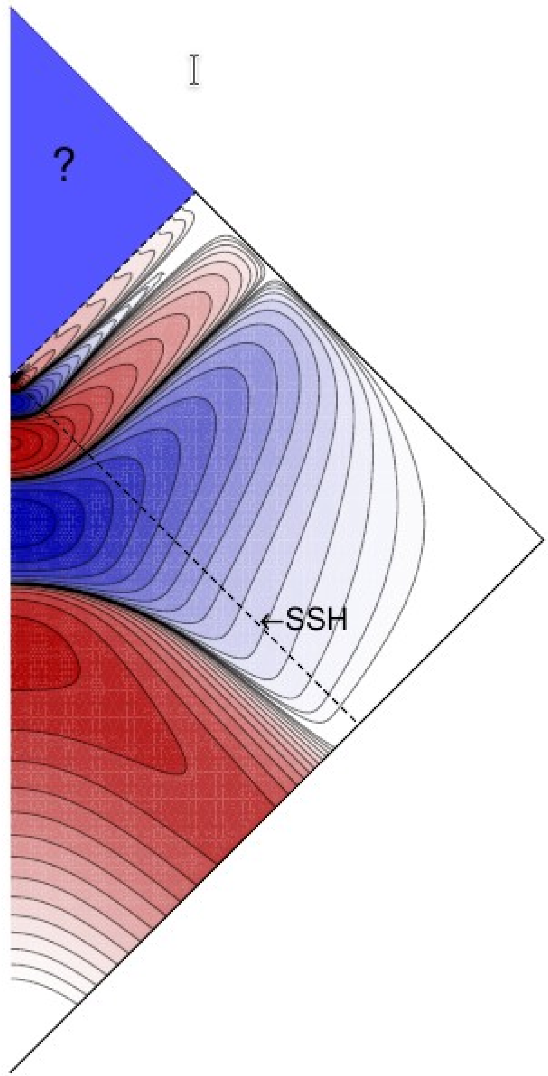


FIG. 3. Illustration of the conjectured universality of critical evolution in the model problem. Each group of four lines consists of one profile, at a particular instant, τ, from near-critical evolution of each of the families listed in Table I [families (a)–(d), front to back]. For each family, I chose a single overall scaling constant, k, to maximize agreement among the first (foreground) group of curves. Agreement of the profiles at later times (towards the back of the plot) demonstrates universality of the evolution, regardless of initial pulse shape. Since these plots show less than one full cycle of evolution, echoing is not apparent here.

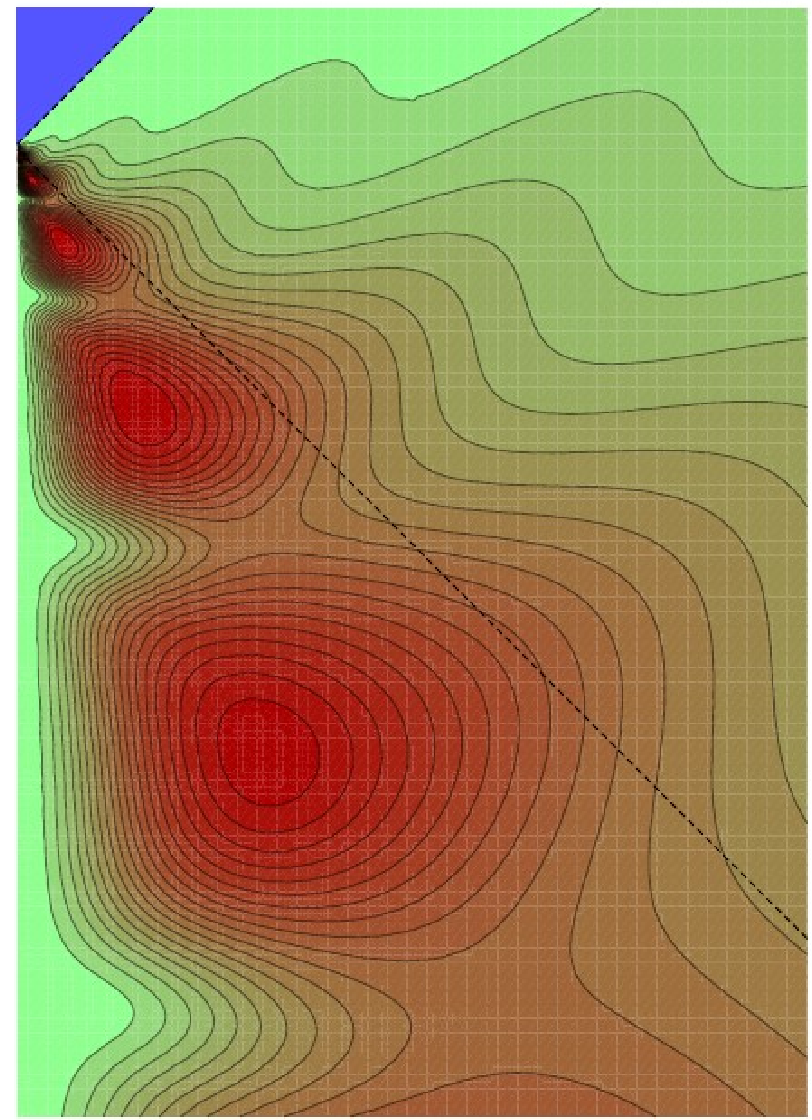
Critical solution



mass



phi



2m/r

Critical behavior

- $M_{BH} = (A\text{-}A^*)^\gamma$ for initial data with amplitude A
- M_{BH} can be arbitrarily small
- Exponent γ independent on the initial data shape
- oscillating fields
- periods of oscillation, fields, etc. are discretely self-symmetric with scale ratio e^{Δ}
- also field configuration etc. are independent on initial data shape
- γ and Δ depend on the field type (e.g. Young-Mills vs scalar)

Critical collapse of GW

- No GW in spherical symmetry (3+1 gravity)

  Computationaly more demanding

- Numerical relativity “workhorse“ failed

  (coordinate singularities appear)

- Also some irreproducible results ...

Initial data for evolution equations

- For a simple wave eq. $\Box u = 0$ one needs initial $u(x)$, $\partial u(x)/\partial t$ at $t=0$
- In GR we need more quantities than DOF

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} dx^i dx^j$$

- Non-linear constraints

$$R + K^2 - K^{ij} K^{ij} = 0$$

$$\mathrm{D}_j K^{ij} - \mathrm{D}^i K = 0$$

- Evolution of vacuum spacetime with GW

$$\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij}, \\
\partial_t K_{ij} &= -\mathrm{D}_i \mathrm{D}_j \alpha + \alpha \left(R_{ij} + K K_{ij} - 2 K_{ik} K^k_j \right)
\end{aligned}$$

- BH may appear in strong initial data

ANNALS OF PHYSICS: 7, 466–483 (1959)

On the Positive Definite Mass of the Bondi–Weber–Wheeler Time-Symmetric Gravitational Waves*†

DIETER R. BRILL

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

After the work of Weber and Wheeler there were four developments, three of them unpublished, which led up to the further analyses of the present paper: (1) J. A. Wheeler pointed out in a lecture at London in March, 1958 that one can reduce the complexity of the time-symmetric initial value equation $^3R = 0$ and can give special attention to toroidal and other simple gravitational waves if one will limit attention to metrics with axial symmetry. The physical meaning

III. PROOF OF THE POSITIVE DEFINITE NATURE OF THE TOTAL MASS-ENERGY OF TIME-SYMMETRIC, AXIALLY SYMMETRIC GRAVITATIONAL WAVES

