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TOWARDS AN EFFECTIVE-ONE-BODY MODEL FOR EXTREME-MASS-RATIO INSPIRALS

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Based on: arXiv:2208.01049 [gr-qc], arXiv:2208.02055 [gr-qc]
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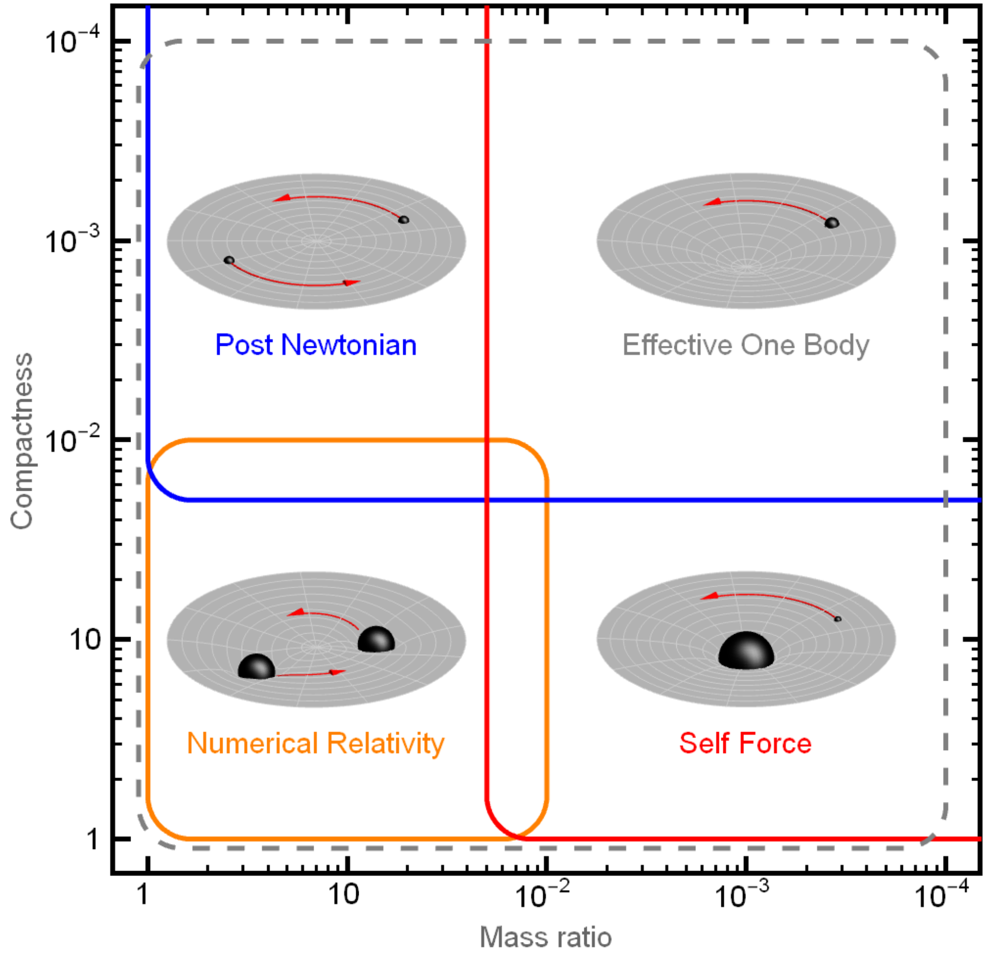
OUTLINE

- Brief intro to the effective-one-body (EOB) approach
  to the two-body problem in general relativity
- Comparison between the EOB model TEOBResumS and
  gravitational self-force (GSF) results:
  - quasi-circular equatorial motion, nonspinning black holes
  - intermediate- and extreme-mass-ratio inspirals (IMRIs and EMRIs)
- Modifying TEOBResumS for IMRIs and EMRIs
- Discussing features to be added: spin, eccentricity,
  environment & beyond GR

THE EFFECTIVE-ONE-BODY FORMALISM



Theoretical results from classical approaches

Information from **N**umerical **R**elativity (NR)

EOB

flexible analytical approach,
mapping the two-body dynamics in the motion of a particle with the reduced mass of the system moving in an effective metric (deformation of Schwarzschild/Kerr)

