Springy relativistic toys

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Geodesics? What geodesics?

- Locomotion in spacetimes.
- Extended bodies.
- Periodic change of configuration.
- Motion of the system?

Swimming robots?

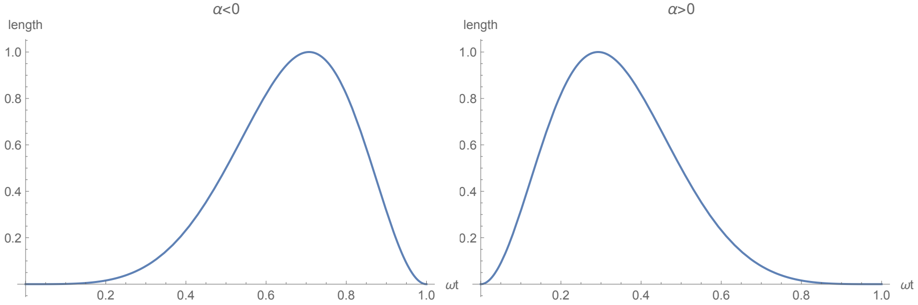
- Controlled Lagrangians.
- Prescribed deformation.
- Active dumbbell.
- Schwarzschild background of mass M.

$$L_d = -m\sqrt{1 - \frac{2M}{r_1} - \frac{\left(\frac{\mathrm{d}r_1}{\mathrm{d}t}\right)^2}{1 - \frac{2M}{r_1}}} - m\sqrt{1 - \frac{2M}{r_1 + l} - \frac{\left(\frac{\mathrm{d}r_1}{\mathrm{d}t} + \frac{\mathrm{d}l}{\mathrm{d}t}\right)^2}{1 - \frac{2M}{r_1 + l}}}$$

$$l(t) = \frac{\delta l}{2}(1 - \cos\left[2\pi\omega t\{\alpha(1 - \omega t) + 1\}\right])$$

- Frequency ω, maximum length δl, shape parameter α.

Swimming robots?

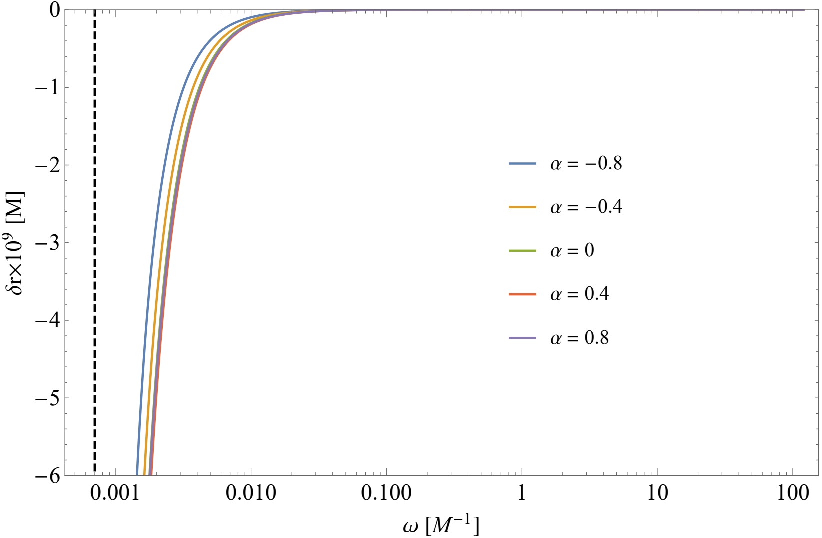


- Point → point.
- $\delta r = r_{\text{final}} - r_{\text{geodesic}}$.

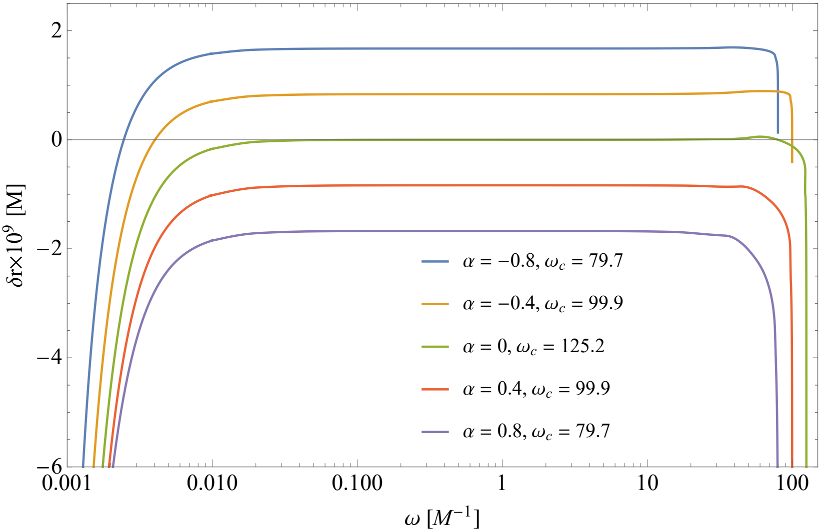
Swimming robots?