A. Boundary Conditions on the Metric Coefficients

Let the axially symmetric metric be written in the form

$$ds^2 = \psi^4[e^{2q}(d\rho^2 + dz^2) + \rho^2\, d\varphi^2], \tag{19}$$

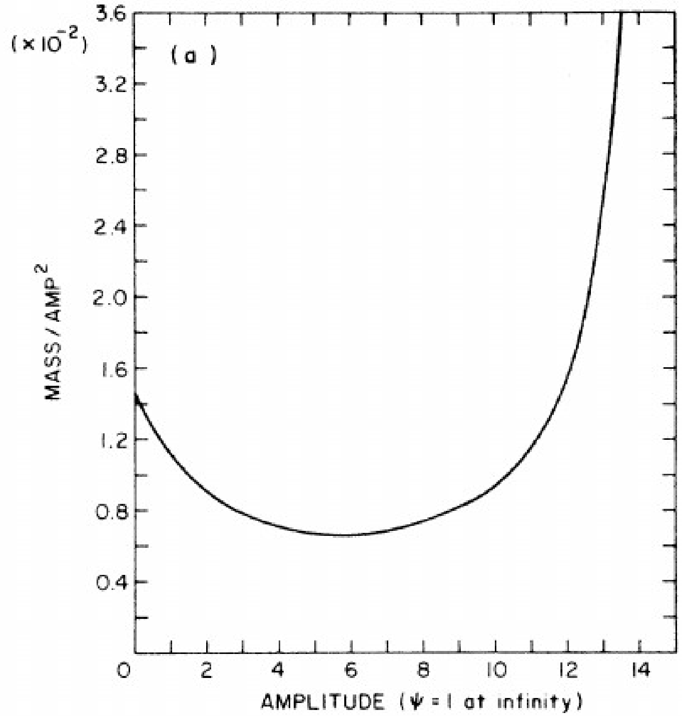
$$\Delta\psi + \frac{1}{4}\left(\partial_{\rho\rho}q + \partial_{zz}q\right)\psi = 0,$$

$$M = (c^2/2\pi G)\int (\nabla \log \psi)^2 \quad dV.$$

PHYSICAL REVIEW D VOLUME 16, NUMBER 6 15 SEPTEMBER 1977

Evolution of time-symmetric gravitational waves: Initial data and apparent horizons*

Kenneth Eppley
University of Maryland, College Park, Maryland
(Received 4 March 1977)



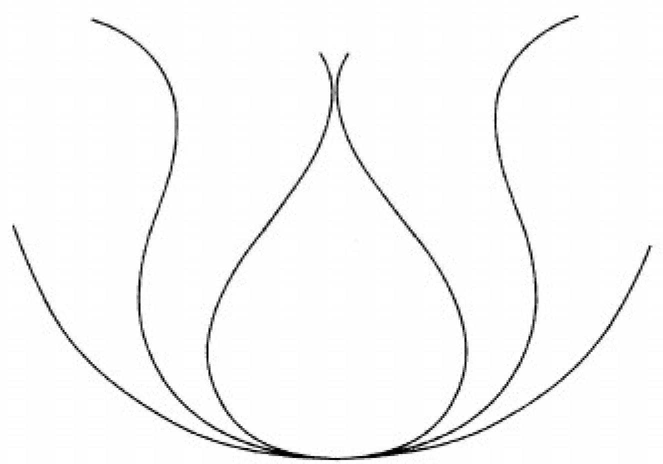


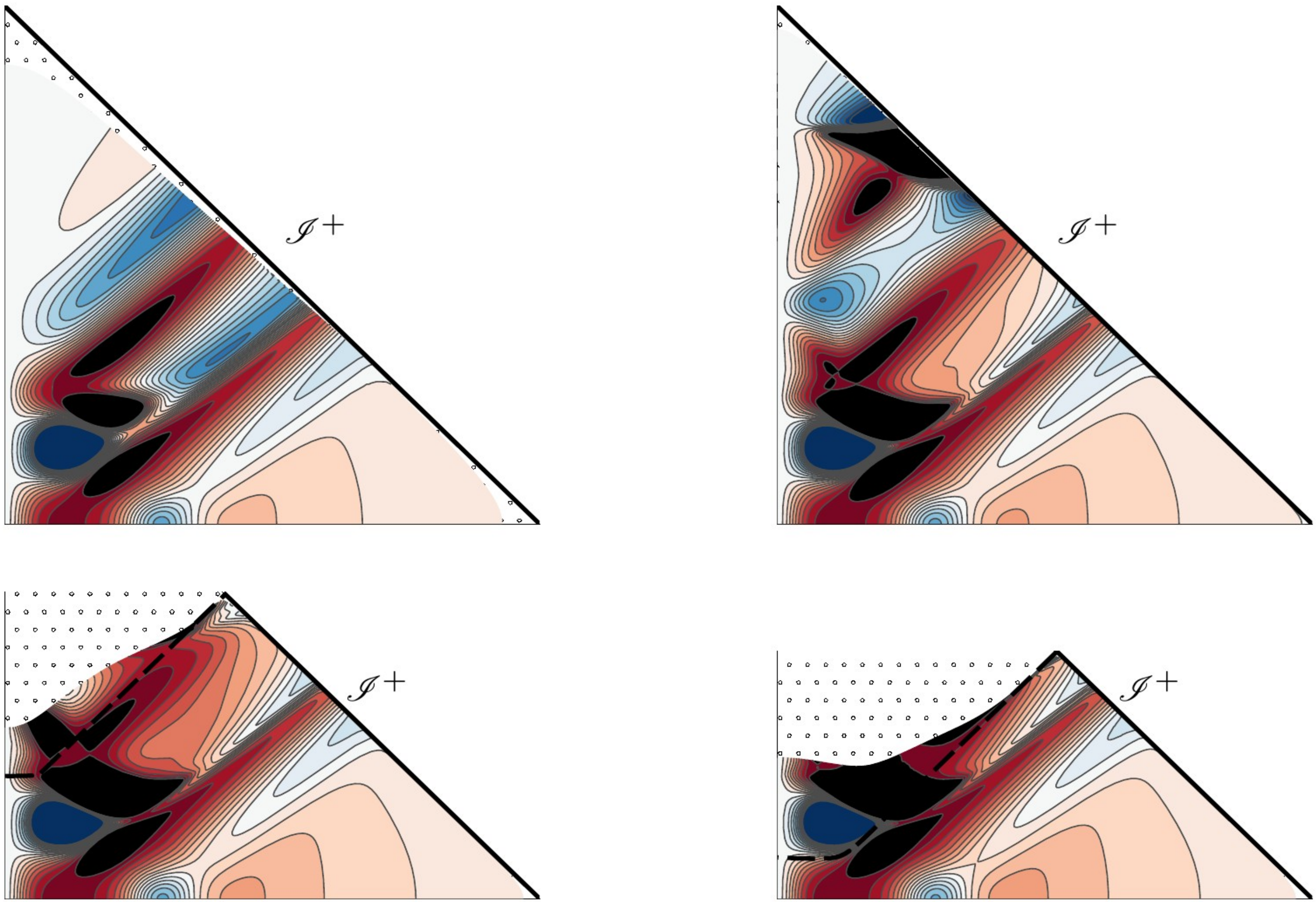
FIG. 2. Embedding of the equatorial geometry into Euclidean space. The three embeddings are for amplitudes $A=2$, 5, and 15, respectively.

The error introduced by the nonconservative form (20) was not trivial, even on quite fine grids, and could amount to almost an order-of-magnitude difference between the masses given by (11) and (12). However, even using a uniform grid on which these integrals agreed, they still differed from (14) by over another order of magnitude. It was necessary to go to a fourth-order accurate scheme to get all the masses to agree. There is no unique way of

IOP Publishing Classical and Quantum Gravity
Class. Quantum Grav. **35** (2018) 215003 (19pp) https://doi.org/10.1088/1361-6382/aae1bc