THEORETICAL FRAMEWORK

conservative sector

- Hamiltonian: found by mapping the "energy levels" of the real problem at a given PN order to the effective ones

dissipative sector

- Hamiltonian equations of motion complemented by the **radiation reaction**

} dynamics

- Waveform: (inspiral + plunge) + ringdown
  decomposed on spin-weighted spherical harmonics

DYNAMICAL BACKGROUND

$G = c = 1$

Mass ratio $q = \frac{m_1}{m_2}$, $m_1 > m_2$ Symmetric mass ratio $\nu \equiv \frac{\mu}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2}$

Continuous deformation in ν of a Schwarzschild metric:

$$ds^2_{\text{eff}} = g^{\text{eff}}_{\mu\nu} dx^\mu_{\text{eff}} dx^\nu_{\text{eff}} = -A(r)dt^2 + B(r)dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad u = 1/r$$

$$A^{\text{PN}}_{\text{orb}}(u) = 1 - 2u + 2\nu u^3 + \nu a_4 u^4 + \nu\left[a^c_5(\nu) + a^{\log}_5 \ln u\right] u^5 + \nu\left[a^c_6(\nu) + a^{\log}_6 \ln u\right] u^6$$

EOB Hamiltonian for nonspinning binaries:

$$\hat{H}_{\text{EOB}} \equiv \frac{H_{\text{EOB}}}{\mu} = \frac{1}{\nu}\sqrt{1 + 2\nu\left(\hat{H}_{\text{eff}} - 1\right)} \qquad \hat{H}_{\text{eff}} = \sqrt{p^2_{r_*} + A(r,\nu)\left(1 + \frac{p^2_\phi}{r^2} + 2\nu(4 - 3\nu)p^4_{r_*}\right)}$$

HAMILTONIAN EQUATIONS OF MOTION

$$\frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} = \Omega$$

Orbital frequency

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

solved numerically with ODE solver

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi = \hat{\mathcal{F}}^\infty_\varphi + \hat{\mathcal{F}}^{\rm H}_\varphi$$

Radiation reaction:
asymptotic contribution + horizon contribution

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r}$$

INSPIRAL (+ PLUNGE) WAVEFORM

- Strain: $$h_+ - ih_\times = \frac{1}{D_L}\sum_\ell \sum_{m=-\ell}^{\ell} h_{\ell m}\,{}_{-2}Y_{\ell m}(\iota,\phi)$$

multipoles

Newtonian prefactor

Resumed PN correction

$$h_{\ell m} = h^{(N,\epsilon)}_{\ell m}\,\hat{S}^{\epsilon}_{\text{eff}}\,\hat{h}^{\text{tail}}_{\ell m}(x)\,\left[\rho_{\ell m}(x)\right]^{\ell}\,\hat{h}^{\text{NQC}}_{\ell m}$$

Next-to-Quasi-Circular corrections

- Regge-Wheeler-Zerilli normalized waveform:

waveform frequency

$$\Psi_{\ell m} \equiv \frac{h_{\ell m}}{\sqrt{(\ell+2)(\ell+1)\ell(\ell-1)}} \qquad \Psi_{22} = A_{22}e^{-i\phi_{22}} \qquad \omega_{22} \equiv \dot{\phi}_{22}$$

TEOBRESUMS

- EOB model built for **comparable-mass binaries** (versions: quasi-circular, eccentric, precessing)
- Incorporates analytical information (PN expansions for potentials & waveform/flux, resummed in some way)
- Some parameters are tuned to NR results (orbital sector, spin-orbit, merger & ringdown)

BLACK HOLE BINARIES: HIGHER MASS RATIOS

- Intermediate and extreme mass ratio black hole binaries are among the sources of the next generation of gravitational wave detectors (ET, CE, LISA)
- Regime scarcely explored by NR
- Apart from EOB, **gravitational self-force** (GSF) theory is the only other available tool to probe the inspiral
- ➡ comparing EOB and GSF

SECOND ORDER GRAVITATIONAL SELF-FORCE

- GSF: taking into account the deviation from the test-mass case due to the second object's gravitational field
- Expanding the metric to 2nd order in the small mass ratio: $\epsilon \equiv m_2/m_1 \leq 1$
- Two-timescale expansion: slow radiation reaction timescale vs fast orbital timescale
- We consider here the post-adiabatic (PA) model presented in Wardell et al. 2021 (arXiv:2112.12265v2)

