EMRI waveforms: a status update

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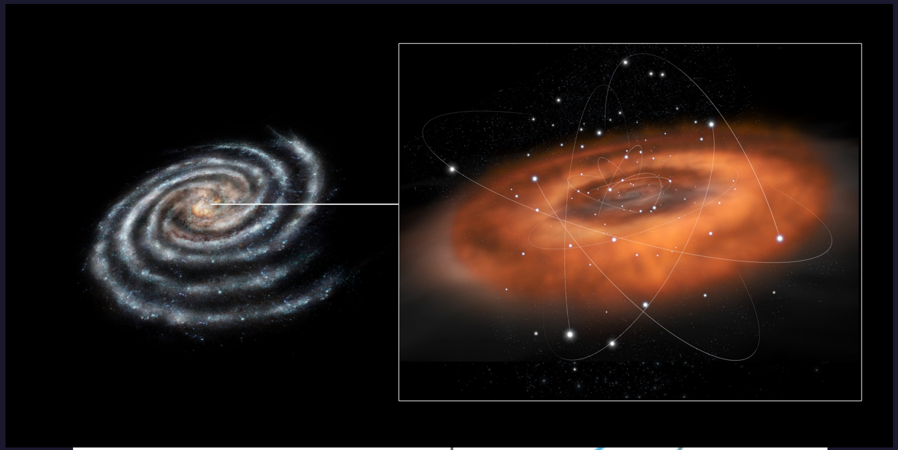
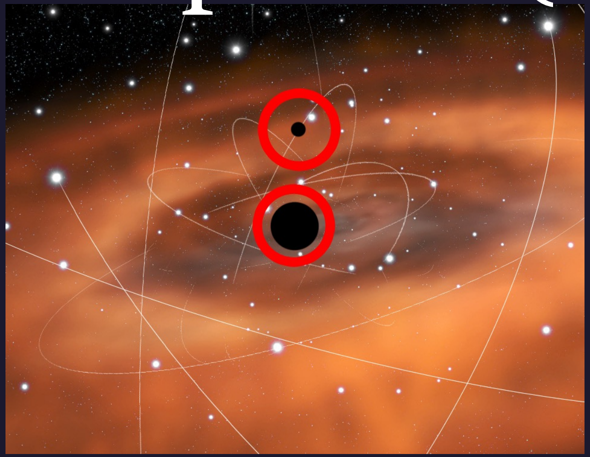
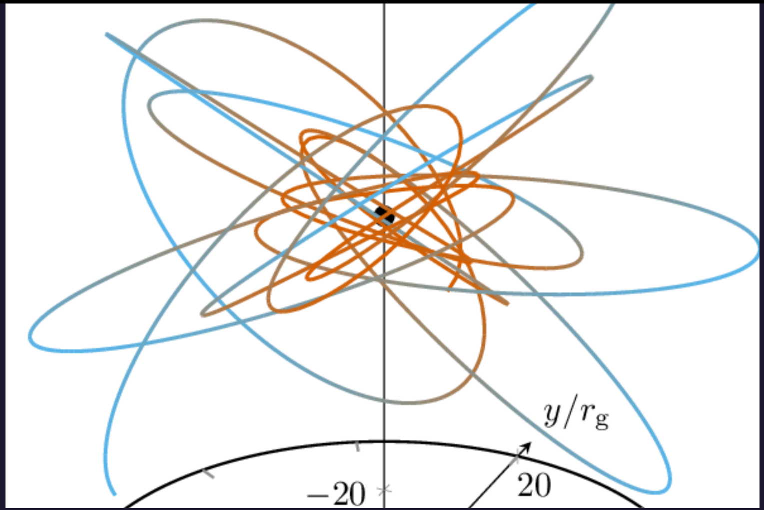
Plan of talk