Slicing conditions for axisymmetric gravitational collapse of Brill waves

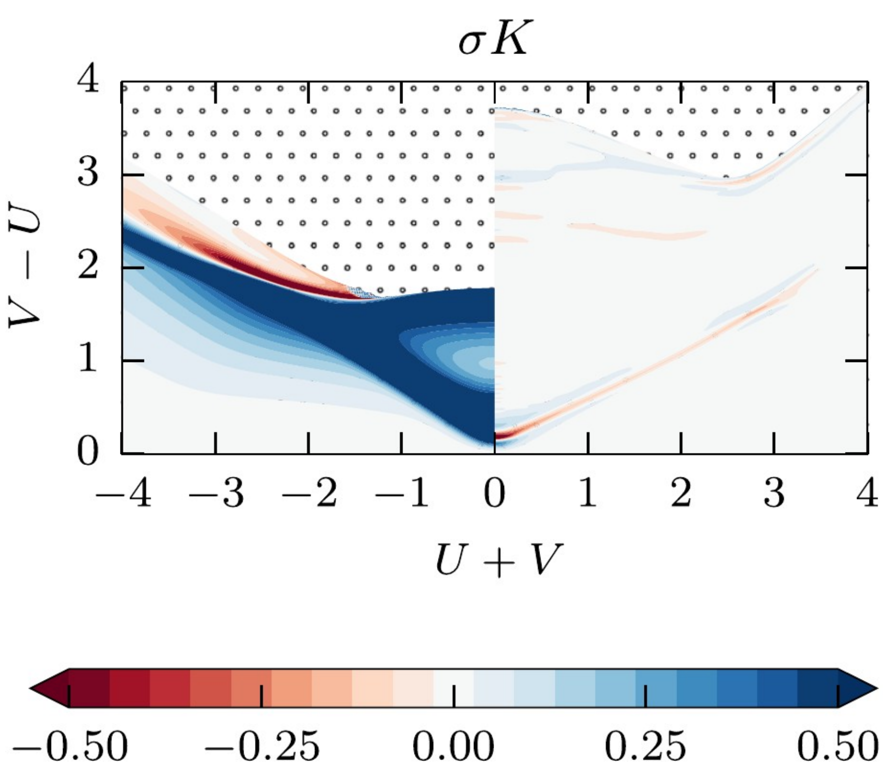
Anton Khirnov[1] and Tomáš Ledvinka

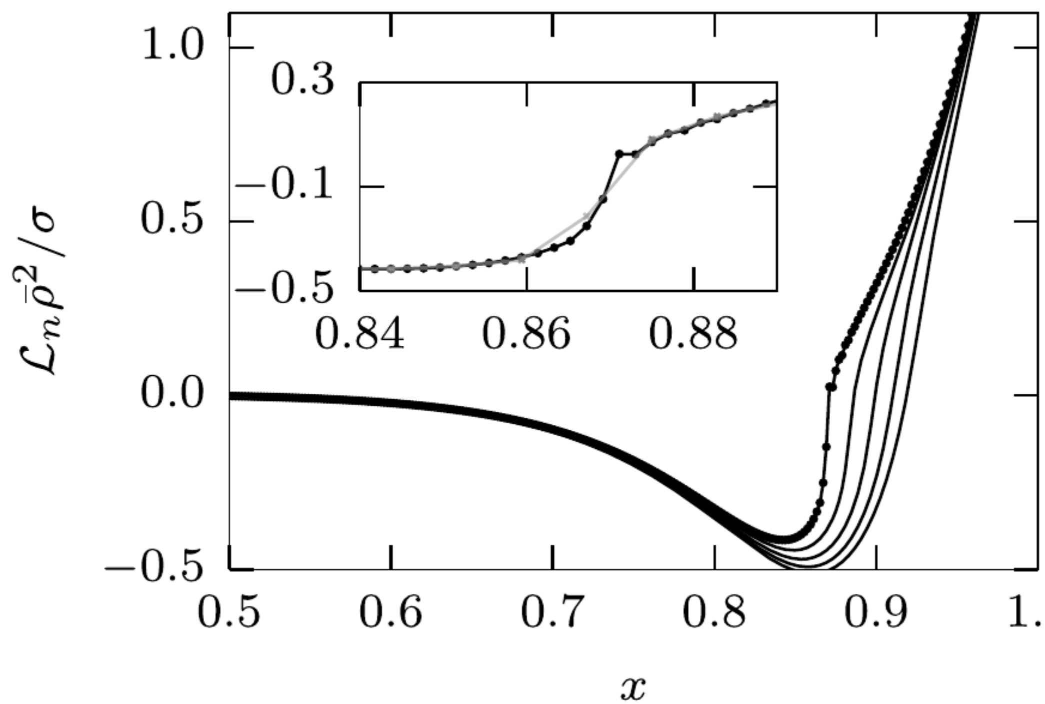


IOP Publishing Classical and Quantum Gravity
Class. Quantum Grav. 35 (2018) 215003 (19pp) https://doi.org/10.1088/1361-6382/aae1bc

Slicing conditions for axisymmetric gravitational collapse of Brill waves

Anton Khirnov[1] and Tomáš Ledvinka





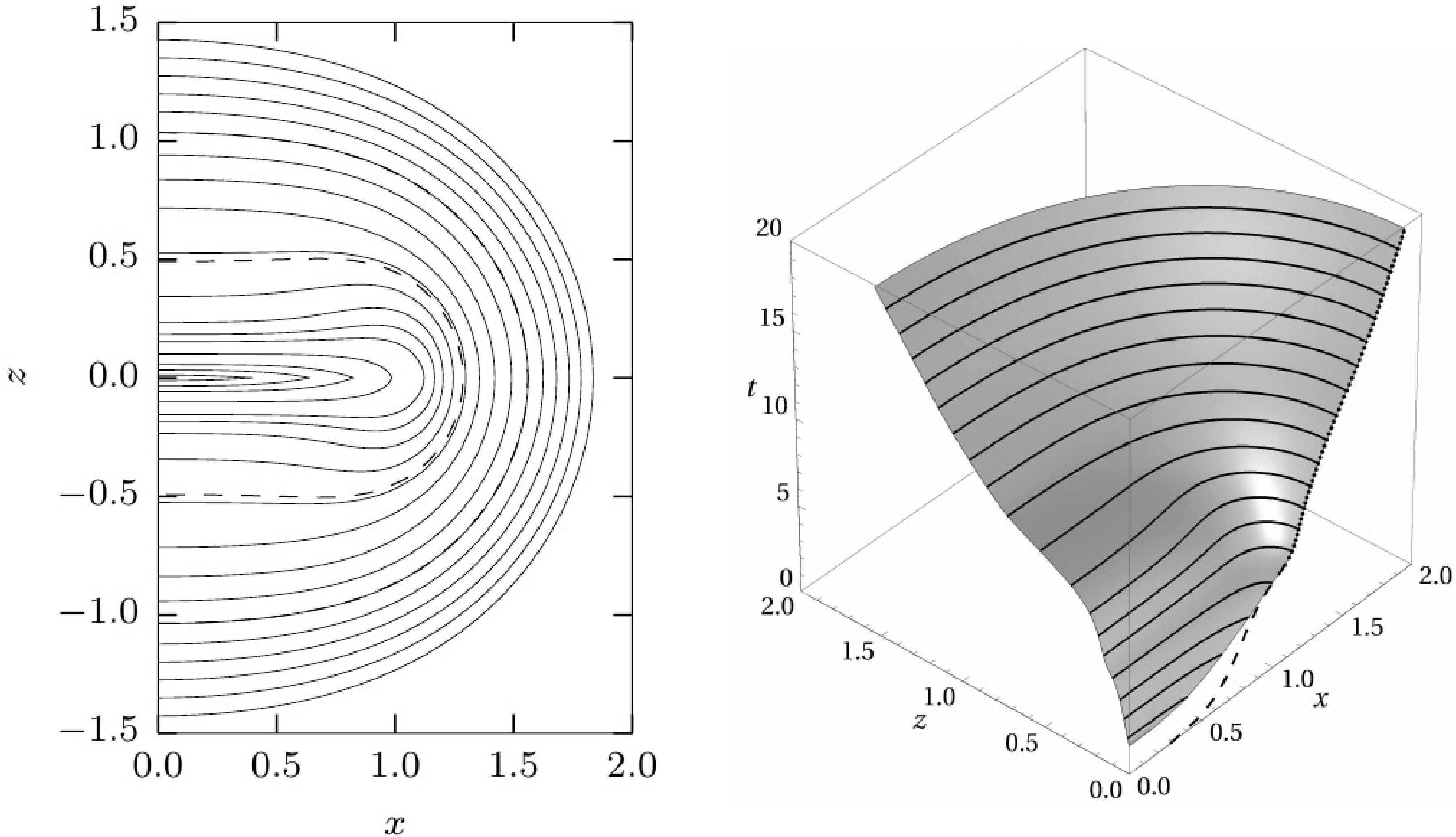
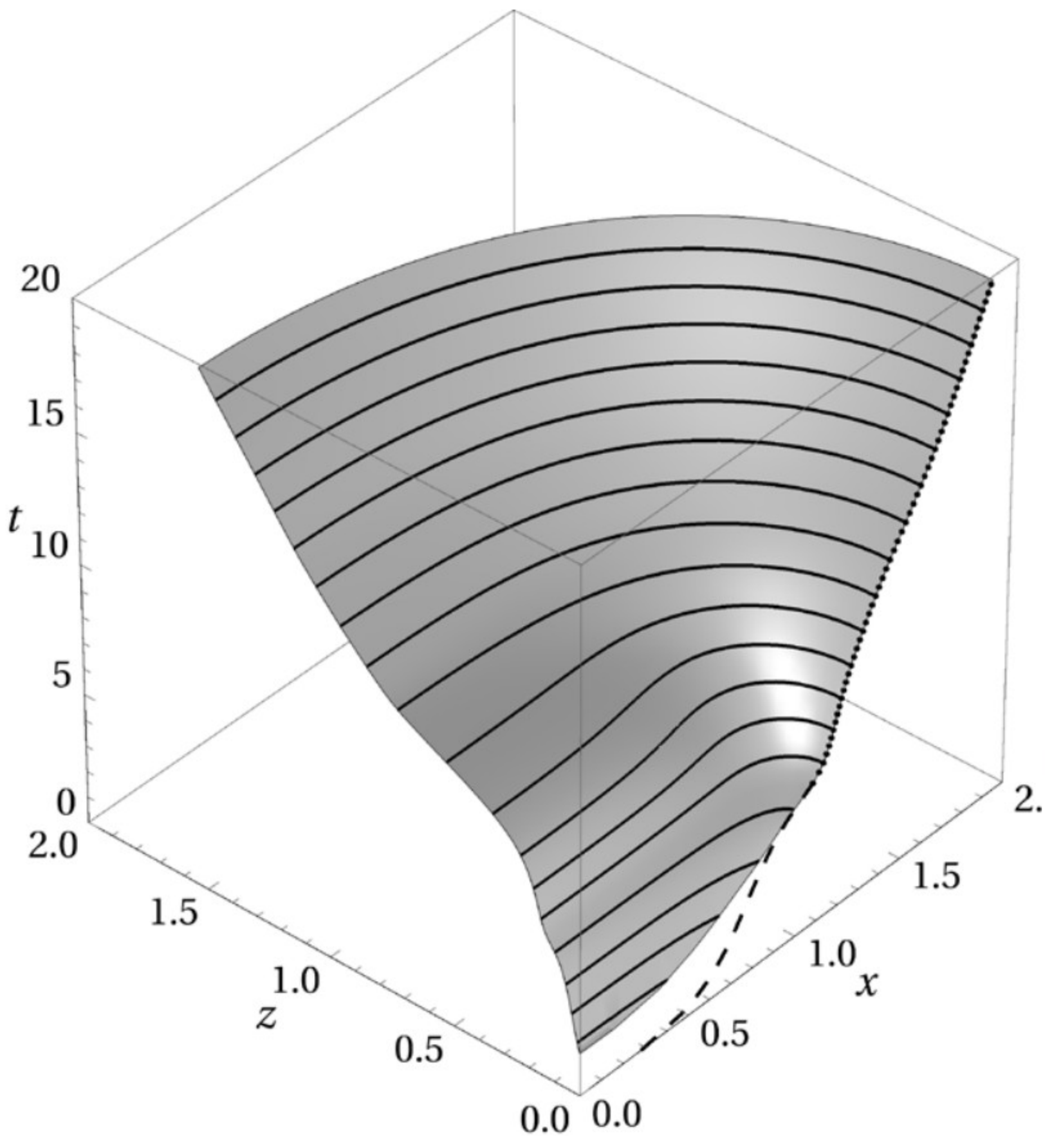
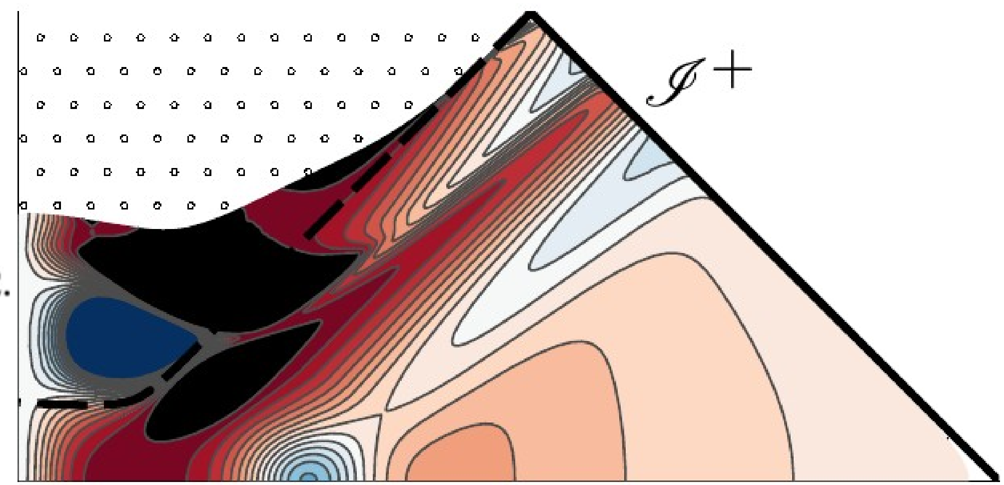


