Gravitational Waves, Neutron Star Equation of State inference and Phase Transitions

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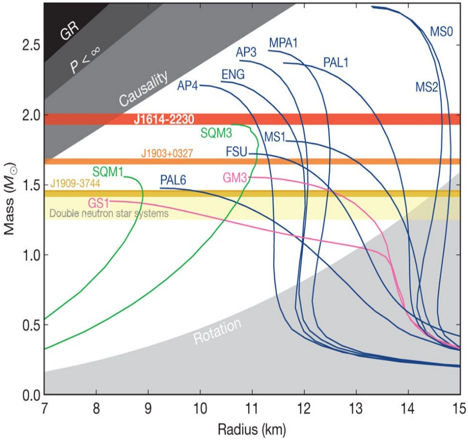
Overview

- A super brief introduction to Neutron Stars (NS) and the Equation of State (EoS)
- Gravitational Waves (GWs) measurements: the mass and Love (M, Λ)
- EoS inference & the Universal relations
- Phase Transitions – and their signatures

Neutron Stars and the Equation of State

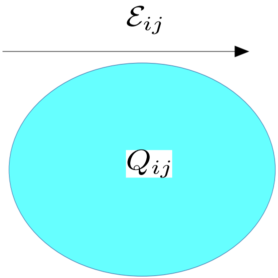


Demorrest et. al, 2010.

- Neutron Stars (NS) – remnants of collapsed stellar cores.
- Stellar structure of NS – gravitational collapse supported by neutron degeneracy pressure, strong nuclear forces …
- Density ~ 10^{15} g/cc – Equation of State (EoS) unknown
- Field equations – TOV sequences
- Sequences give the 'M-R curves'
- M – R curves constraints.
- **Purely exotic matter EOS 'do not' satisfy 2 $M_\odot$ pulsar observation.**

Introduction to Love Number Λ

- Tides discussed mathematically by geophysicist and mathematician A.E.H. Love in 1911.
- Tides – deformation of bodies under differential stresses.
- Generalised to context of NS in 2008.
- Measures the flexibility of a body under tidal stress.
- **Tidal parameter carries signature of NS EoS.**
- Astrophysical equivalent of Hooke's elasticity.
- $\Lambda = \lambda/M^5$



$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

Tanja Hinderer, ApJ, 677, 1216, (2008)

GW measurements: (M, Λ)

- GWs $\longrightarrow$ generated by time varying ($l \geq 2$) modes

$$\mathrm{h}_{\alpha\beta} = \partial_t^2 Q_{\alpha\beta} \quad \text{Quadrupole Formula}$$

- $Q_{\alpha\beta}$ $\longrightarrow$ From orbital evolution – dependence on (M, η, χ, ...)

  $\searrow$ From NS deformation – dependence on Λ
- We have both (M,Λ) dependence in GW phasing
- (M,Λ) inferred by 'Matched Filtering'
- Inference :(M, δM) (Λ, $\delta\Lambda$) because of noise in detector.

$(M, \Lambda) \rightarrow (M, R)$

- No R information in GW raw posterior
- Λ cannot uniquely pin an EoS – degeneracies.
- Two possible solutions

EoS Insensitive Universal Relations
1. Convert Λ to M
2. C- Λ relations
3. M-R plots
4. MODEL INDEPENDENT ....!!!

Parametrised EoS
1. Parametrises p = p(ρ) directly
2. M-R plots
3. MODEL DEPENDENT ...!!!

Model independent seems the more favourable choice ...

The (M, R) posterior: GW170817

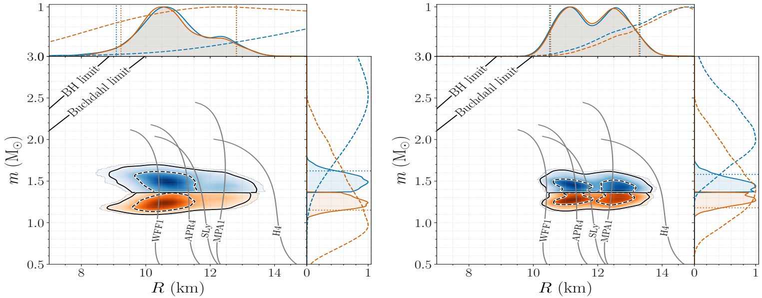
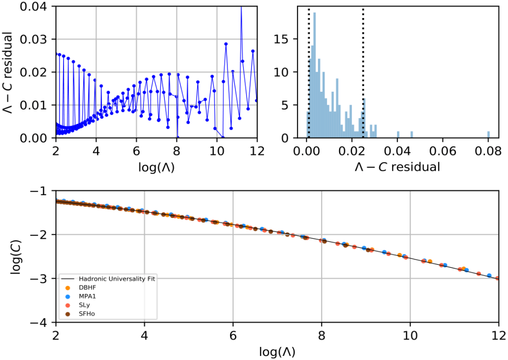