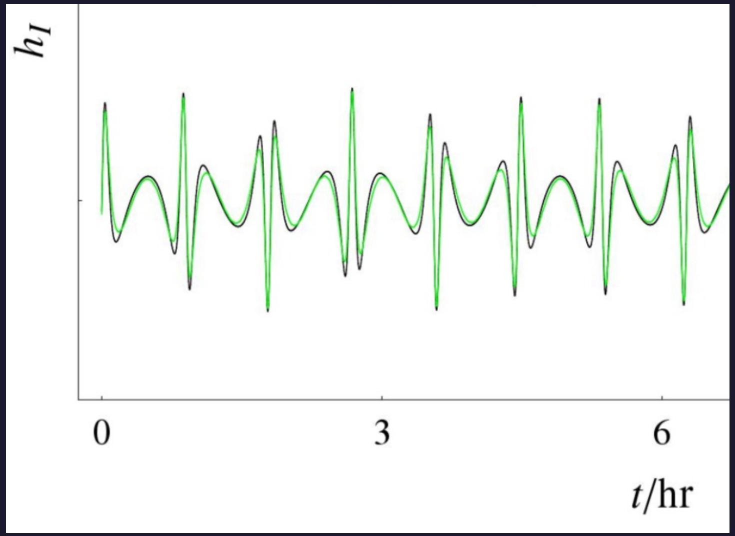
- What are EMRIs and how will LISA see them
- The modelling challenge
- Progress of the self-force program
- Spin
- External matter fields and modified gravity?
- Large mass ratios with EM counter-parts?

Extreme mass ratio inspirals (EMRIs)





