# Slowly rotating black holes in nonlinear electrodynamics

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# Motivation

Extend static spherically symmetric solution to rotating solutions,

NE version of Kerr–Newman

# Introduction about NE

The idea of Non-Linear Electrodynamics is about a century old but it was made popular in 1930s by Born and Infeld.

The main goal was solving the point charge singularity:

$$\frac{q}{r^2} \Rightarrow \frac{q}{r^2 + r_0^2}$$

# Introduction about NE

- Resolve the spacetime singularity
- Wide application in different theories
- Different forms of Nonlinear Electrodynamics

# Generating rotating solutions

- Using Kerr metric as an anzats $\to$ NOT an easy task!
- Using Newmann–Janis algorithm $\to$ Does NOT work for arbitrary source nor vacuum solutions in modified gravity theories

$$\Downarrow$$

Slow Rotation Approximation

# Newmann–Janis algorithm

General static spherical symmetric spacetime,

$$ds^2 = -f(r)\,dt^2 + \frac{dr}{f(r)} + g(r)\,d\Omega^2$$

Applying the Newmann–Janis algorithm (complex coordinate transformation in Eddington–Finkelstein coordinates) we get

$$\begin{aligned} ds^2 &= -\frac{\Delta}{\Sigma}(dt - a\,\sin^2\theta d\varphi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \\ &\quad + \frac{\sin^2\theta}{\Sigma}\left[(r^2 + a^2)d\varphi - a\,dt\right]^2 \end{aligned}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta\,, \quad \Delta = r^2 f + a^2$$

This trick successfully leads to Kerr–Newman solution!

# Can we use JN algorithm for NE?

JN algorithm is frequently used to obtain rotating solution for NE models. To prove these are wrong, we apply the limit of small a in Kerr metric,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + 2\,a\,r^2\,h(r)\,\sin^2\theta dt d\varphi + r^2 d\Omega^2$$

where

$$h = \frac{f-1}{r^2}$$

And the vector potential for magnetic charge after limit is

$$A = p\cos\theta\left(d\varphi - \frac{a}{r^2}dt\right)$$

We show that JN algorithm fails to produce rotating solutions for any non-trivial NE.

# Set up: slowly rotating solution in NE theory

Electromagnetic Field Invariants:

$$F = \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \,, \quad \mathcal{P} = \frac{1}{2} F_{\mu\nu} (*F)^{\mu\nu}$$

The modified Maxwell field equations for $\mathcal{L}(F)$,

$$\partial_\mu (\sqrt{-g}\, \mathcal{L}_F \, F^{\mu\nu}) = 0$$

Metric ansatz, *generalized Lense–Thirring*,

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + 2\, a\, r^2\, h(r)\, \sin^2\theta dt d\varphi + r^2 d\Omega^2 \qquad (1)$$

- $f$ is the static solution
- $a$ is the rotation parameter
- $h$ should be found

# Electric solution

The vector potential:

$$A = e\,\phi(r)\,(dt - a\,\omega(r)\,\sin^2\theta d\varphi) \tag{2}$$

- $\phi$ is the static solution
- $\omega$ captures the effect of rotation

Electromagnetic Field Invariants:

$$F = -e^2\phi'^2 + O(a^2)\,, \quad \mathcal{P} = -\frac{4e^2\phi\phi'\,\omega\cos\theta}{r^2}\,a + O(a^3)$$

# Electric solution

How to find $h(r)$, $\omega(r)$?

- For specific NE model, e.g. Born–Infeld $\to$ too complicated

  Hint from JN algorithm, for a general NE model, we can assume
- $h = \frac{f-1}{r^2}$
  or
- $\omega = 1$

# Electric Solution: $h = \frac{f-1}{r^2}$

If we assume $h = \frac{f-1}{r^2}$ , then from modified Maxwell equations we find $\omega = 1$ and the electric field would be Maxwell one.

**Theorem.** *For restricted class of theories, $\mathcal{L}(F)$, the only NE consistent with $h = (f-1)/r^2$ for the ansatz* (1) *and* (2) *is the Maxwell theory.*

**Collorary.** *Electrically charged NE spacetimes generated by the standard Newmann–Janis algorithm do not solve the corresponding NE equations following from $\mathcal{L}(F)$, not even at the linear $O(a)$ level.*

# Electric Solution: $\omega = 1$

With considering $\omega = 1$, we get

$$h = \frac{f-1}{r^2} - \frac{2\,M_0}{r^3}$$

- If $M_0 = 0 \rightarrow$ Maxwell theory
- If $M_0 \neq 0 \rightarrow$ NEW Lagrangian

Among restricted NE theories, $\mathcal{L}(F)$, there are two theories that yield the Lense–Thirring solutions with $\omega = 1$: the Maxwell theory and the theory defined by the New Lagrangian.