- Maximum length: $\delta l = 5 \times 10^{-3} M$.
- Initial height: $r(0) = 120M$.
- Initial velocity—none.
- Drop!

No Newtonian swimming



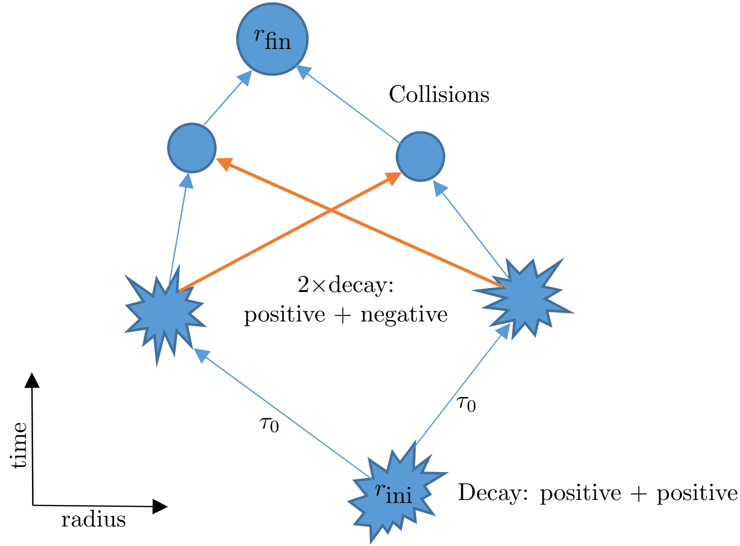
Relativistic swimming?



. . . no swimming

- Superluminal communication.
- Better model?
- Spring.
- No relativistic spring.
- Discrete model.

Discrete spring



Discrete spring

- Decay, fusion, geodesics.
- Local interaction only.
- Negative-mass interaction particles.
- Negative momentum.
- Tension.
- Needed: geodesics, conservation of 4-momentum.
- Radial motion only—symmetry.

Discrete spring

- Conservation of 4-momentum.

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = \mu_1 \frac{\mathrm{d}t_1}{\mathrm{d}\tau_1} + \mu_2 \frac{\mathrm{d}t_2}{\mathrm{d}\tau_2}$$
$$\frac{\mathrm{d}r}{\mathrm{d}\tau} = \mu_1 \frac{\mathrm{d}r_1}{\mathrm{d}\tau_1} + \mu_2 \frac{\mathrm{d}r_2}{\mathrm{d}\tau_2}$$

- Normalization of 4-velocity.

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{1}{\sqrt{-g_{tt} - g_{rr}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2}}$$

- Outcome:

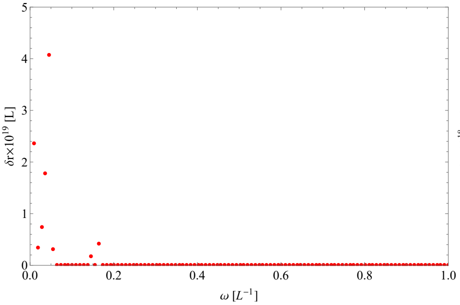
$$4 \quad \mu_1^2 g_{rr}\left(\frac{\mathrm{d}r_1}{\mathrm{d}\tau_1}\right)^2 - 2(\mu_1^2 - \mu_2^2 + 1)(2\mu_1 g_{rr}\frac{\mathrm{d}r}{\mathrm{d}\tau})\frac{\mathrm{d}r_1}{\mathrm{d}\tau_1}$$
$$-(\mu_1^2 - \mu_2^2 + 1)^2 + 4\mu_1^2 + 4\mu_1^2 g_{rr}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 = 0 \qquad (1)$$

