Progress in the Field of Gravitational Self-Force

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Introduction

- Self-force overview
- Issues encountered at second order
- Overview of the highly regular gauge and advantages
- Derivation of second-order stress-energy tensor – the *Detweiler stress-energy tensor*
    - Using Detweiler canonical defintion in EFEs

Self-Force Overview

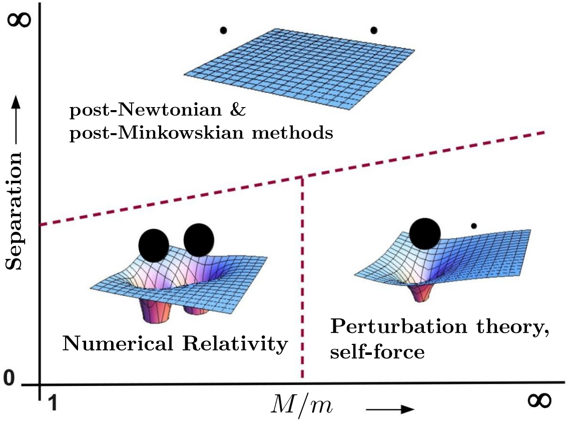

Image adapted from Barack & Pound, 2018, arXiv:1805.10385

- Power series in mass ratio $\epsilon := m/M$

$$g^{\text{exact}}_{\mu\nu} = g^{\text{bg}}_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \dots$$

- Equation of motion

$$\frac{D^2 z^\mu}{d\tau} = \epsilon f^\mu_1 + \epsilon^2 f^\mu_2 + \dots$$

$$f^\mu_n = f^\mu_{n,\text{cons}} + f^\mu_{n,\text{diss}}$$

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  - Thus we need to calculate the force to *second order* in the mass ratio

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$$\varphi = \frac{1}{\epsilon}\Big(\varphi_0 + \epsilon\varphi_1 + \mathcal{O}\big(\epsilon^2\big)\Big)$$

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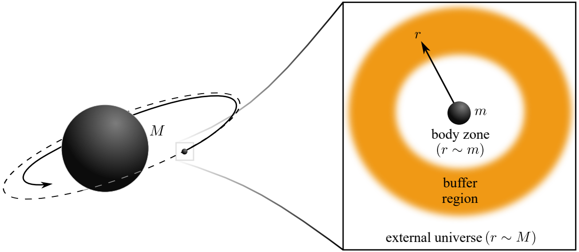
- For error $\ll 1$, we need φ_0 and φ_1

Matched Asymptotic Expansions



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g^{\mathsf{exact}} & = & g^{\mathsf{obj}} & + & \epsilon H^1 & + & \epsilon^2 H^2 \\
\| & & \wr & & \wr & & \wr \\
g^{\mathsf{bg}} & \sim & 1 & & r & & r^2 \\
+ & & & & & & \\
\epsilon h^1 & \sim & \frac{\epsilon}{r} & & \epsilon & & \epsilon r \\
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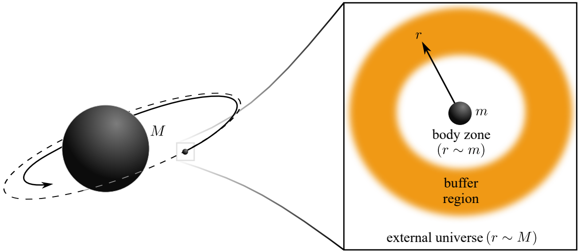

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  - Form of Taylor expansion

$$h^{\mathrm{R}n}_{\mu\nu} = h^{\mathrm{R}n}_{\mu\nu}|_\gamma + h^{\mathrm{R}n}_{\mu\nu,a}|_\gamma x^a + \tfrac{1}{2} h^{\mathrm{R}n}_{\mu\nu,ab}|_\gamma x^a x^b + \mathcal{O}\left(r^3\right)$$

Second-Order EFEs and EoM

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$$\delta G^{\mu\nu}[h^1] = 8\pi T_1^{\mu\nu}$$
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- Equation of motion for non-spinning, spherically symmetric small object given by [Pound, 2012, 1201.5089 & 2017, 1703.02836]

$$\frac{D^2 z^\mu}{d\tau^2} = -\frac{1}{2} P^{\mu\alpha}\left(g_\alpha{}^\delta - h^{\mathrm{R}\delta}_\alpha\right)\left(2h^{\mathrm{R}}_{\delta\beta;\gamma} - h^{\mathrm{R}}_{\beta\gamma;\delta}\right)u^\beta u^\gamma + \mathcal{O}\left(\epsilon^3\right)$$