Figure: M, R posteriors as reproduced by using the EoS insensitive relations (left) & the parametrised EoS formulation

Abbott et. al., (2018)

The Universal Relations

Yagi & Yunes (2013) -- trend in numerical simulations.



$$\log C = -0.007\,(\log\Lambda)^2 - 0.072\,(\log\Lambda) - 1.067$$

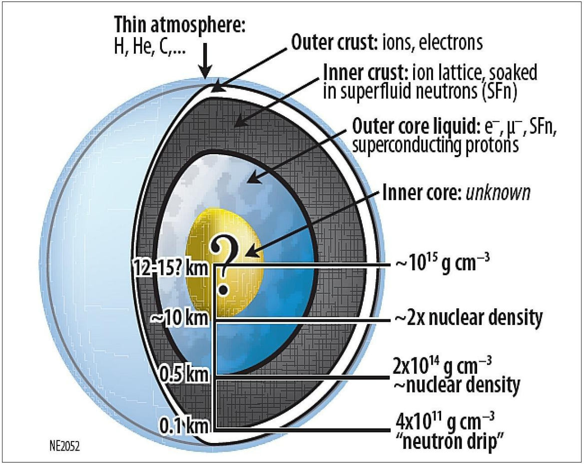
Universal relation of the first kind.

Love number Λ and compactness C = M/R show remarkable fits

The Universal Relations: Why ?

- Seen through numerical TOV sequences.
- Some unresolved debate on why ? Two propositions
- Outer NS layers contribute most to $\Lambda - C$ behaviour - EoS fairly universal in outer layers.
- Matter equivalent of the BH no-hair

Phase Transitions (PT): Introduction

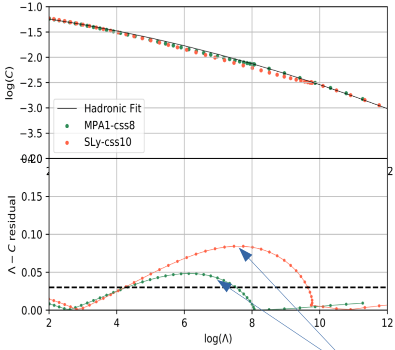


- Fundamental inter – particle interactions unknown at $\rho \sim$ saturation density.
- Energetically more favourable for matter to exist in "exotic phases"
- Examples – (quark deconfinement, pion condensate…)
- First order Phase Transition (PT) $\rightarrow$ p continuous ρ discontinuous across boundary.
- Boundary defined by equating some Free Energy like in thermodynamics…
- Exotic phase implementation $\rightarrow$ parametrisation with constant value of sound speed c_s^2.

Image credit: NASA/NICER

PT: Why bother about it ?

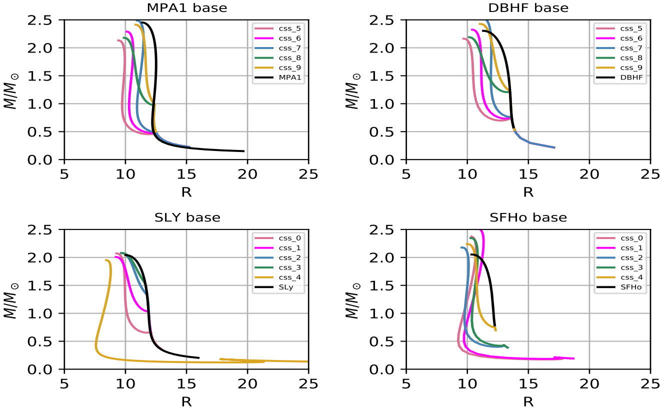
Phase transition brings a problem...!!!



Deviations greater than largest of ordinary residuals

- With PT, we see breakage of the Λ – C universal relations.
- Nothing new: also have been previously observed in Chatziioannou & Han (2019)
- Cannot use Λ – C for (M,Λ) → (M,R).
- No model independent way for EoS inference.
- At the very least – can we separate out PT at the level of (M,Λ) ?
- If not – how far are we ?