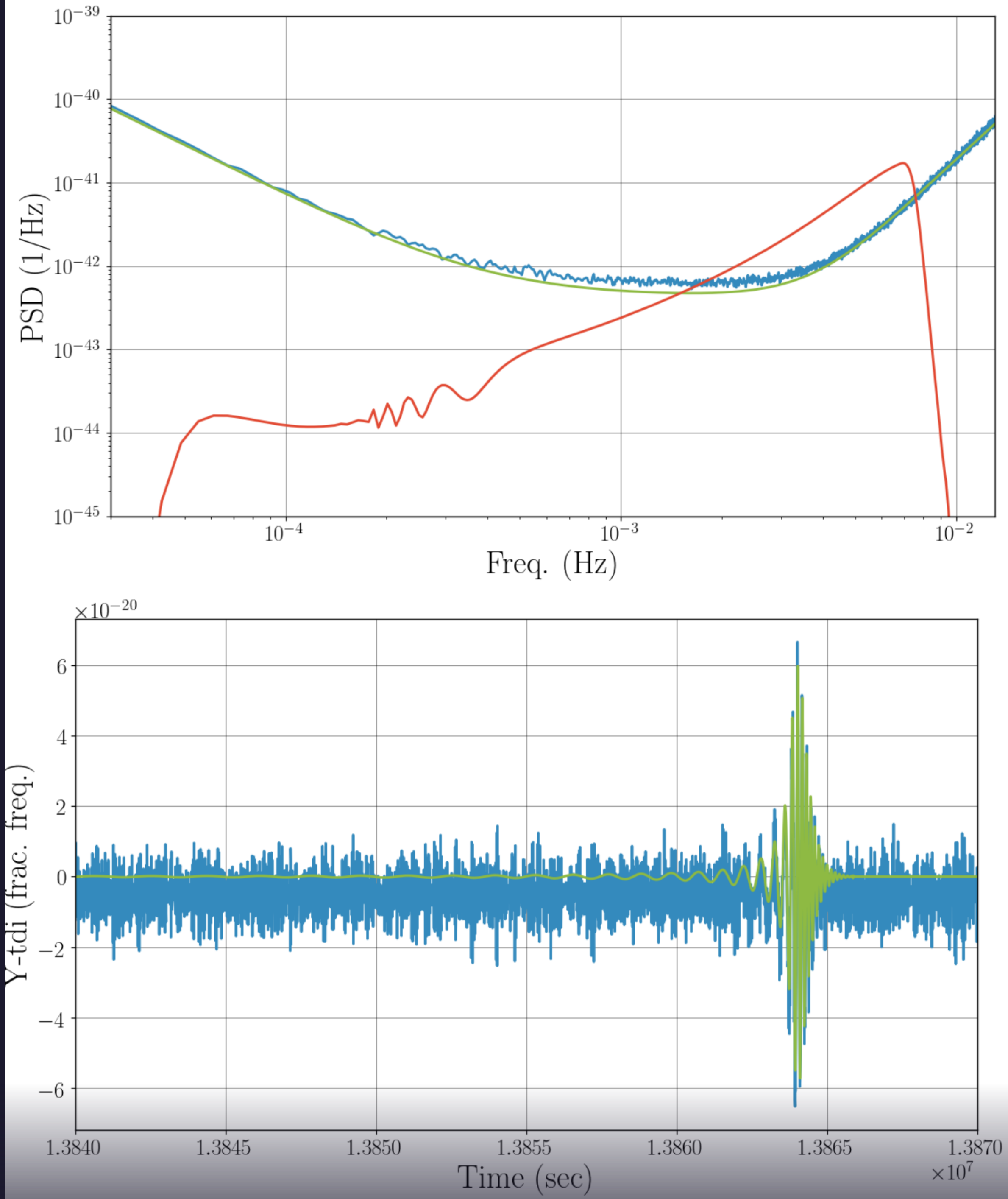


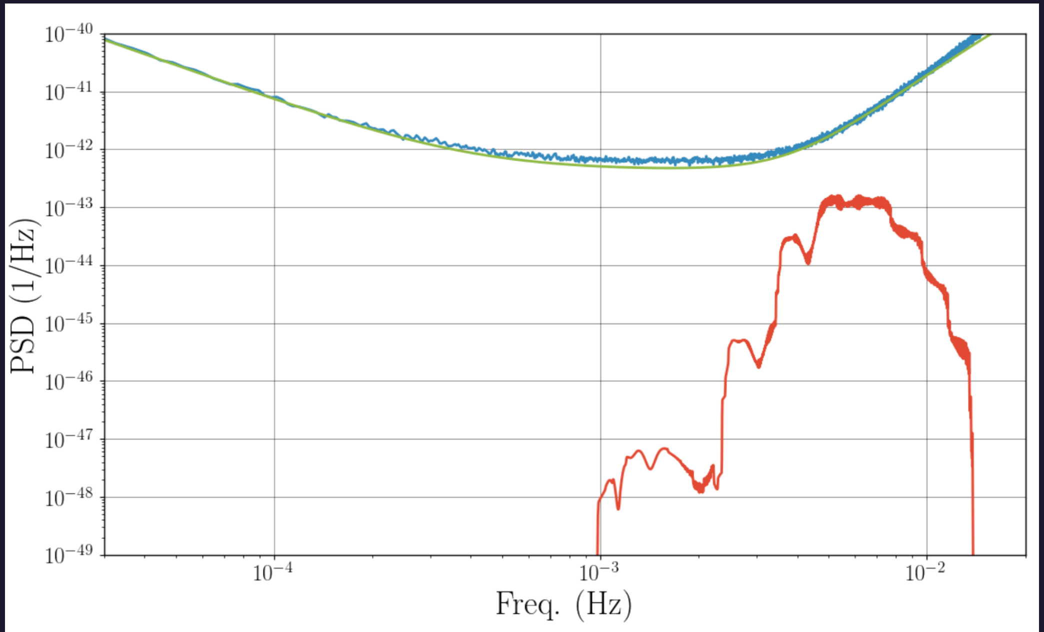


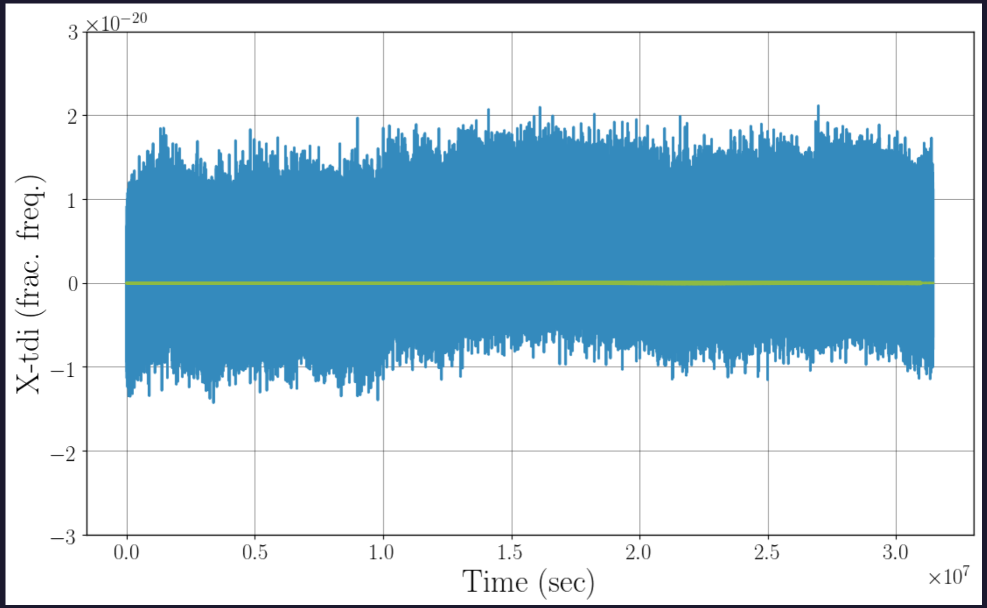
- Direct capture gives eccentric EMRIs (when entering LISA band). Hills mechanism low eccentric, disk migration essentially circular. Inclination generic in essentially all cases.
- Direct capture is assumed to dominate, gives e~0.7 when entering LISA band, e~0.2 when plunging (Babak et al. 2017), this can change with model assumptions (e.g. Pan et al. 2021)

How will LISA see EMRIs? For contrast: Massive black hole binary in LISA

- Comparable mass ratio, total mass $\sim 3\times10^6 M_\odot$, at redshift ~3
- Visible by naked eye (immediate amplitude reaches above noise)
- Done in ~ 8 hours
- Source: Antoine Petiteau and Stas Babak/LDC