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- Also requires the use of a two-timescale expansion based on a "fast time" and "slow time" to capture processes happening on the orbital timescale and the radiation-reaction timescale [Miller & Pound, 2021, 2006.11263]

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- Effect of resonances is being investigated e.g. [Lukes-Gerakopoulos & Witzany, 2021, 2103.06724]

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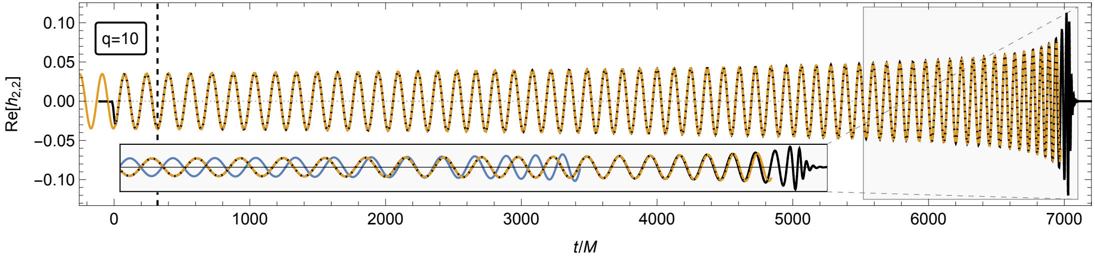

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  - Ill-defined on any domain including the worldline, $r = 0$

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- Linear in $1/r$

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- However, in a class of highly regular gauges, this is less singular
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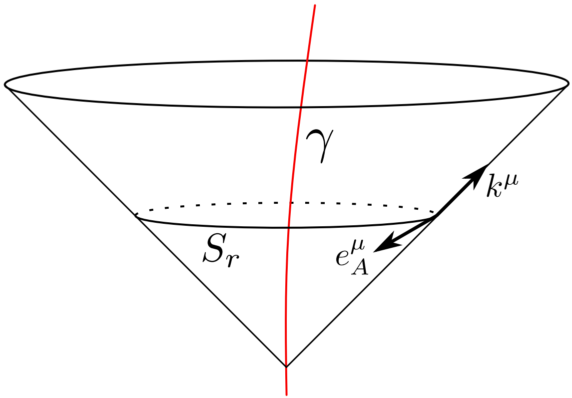
- Expression is in Fermi–Walker coordinates, upcoming paper will provide the expressions in covariant form using methods of [Pound & Miller, 2014, 1403.1843]

Structure of Highly Regular Gauge

- Based on preserving local lightcone structure



Local lightcone structure around worldline, γ. Based on image by Adam Pound.

Structure of Highly Regular Gauge

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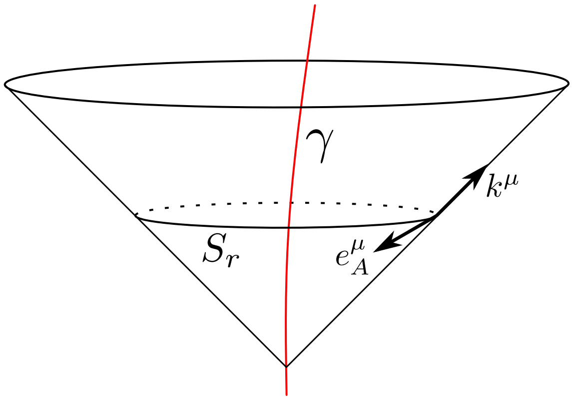
- Gauge conditions:

$$h^{\mathrm{HR}}_{\mu\nu} k^\nu = 0 \qquad h^{\mathrm{HR}}_{\mu\nu} e^\mu_A e^\nu_B \Omega^{AB} = 0$$

k^μ is a future-directed null vector and Ω_{AB} is metric on S^2



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Detweiler Stress-Energy Tensor [SDU & Pound, 2021, 2101.11409]

- Express $T^{\mu\nu}_{2,\mathrm{HR}}$ in terms of transformation from light-cone gauge

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Distributional Sources (cont.)

- Leads to distributionally well-defined equations in Lorenz gauge

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- $\delta^2 G_{\mu\nu}[h^1, h^1]$ will diverge when integrating over it to solve for $h^2_{\mu\nu}$
- Using distributional definition, we get a counter term that cancels this divergence

$$\delta G_{\mu\nu}[h^2] = 8\pi(T^2_{\mu\nu} - T^{Q\flat\flat}_{\mu\nu}) + \lim_{s\to 0}\Big[8\pi T^{\text{counter}}_{\mu\nu} - \theta(r-s)\delta^2 G_{\mu\nu}[h^1, h^1]\Big]$$

where

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  - Derived the Detweiler stress-energy tensor – point particle in effective spacetime
  - Showed how this can be used in a practical way to solve for the second-order metric perturbations