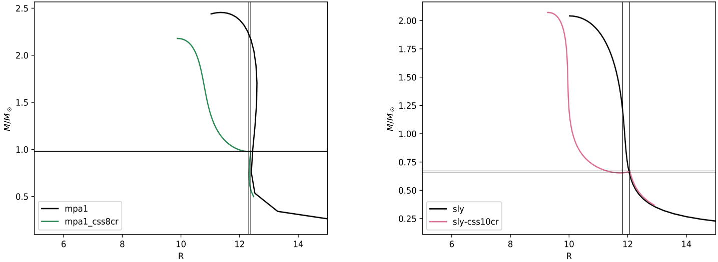
PT: EoS curves



Salient feature → Splitting of the hybrid branch sequences from the purely hadronic branch.

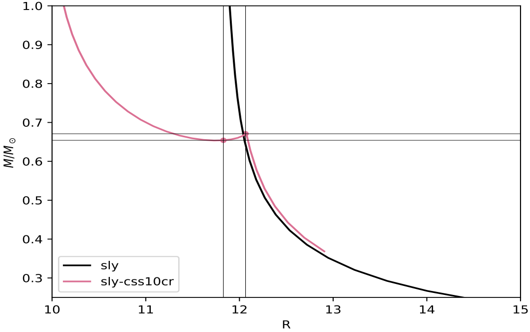
PT EoS curves : A closer look

- Hybrid sequences join the main sequence at some point.
- Focus on the morphology of join.



Mass ranges which support more than 1 radii

- Multiple radii R for same mass value M - a beautiful result.
- $\Lambda \propto C^{-5} \rightarrow$ Shows up in (M, R) data.
- Unstable regions...

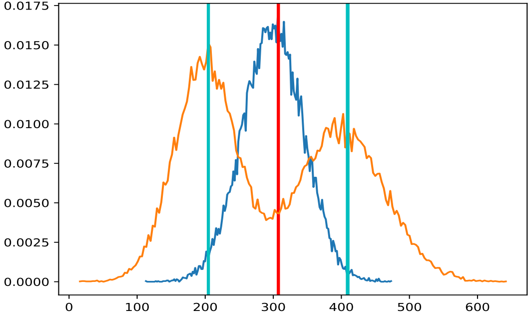


Genysis of a gap in values of Λ – another very ubiquitous signature.

(M, Λ) posteriors & the PT smoking gun...

Implications on the Λ posterior

- Characteristic ‘parting’ of posterior.
- Unimodal Λ posteriors turn bimodal.

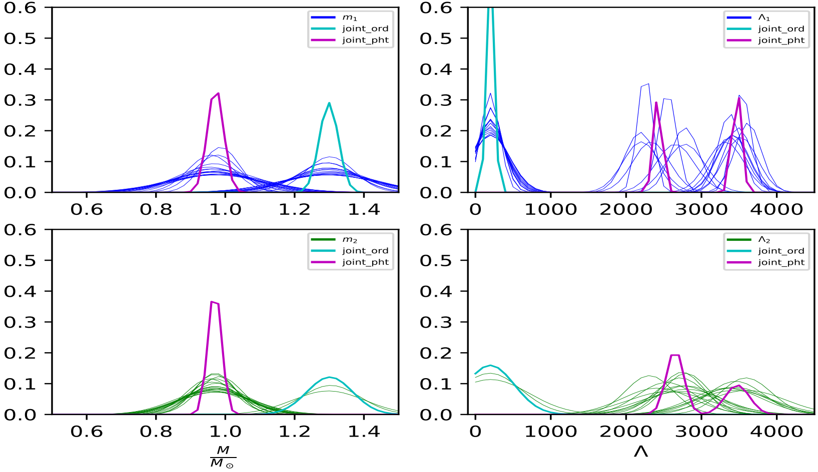


- Smoking gun for the evidence of a phase transition....!!

Mock population studies

- Study the marginalised (M, Λ) posteriors for a mock population of events.
- Scale $1/\text{SNR}$ to give errors in $(M, \Lambda) \to (\delta M, \delta\Lambda)$
- Assume a probability distribution of SNR.
- Assume a specific EoS to interpolate (M, Λ) – without loss of generality. Qualitative feature is the same.

Result



m_1
joint_ord
joint_pht
Λ_1
joint_ord
joint_pht
m_2
joint_ord
joint_pht
Λ_2
joint_ord
joint_pht
$\frac{M}{M_\odot}$
Λ

To Conclude

- Mixed phases not conclusively ruled out... can happen in NS populations.
- PT violate EoS insensitivity - model dependence.
- Bi-modality in (M, Λ) - the PT smoking gun.
- Essential to integrate in EoS inference pipelines.