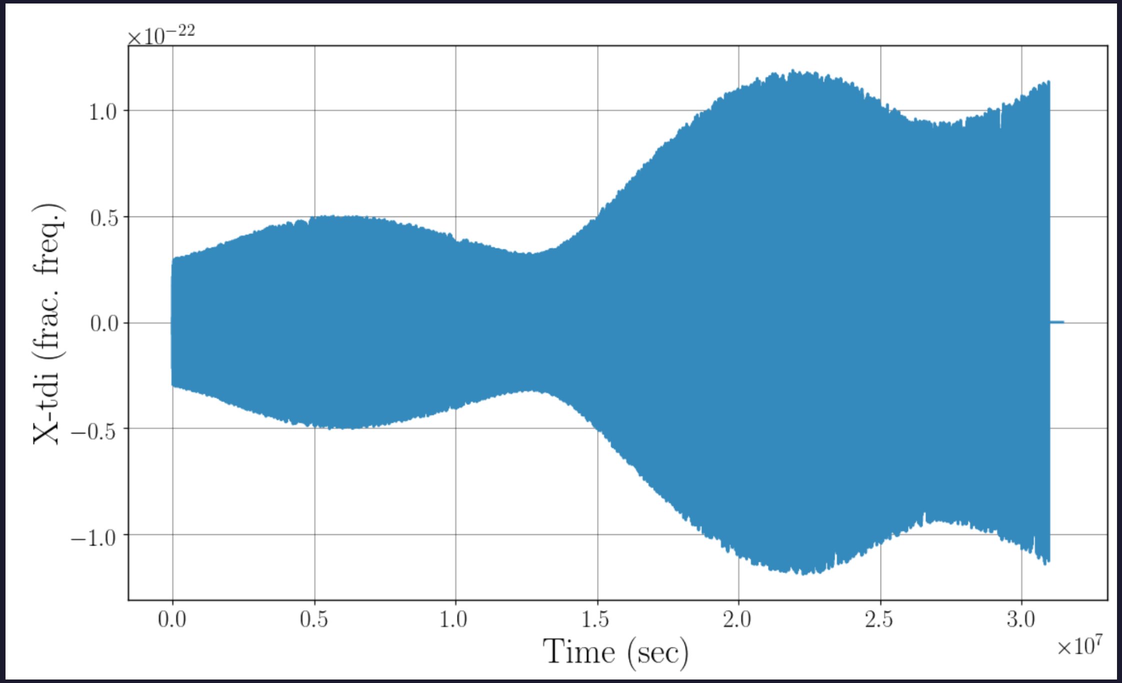






How will LISA see EMRIs
A realistic example

- Mass of primary ~ $1\times10^{6} M_{\odot}$, secondary ~ $10\ M_{\odot}$, distance 1 Gpc (redshift ~0.3)
- Impossible to notice without analysis (immediate amplitude two orders below the noise)
- Takes roughly a year, order of 10^{5} cycles!
- LISA data analysis will need waveforms that stay in phase with the real signal during this time!
- Source: Antoine Petiteau and Stas Babak/LDC



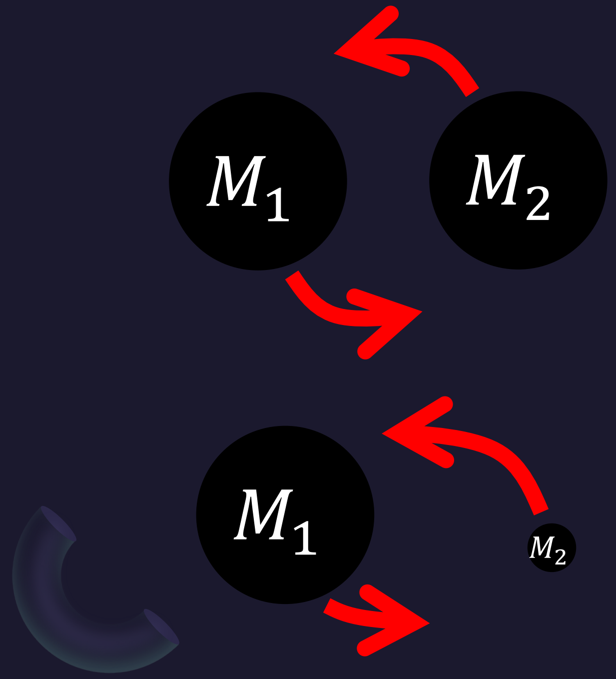
Why is this?

$$L \sim \frac{M_1^2 M_2^2 (M_1+M_2)}{r^5},$$
$$\eta = \frac{M_1 M_2}{M^2}, M = M_1 + M_2$$

Peak luminosity only(!) $\sim \eta^2$

Peak strain $\sim \eta M / D_{\rm L}$

Time spent around peak $\sim M/\eta$



(at fixed total M)