Figure 1.15: *Left:* Event horizon (solid curves) in $x - z$ plane at simulation times $t = 2, 3, ..., 19$ in $\sigma = 1, A^{\text{Brill}} = 5$ simulation with $\beta^i = 0$. The dashed curves show apparent horizon at $t = 10$ (first MOTS appears at $t \approx 9.5$) and $t = 14$. Much of the horizon growth is a coordinate effect — when coordinates with vanishing shift are used, the horizon coordinate radius grows even when its area stays constant. The early event horizon is not smooth. It has a rim which worldsheet is a counterpart of the single vertex at the "beginning" of the spherical symmetric event horizon in Fig. 1.10. *Right:* The same plot with added time dimension. It shows that the horizon becomes smooth when null rays propagating radially in the equatorial plane (dashed



20
15
t
10
5
0
2.0
1.5
1.0
z
0.5
0.0
0.0
0.5
1.0
x
1.5
2.



$\mathscr{I}^+$

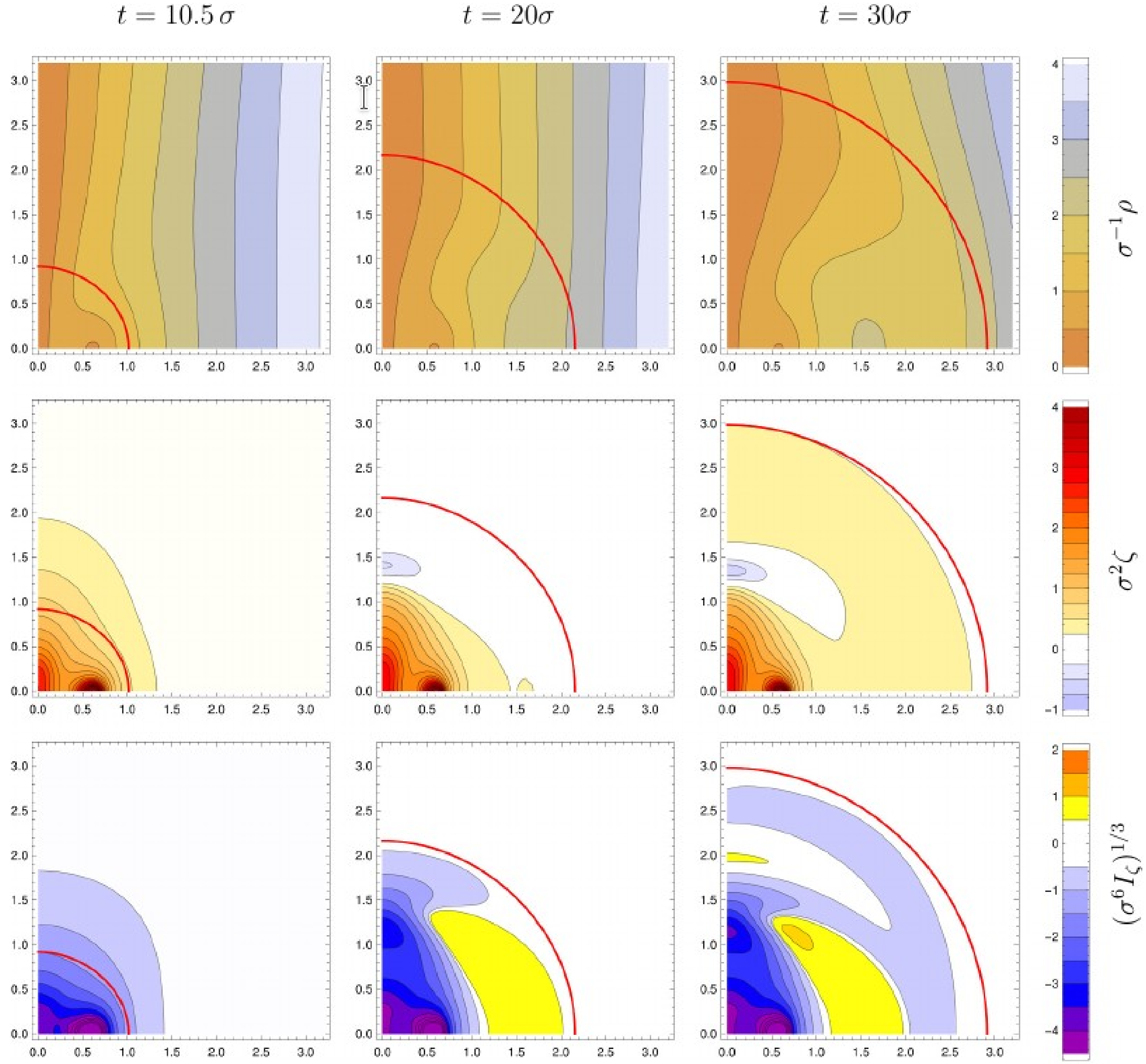


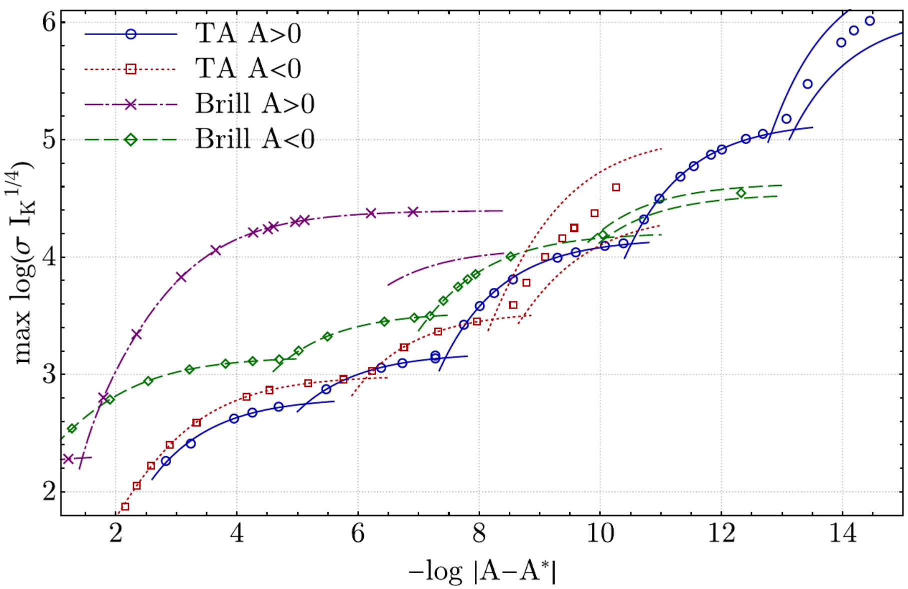
Figure 1.14: Invariant quantities ρ, ζ and I_ζ in the $x{-}z$ plane as the newly born black hole settles toward the spherically symmetric state in an approximately maximal slicing. TA initial data with $\sigma = 1, A = \overline{0.9}$. The simulation coordinates x, z do not have direct meaning in the central region. Each column shows fields at given simulation time (indicated at the top). The top row shows the circumferential radius ρ and the usual horizon expansion typical for coordinates with $\beta^i = 0$. Red segments indicate MOTS at given times. Notice the difference between coordinate radius and circumferential radius and the fact that all three apparent horizon areas are roughly the same; their masses are 1.03, 1.19, 1.20, spacetime $M_{\mathrm{ADM}} \doteq 1.24$. The invariant ζ is in the middle row, and I_ζ at the bottom. Departures from the "trivial" Schwarzschild geometry are either radiated or end up inside the black hole.

PHYSICAL REVIEW LETTERS **127,** 011104 (2021)

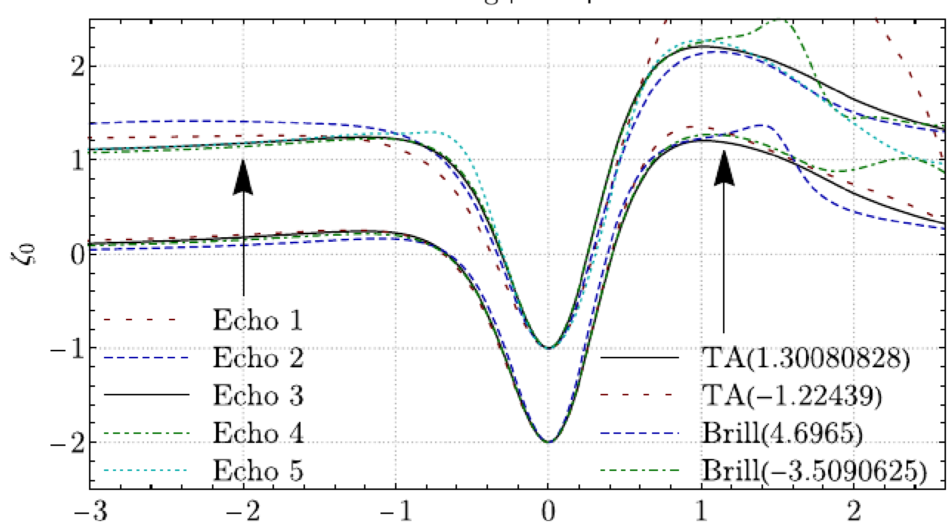
Universality of Curvature Invariants in Critical Vacuum Gravitational Collapse

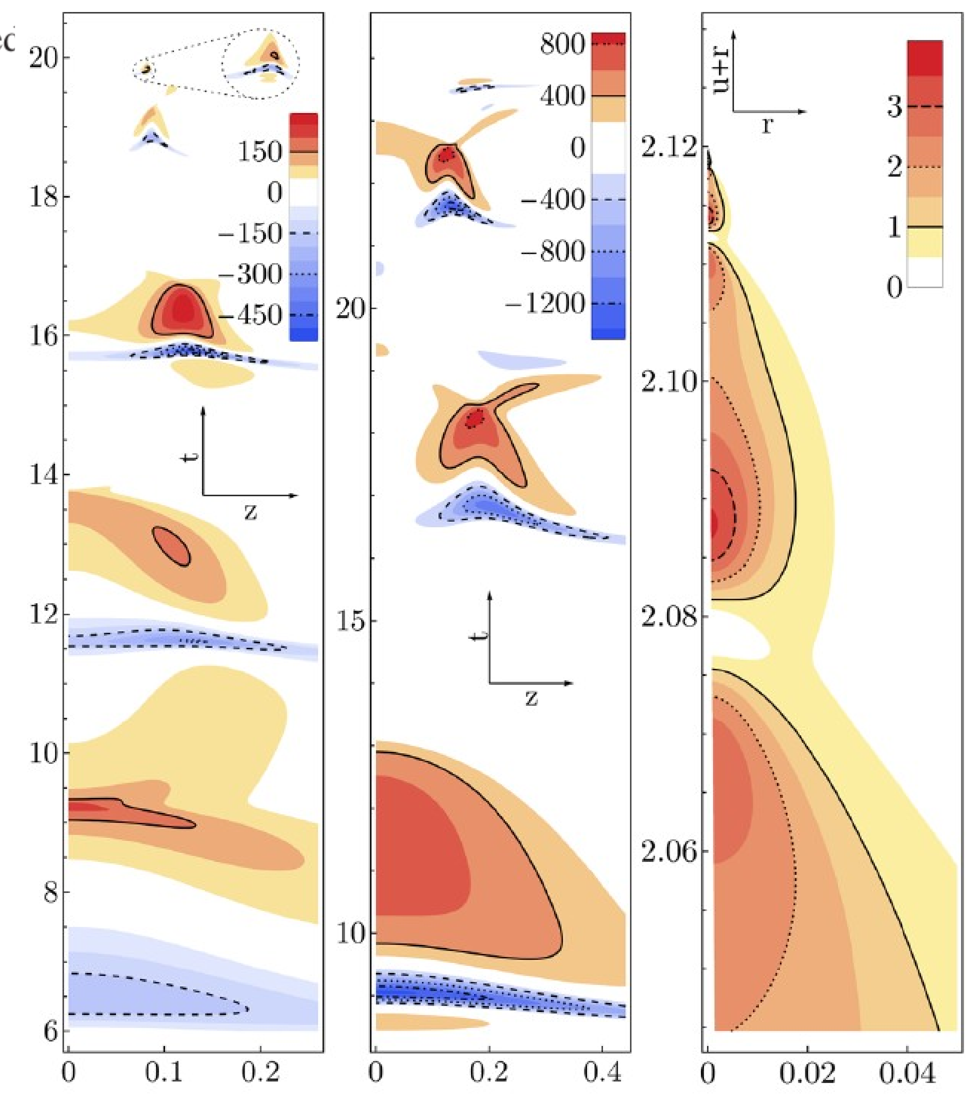
Tomáš Ledvinka* and Anton Khirnov†

Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, CZ-180 00 Prague, Czech Republic



accepted





Universality of Curvature Invariants in Critical Vacuum Gravitational Collapse

Tomáš Ledvinka[*] and Anton Khirnov[†]
Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, CZ-180 00 Prague, Czech Republic

(Received 18 February 2021; accepted 8 June 2021; published 2 July 2021)