WAVEFORM ALIGNMENT IN THE TIME DOMAIN

- We focus on the $\ell = m = 2$ strain multipole
- Phasing: finding the **time and phase shift** by minimizing the root-mean-square of the phase difference on a certain interval

warning: a bit arbitrary!

$$\Delta\phi(t_i, \tau, \alpha) = \left(\phi_2(t_i - \tau) - \alpha\right) - \phi_1(t_i)$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\Delta\phi(t_i, \tau, \alpha))^2}$$

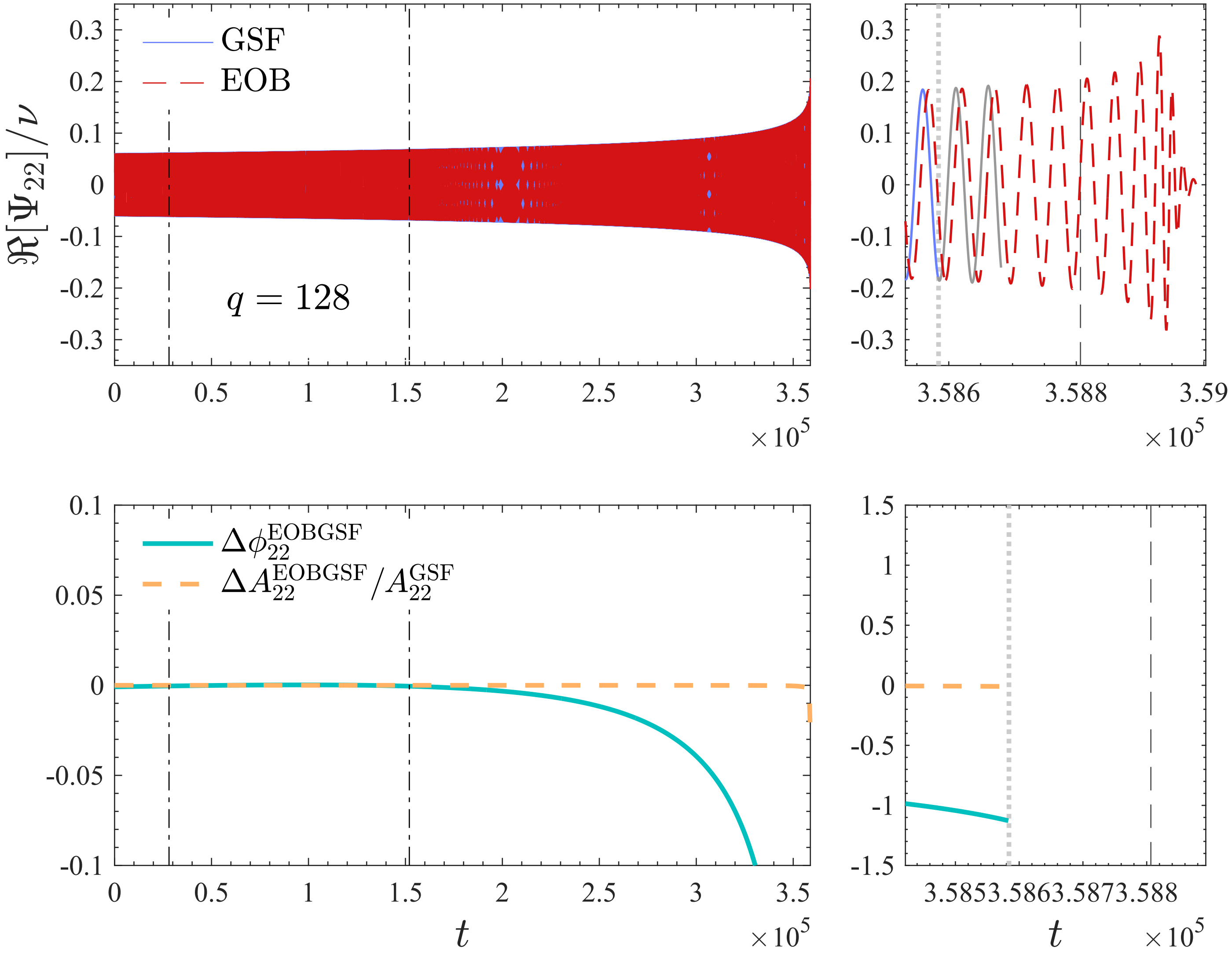
$$\Psi^1_{22} = A^1_{22}(t_1)e^{-i\phi_1(t_1)}$$