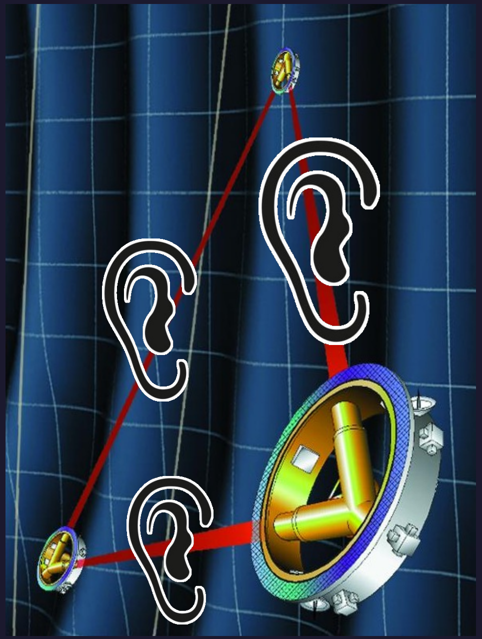
$1/4$

SHORT & LOUD

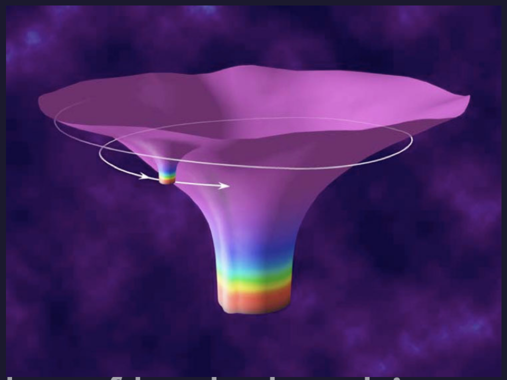
η

LONG & QUIET

0^+



Self-force program



- Numerical relativity needs to resolve scales associated with the length and time-scales of both the objects → steep inverse scaling of needed resources with small mass ratio $q = \frac{M_2}{M_1} \sim 10^{-5}$
- Post-Newtonian expansions converge badly for large mass ratios (inspiral lingers in strong field)
- Self-force: Black hole perturbation theory in the field of the primary, (Regge-Wheeler-Zerilli equation in Schwarzschild, Teukolsky in Kerr, metric reconstruction in radiation gauge,…), formal singularities arise and must be regularized to obtain self-force (see talk by Samuel Upton)
- O(1) accuracy is achieved by use of the two-timescale expansion (post-adiabatic iteration)

$$\phi_{\text{fin}} = \underbrace{\phi^{(1)}_{\text{radi}}}_{\mathcal{O}(q^{-1})} + \underbrace{\phi^{(1)}_{\text{reso}}}_{\mathcal{O}(q^{-1/2})} + \underbrace{\phi^{(1)}_{\text{cons}} + \phi^{(1)}_{\text{spin}} + \phi^{(2)}_{\text{radi}}}_{\mathcal{O}(q^{0})} + \underbrace{\phi^{(2)}_{\text{reso}}}_{\mathcal{O}(q^{?})} + \mathcal{O}(q)$$

A two-timescale decomposition (1PA)

	Geodesic	Grav. self-force	Spin-curvature
Conservative, orbit evol.	$J_\mathrm{o}(p,e,i)$, $\Omega_\mathrm{o}(J_\mathrm{o})$	$\langle\delta^{\mathrm{gsf1}}\Omega_\mathrm{o}\rangle(J_\mathrm{o})$ $\delta^{\mathrm{gsf1}}x^\mu$	$\langle\delta^{s}\Omega_\mathrm{o}\rangle(J_\mathrm{o},J_\mathrm{s})$ $\delta^{s}x^\mu$
Dissipative, orbit evol.		$\langle\dot{J}_\mathrm{o}\rangle^{x_\mathrm{geo}}_{\mathrm{gsf1}}(J_\mathrm{o})$, $\langle\delta\dot{J}_\mathrm{o}\rangle^{\delta^{\mathrm{gsf1}}x}_{\mathrm{gsf1}}(J_\mathrm{o})$, $\langle\dot{J}_\mathrm{o}\rangle^{x_\mathrm{geo}}_{\mathrm{gsf2}}(J_\mathrm{o})$	$\langle\delta\dot{J}_\mathrm{o}\rangle^{\delta^{s}x}_{\mathrm{gsf1}}(J_\mathrm{o})$, $\langle\delta\dot{J}_\mathrm{o}\rangle^{\mathrm{Sp.source}}_{\mathrm{gsf1}}(J_\mathrm{o})$
Conservative, spin evol.	$J_\mathrm{s}(p,e,i,S^{\mu\nu})$, $\Omega_\mathrm{s}(J_\mathrm{o},J_\mathrm{s})$	negligible	negligible
Dissipative, spin evol.		$\langle\dot{J}_\mathrm{s}\rangle^{x_\mathrm{geo}}_{\mathrm{gsf1}}(J_\mathrm{o})$	