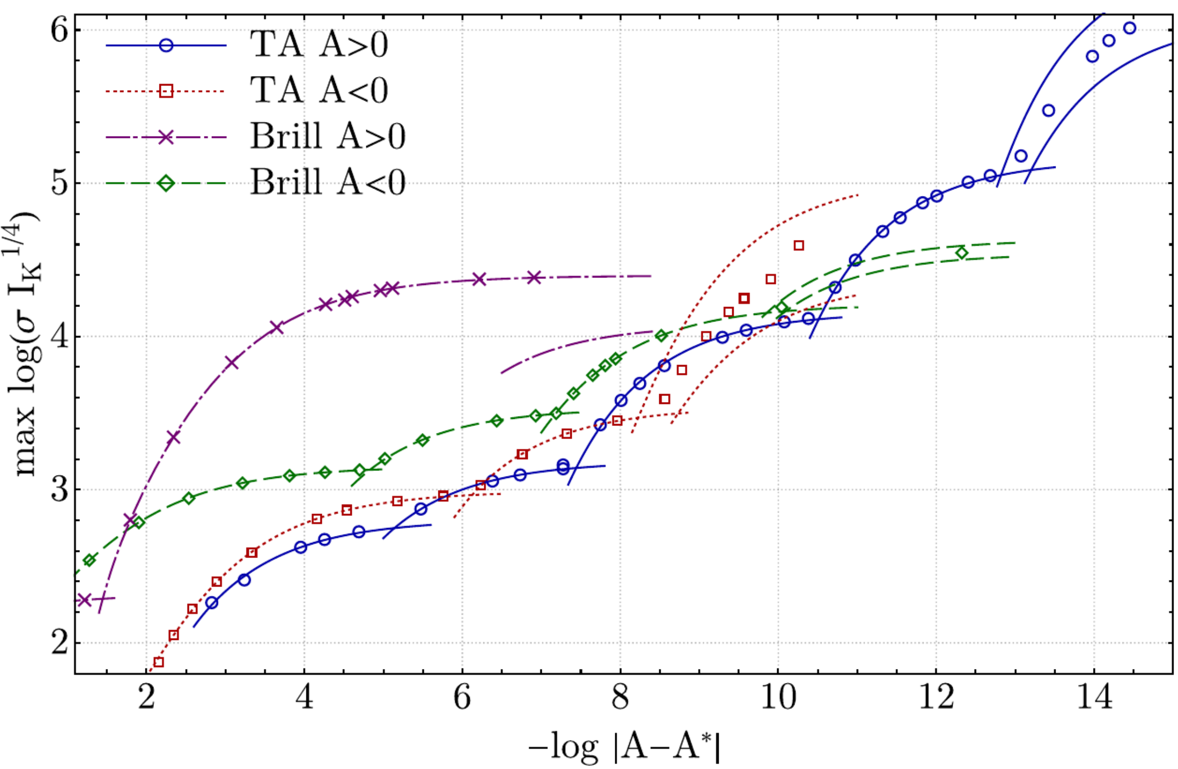


FIG. 1. Global maxima of the Kretschmann invariant in subcritical spacetimes with four families of initial data depending on a parameter A. As A approaches the critical value A_*, the maxima get ever larger, as newly appearing local extrema overtake earlier ones. To illustrate the smooth dependence of these local extrema (echoes) on the parameter A, we fit the simulation results shown as points with a polynomial—typically a simple linear dependence $\log I_K^{\max} = pA + q$. The plotted curves are thus composed of segments, each corresponding to a specific local maximum being the strongest one. An effect of the uncertainty of A_* within the final bisection interval is indicated in the rightmost segments; $A_*^{\mathrm{Brill+}}$ is taken from Ref. [12].

Initial data

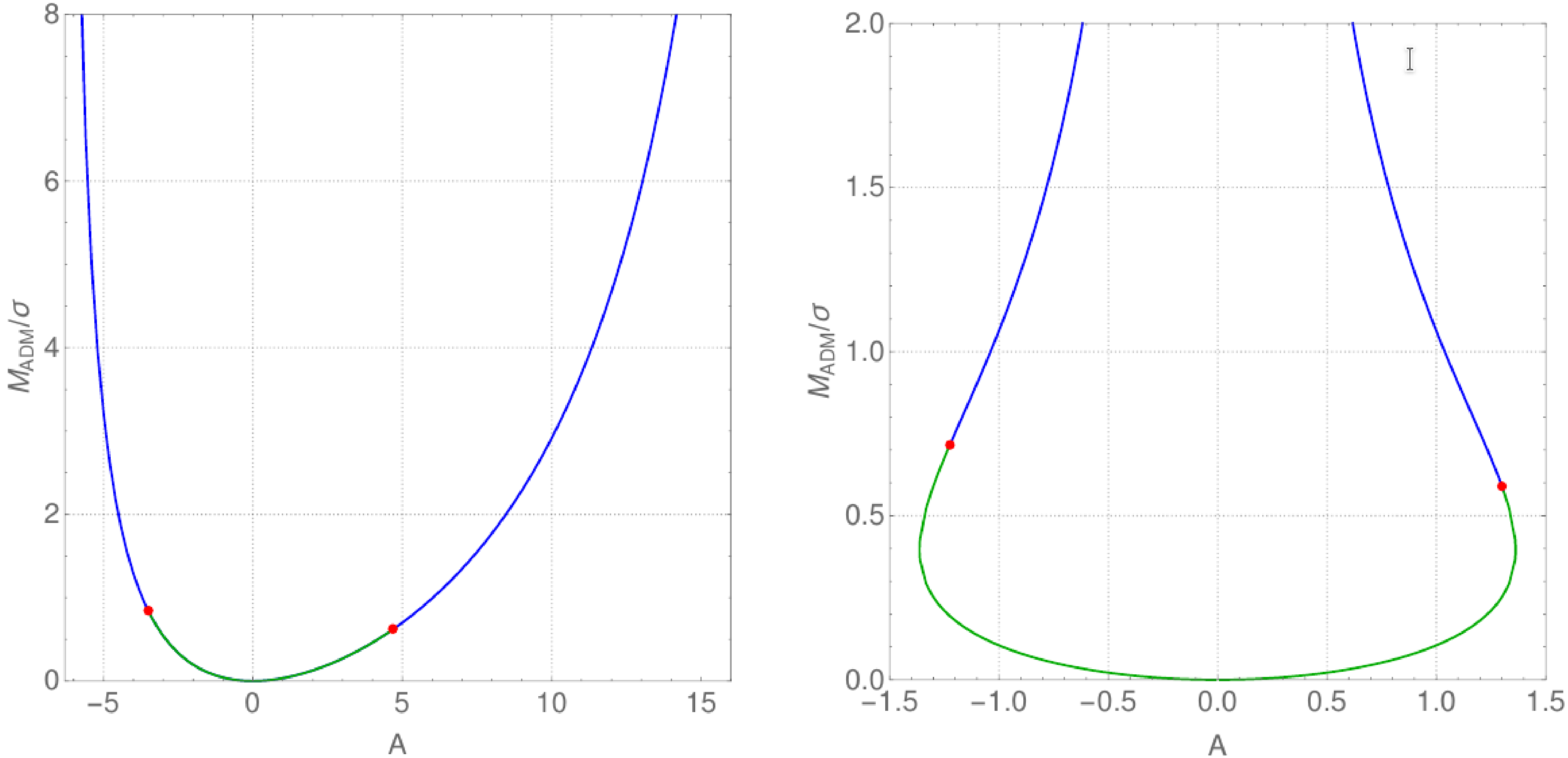


Figure 1.11: The dependence of the ADM mass of the spacetime with gravitational waves on the amplitude A of the Brill (left) and TA (right) initial data. Points on both curves indicate critical values of the parameter A. For smaller $|A|$ (green curve) the gravitational disperse and leave behind an empty space. Stronger initial data lead to black holes.