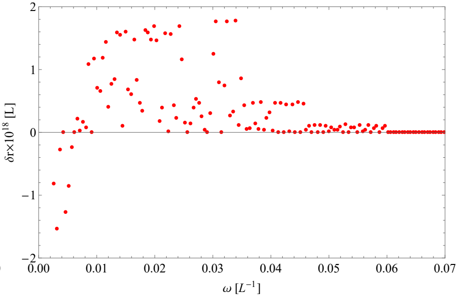
Discrete spring

- Decay time: τ_0.
- Relative masses of product particles: $\mu_1 = \frac{3}{2}, \mu_2 = -\frac{2}{5}$.
- Shape of the deformation curve.
- Initial decay velocities: $u_0 \to \delta l \approx 2u_0\tau_0$.
- Total coordinate time to collapse back: $\Delta t \to \omega := \frac{1}{\Delta t}$.

de Sitter

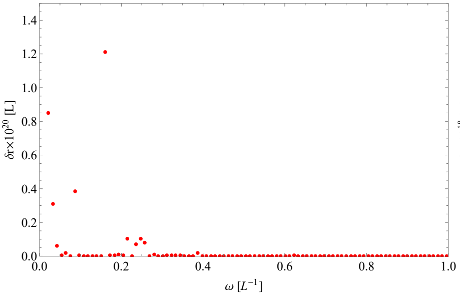
- Test: Dixon in dS $\rightarrow$ no swimming.
- Initial position $r_0 = 120\frac{1}{\sqrt{30000\Lambda}}$.

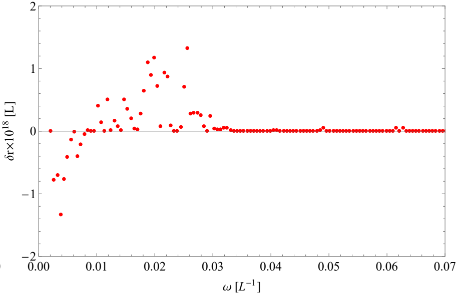




anti de Sitter

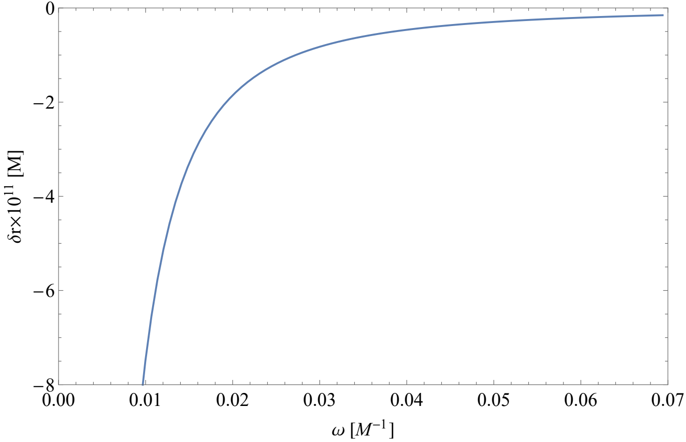
- Test: Dixon in AdS $\rightarrow$ no swimming.
- Initial position $r_0 = 120 \frac{1}{\sqrt{-30000\Lambda}}$.





Schwarzschild

- Small frequencies.
- Drowning only.
- Initial position $r_0 = 120M$.



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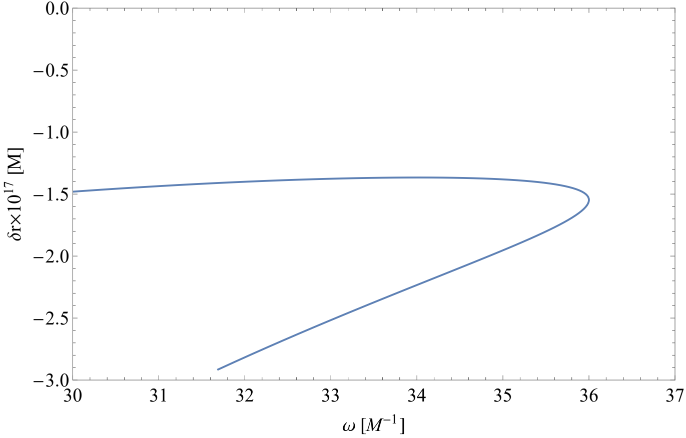

What next?

- Simple model.
- No swimming: $M_{\text{final}} > M_{\text{initial}}$.
- Other situations?
- Discrete spring passive.
- Active swimmer?