Referring to the two-timescale decomposition of Hinderer & Flanagan (2008, 0805.3337)

Status of self-force program

- Full self-force on generic orbits in Kerr (van de Meent 2018, 1711.09607), but not very efficient yet
- Adiabatic waveforms for generic Kerr orbits (Hughes et al. 2021, 2102.02713)
- Second-order fluxes on quasi-circular Schwarzschild orbits (Warburton et al. 2021, 2107.01298)
- Corresponding 1PA waveforms for quasi-circular inspirals (Wardell et al. 2021, 2112.12265)
- Second-order fluxes for *generic* orbits in Schwarzschild: WIP (computationally challenging and laborious implementation)
- Second-order fluxes in Kerr: WIP (also conceptually challenging, some recent progress from Loutrel et al. (2020, 2008.11770) and Toomani et al.(2021, 2108.04273), upcoming Leather et al.)
- Passage through resonances: WIP (Recent progress in Lukes-Gerakopoulos & VW (2021, 2103.06724))
- Spin of secondary: WIP (see next slide)
- Transition to plunge: WIP (laborious implementation for circular, theoretical work needed for generic, WIP Durkan, Zimmerman, Küchler, Compére,…)

Spin of secondary

- Spin-curvature effect on geodesics can be solved analytically (Witzany 2019, 1903.03651) or by a semi-analytical frequency-domain decomposition (Drummond & Hughes 2022, 2201.13334)
- This is the basis of computing corrections to the fluxes of energy, angular momentum and Carter constant, most advanced is Kerr equatorial (Skoupý & Lukes-Gerakopoulos 2022, 2201.07044)
- Generic Kerr is WIP (energy and angular momentum fluxes underway, Carter constant needs analytical work)

External matter and modified gravity

- One of the essential goals of LISA is to test gravity
- We are working hard on ”vanilla” EMRI waveforms, pure GR, no external perturbation
- Model-independent tests of gravity can look for residuals after the best-fit parameter estimate
- Ruling out modifications of gravity requires having a modified gravity waveform and showing it is not consistent with the signal!
- A large residual can also mean strong environmental effects (surrounding matter, non-compact secondary passing through accretion disk, . . .)
- *Even the adiabatic level requires solving wave equations on non-Kerr (Petrov non-D) backgrounds without spherical symmetry. This is hard and very few people are actually doing it. (Some WIP Hussain & Zimmerman, Li et al. . . . (2206.10652); Brito et al.)*

Closing notes

- EMRI/IMRI waveform requirements are currently being formulated for the LISA Red book (definition study report). E.g. what kind of waveform accuracy do we need to fulfil (1702.00786)
- The self-force program/1PA waveforms seem to have a much larger applicability than thought – for quasi-circular binaries it can describe even comparable masses!
- A new type of EM sources has been discovered in 2018, Quasi-periodic X-ray erruptions (QPEs, e.g. Arcodia et al. 2021, [2104.13388](2104.13388)). These are possibly associated with large-mass-ratio binaries (Sukova et al. 2021, [2102.08135](2102.08135)). Do we need special modeling of the waveform?

OR2.4.b: Have the ability to detect unequal mass MBHBs of total intrinsic mass $10^4 - 10^6\,M_\odot$ at $z < 3$ with the lightest black hole (the IMBH) in the intermediate mass range (between 10^2 and $10^4\,M_\odot$) [11], measuring the component masses to a precision of 10%, which requires a total accumulated SNR of at least 20.