$$\rho^2 = \eta^a \eta_a$$

$$I_A = \frac{1}{2}\eta^{a;b}\eta_{a;b} \doteq 1 - 2m\frac{\sin^2\theta}{r}$$

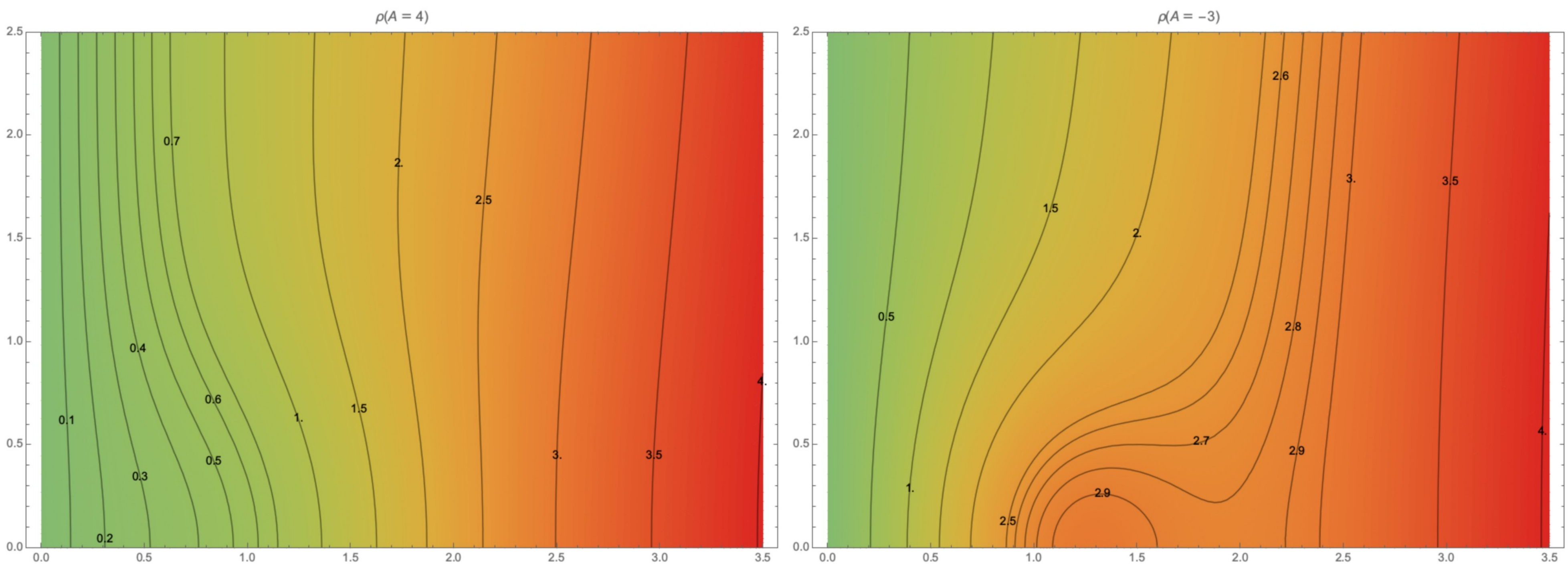
$$I_B = R^{abcd}\eta_{a;b}\eta_{c;d} \doteq 4m\frac{2 - 3\sin^2\theta}{r^3} + 8m^2\frac{\sin^2\theta}{r^4}$$

$$I_C = \eta^{a;bc}\eta_{a;bc} = R^{abcd}R_{abce}\eta_d\eta^e \doteq 12m^2\frac{\sin^2\theta}{r^4}$$

$$I_D = R^{abcd}R_{aecf}\rho_b\rho_d\rho^e\rho^f \doteq 6\,\frac{m^2(3\,\cos^4\theta - 3\,\cos^2\theta + 1)}{r^6} + 12\,\frac{m^3\sin^2\theta(3\,\cos^2\theta - 2)}{r^7} + 24\,\frac{m^4\sin^4\theta}{r^8}$$

$$I_E = R^{abcd}R_{aecf}\eta_{b;d}\eta^{e;f} \doteq 4m^2\frac{1 - 3\sin^2\theta}{r^6} + 16m^3\frac{\sin^2\theta}{r^7}$$

$$I_K = R^{abcd}R_{abcd} \doteq 48\frac{m^2}{r^6} \qquad I_{AA} = (1 - I_A)/\rho^2$$



The circumferential radius ρ for $A = 4$ (left) and $A = -3$ (right), when it is no longer monotonous function of radial coordinate.

Universality of Curvature Invariants in Critical Vacuum Gravitational Collapse

Tomáš Ledvinka * and Anton Khirnov †
Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, CZ-180 00 Prague, Czech Republic

(Received 18 February 2021; accepted 8 June 2021; published 2 July 2021)

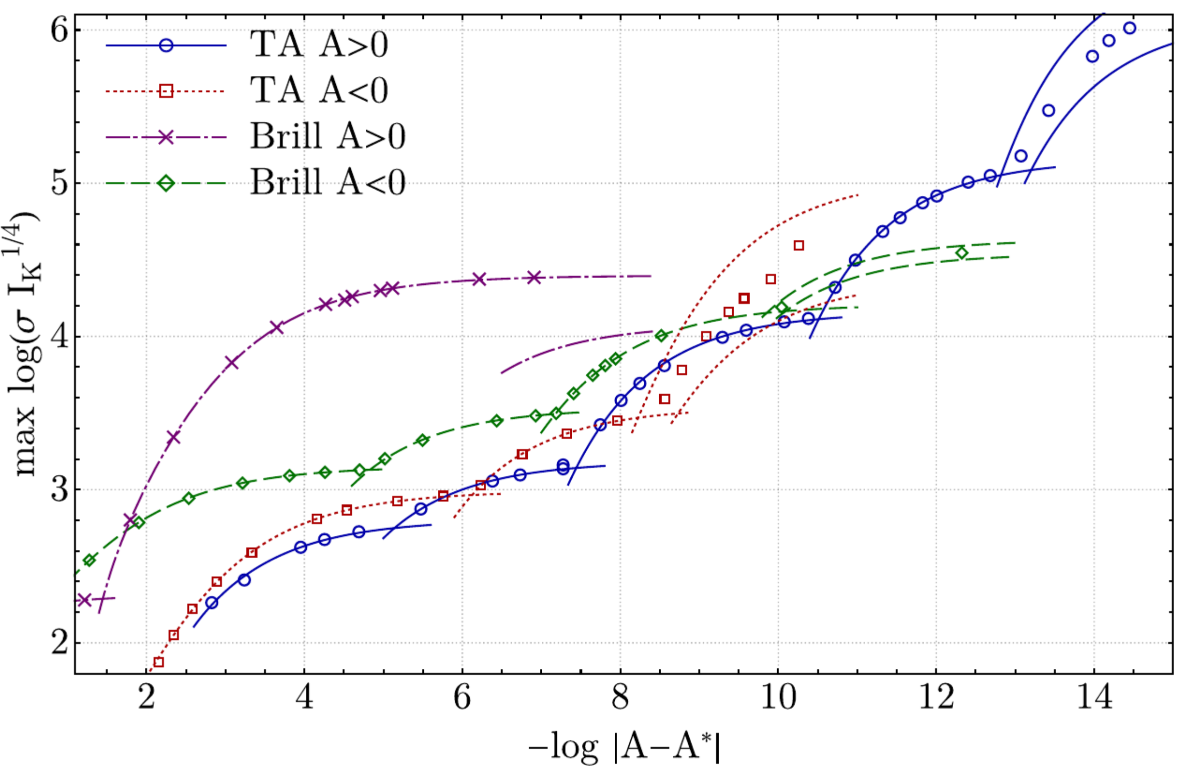
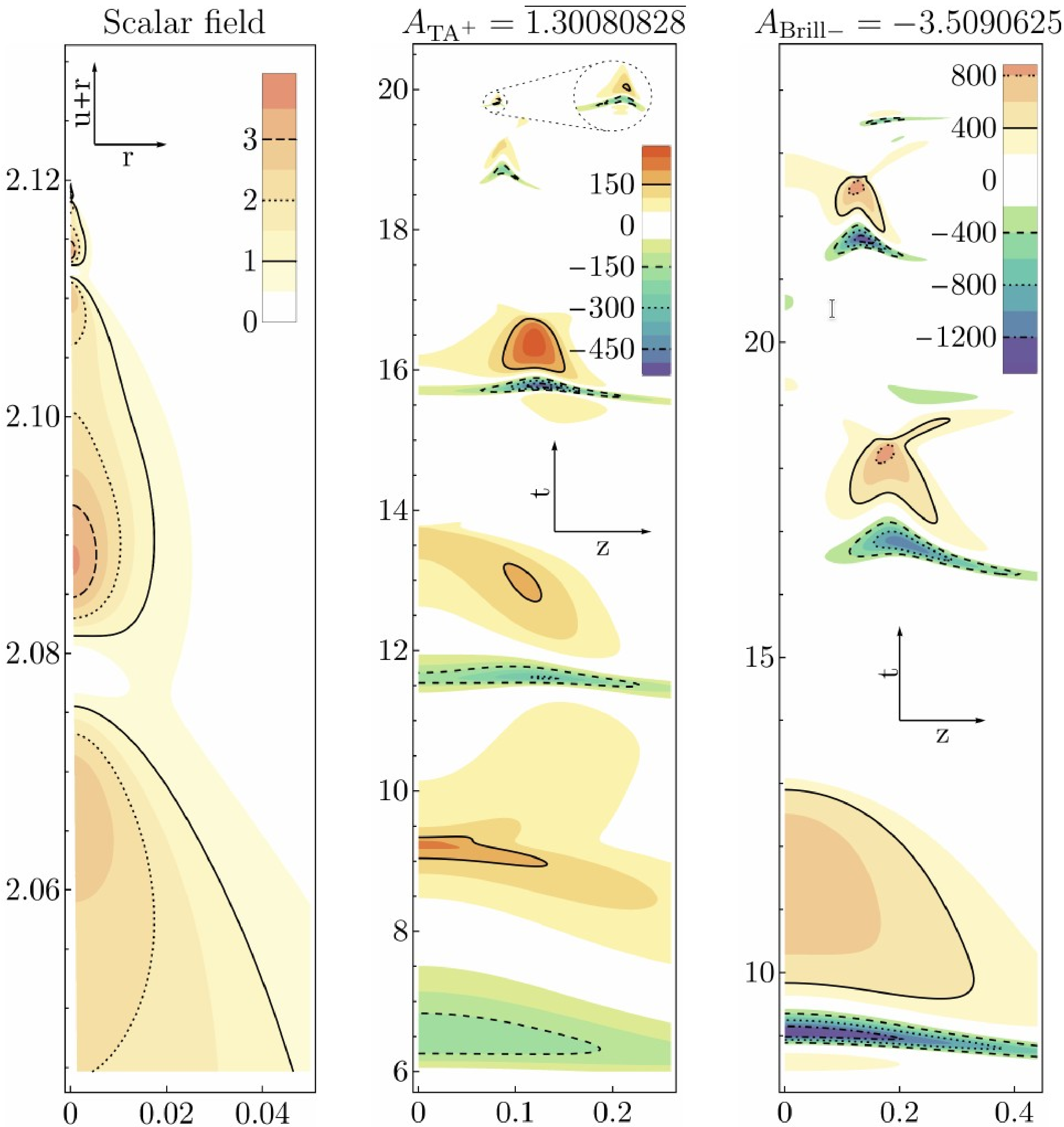