# New Lagrangian

From $h = \frac{f-1}{r^2} - \frac{2\,M_0}{r^3}$, we get

$$\phi = \frac{1}{r + 3\,M_0}$$

And we find the

$$\mathcal{L} = 2\mathcal{S}_0 \left( \frac{u^3 + 3u^2 - 4u - 2}{2(1-u)} - 3\ln(1-u) + 1 \right)$$

where $u = \left(-\frac{F}{\mathcal{S}_0}\right)^{\frac{1}{4}}$ and $\mathcal{S}_0 = \frac{e^2}{(3\,M_0)^4}$.

Note: $M_0$ seems physical as it 'redefines' the asymptotic angular momentum.

# New Lagrangian and the metric solution

The corresponding metric function $f$

$$
\begin{aligned}
f &= 1 + \frac{4\beta e^2}{9M_0^2} - \frac{2M + 8\beta e^2/(9M_0)}{r} - \frac{8r\beta e^2}{27M_0^3} \\
&\quad + r^2\Big(\Lambda - \frac{8\beta e^2}{81M_0^4}\lg(\frac{r}{r+3M_0})\Big), \qquad (3)
\end{aligned}
$$

for large $r$ this has the following expansion:

$$
f \approx 1 - \frac{2M}{r} + \Lambda r^2 - \frac{2\beta e^2}{r^2} + \frac{24\beta e^2 M_0}{5r^3} + O\Big(\frac{1}{r^4}\Big), \qquad (4)
$$

- Setting $\Lambda = 0$ and $\beta = -1/2 \rightarrow$ Reissner–Nordstrom
- For small positive $M_0 \rightarrow$ (up to) two horizons

# Magnetic Solution

The vector potential:

$$A = p \cos\theta \left( d\varphi - \frac{a\,\omega}{r^2} dt \right)$$

- $p$ is the magnetic charge
- $\omega(r)$ new vector potential

The field invariants

$$F = \frac{p^2}{r^4} + O(a^2)\,, \quad \mathcal{P} = \frac{2p^2 \cos\theta (r\omega' - 2\omega)}{r^5}\, a + O(a^3)$$

# Magnetic Solution

If we assume $h = \frac{f-1}{r^2}$ , then from modified Maxwell equations we find $\omega = 1$ and we obtain $\mathcal{L}_F = const.$, which is only consistent with the Maxwell theory.

**Theorem.** *Among all restricted NE theories $\mathcal{L}(F)$, Maxwell theory is the only one that admits the magnetically charged slowly rotating solutions of the form* (1), (5) *with the restriction* $h = (f-1)/r^2$. *In particular, this means that the standard Newmann–Janis algorithm fails to produce solutions already at the linear $O(a)$ level.*

# Square Root Model: $\mathcal{L}(F) = -\beta\sqrt{F}$

Question: Why this model?

- This Lagrangian represents a strong field regime of many models of NE, the Born–Infeld theory for example.
- The confinement potential in gauge theories can be generated from this model.
- Subclass of Power Maxwell Lagrangian

# Magnetic solution for $\mathcal{L} = -\beta\sqrt{F}$

The static metric function $f$,

$$f = 1 - \frac{2M}{r} + 2\beta p$$

For $M = 0$:

$$\begin{aligned} \omega &= \frac{\omega_1}{r} + \omega_2 r^2 + \omega_3 r^{(\frac{1}{2}+q)} + \omega_4 r^{(\frac{1}{2}-q)}\,, \\ h &= -\frac{\omega_1}{r^3} - \omega_2 - 2\omega_3\beta p r^{(q-\frac{3}{2})} - 2\omega_4\beta p r^{(-q-\frac{3}{2})}\,, \end{aligned}$$

where $\omega_i$'s are the integration constants, and

$$q = \frac{\sqrt{4p^2\beta^2 + 36p\beta + 17}}{4p\beta + 2}$$

When $M \neq 0$, the terms with 'strange powers' of $r$ in $h$ are replaced by hypergeometric functions.

# Conclusion and future plans

The standard Newmann–Janis algorithm fails to produce rotating solutions in NE even at the lowest level in rotation parameter $a$.

Future plans

- Extend our results for more general NE, $\mathcal{L}(F, P^2)$
- Modified Newmann–Janis algorithm
- Finding full charged and rotating solutions in some non-trivial NE.

THANK YOU