FIG. 1. Global maxima of the Kretschmann invariant in subcritical spacetimes with four families of initial data depending on a parameter A. As A approaches the critical value A_*, the maxima get ever larger, as newly appearing local extrema overtake earlier ones. To illustrate the smooth dependence of these local extrema (echoes) on the parameter A, we fit the simulation results shown as points with a polynomial—typically a simple linear dependence $\log I_K^{\max} = pA + q$. The plotted curves are thus composed of segments, each corresponding to a specific local maximum being the strongest one. An effect of the uncertainty of A_* within the final bisection interval is indicated in the rightmost segments; $A_*^{\mathrm{Brill}+}$ is taken from Ref. [12].



Scalar field
$A_{\mathrm{TA}^+} = \overline{1.30080828}$
$A_{\mathrm{Brill}-} = -3.5090625$
u+r
r
t
z
t
z
3
2
1
0
150
0
−150
−300
−450
800
400
0
−400
−800
−1200
2.12
2.10
2.08
2.06
20
18
16
14
12
10
8
6
20
15
10
0
0.02
0.04
0
0.1
0.2
0
0.2
0.4

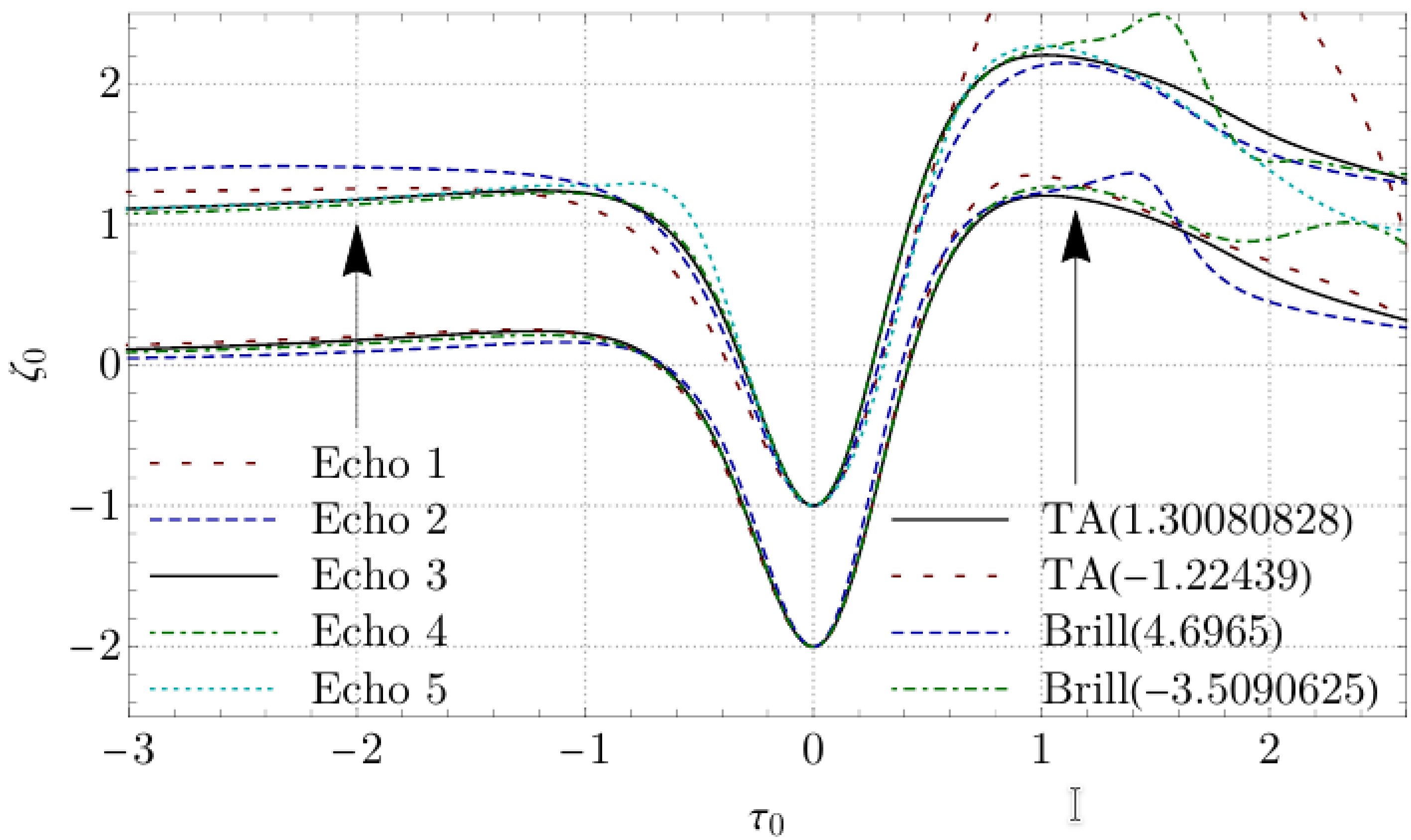


Figure 1.16: Profiles of the invariant ζ along a timelike worldline through 'echo' when rescaled in time and amplitude – $\zeta_0 = \lambda^2\zeta(\tau_e + \lambda\tau_0)$ (see text). The same scale λ is chosen so that so that we get dimensionless both the function value and the argument and at the minimum we get $\min(\zeta_0) = -2$. *Top curves* show the shifted value $\zeta_0 + 1$ of five successive 'echoes' which appear in the simulation with initial data (1.62) with $A_{\mathrm{TA}^+} = \overline{1.30080828}$. To demonstrate the universality of the curvature spikes, the *bottom curves* compare the observed profiles of ζ_0 of four different families of initial data with indicated initial data parameters. The plot is taken

Conclusions

A single scalar invariant is not enough to determine the spacetime geometry unambiguously, but because we know that ζ is the only nonvanishing component of the Riemann tensor at the axis, the echoes also represent approximate scaled copies of the same patch of spacetime. Because near its maximum ζ changes only slowly in the z direction, it is interesting that a similar but time-symmetric profile of ζ appears at the axis for the Weber-Wheeler-Bonnor cylindrical GW pulse [24].

Conclusions.—Critical collapse of gravitational waves has been studied for a long time with the hope that a clear, universal, discretely self-symmetric structure will appear. We showed that the first echoes in a near-critical collapse exhibit only a partial similarity to the DSS behavior of a massless scalar field. While we observed a universal profile of the echo forming patches of strongest spacetime curvature as approximate copies of a universal template, these appear with apparently irregular delays and scales. Thus, we did not observe a universal and regularly self-similar solution in the $A \to A_*$ limit, and the dimensionless characteristics of the near-critical behavior seem to depend on the ID family.

Thank you