$$\Psi^2_{22} = A^2_{22}(t_2 - \tau)e^{-i[\phi_2(t_2-\tau)-\alpha]}$$

PHASE DIFFERENCES

Binaries with mass ratio q = 15, 32, 64, 128 to complement the findings of Nagar et al. 2022 (arXiv:2202.05643v1)

q	$\Delta\phi_{22,t}^{\mathrm{EOBGSF}}$
15	0.3782
32	−0.1267
64	−0.5091
128	−1.1287



GAUGE-INVARIANT ANALYSIS: Q_ω

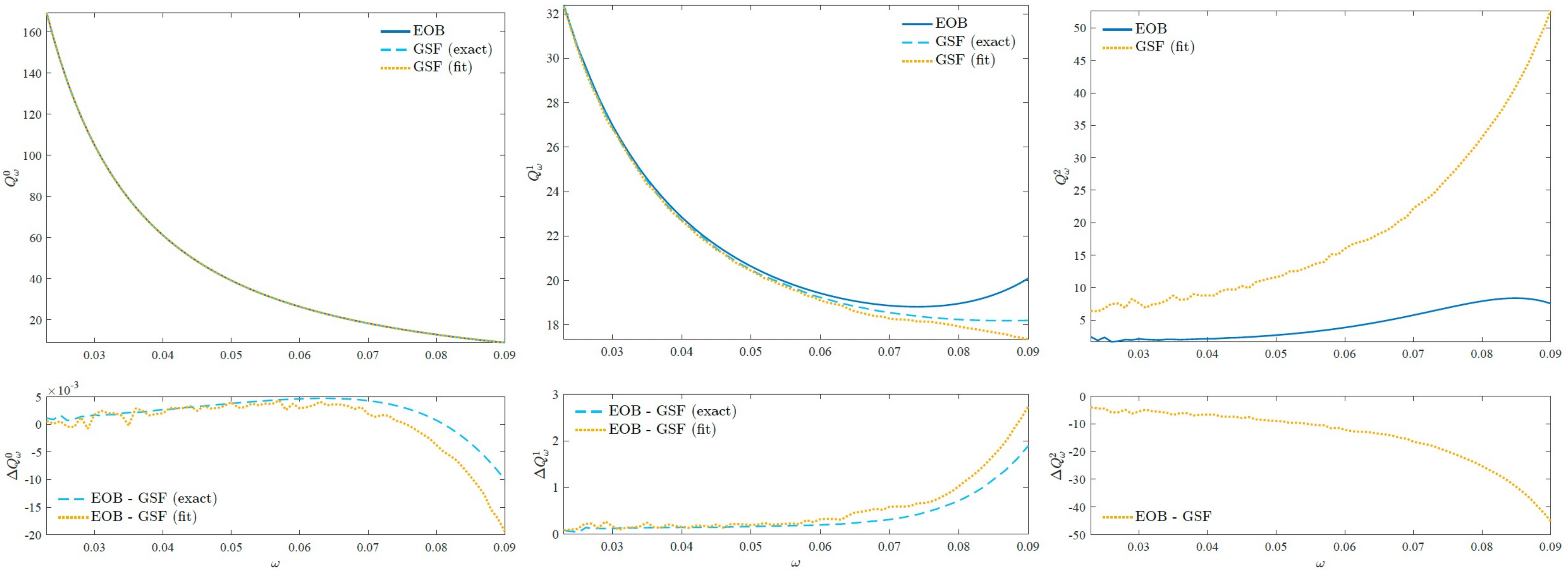
- Adiabaticity parameter: $Q_\omega = \frac{\omega^2}{\dot{\omega}}$ $\qquad \omega = \omega_{22}$
- Q_ω >> 1 adiabatic motion
- Phase difference: $\Delta\phi_{(\omega_1,\omega_2)} = \int_{\omega_1}^{\omega_2} Q_\omega d\log\omega$

- Expanding in the symmetric mass ratio:

$$Q_\omega(\omega;\,\nu) = \underbrace{\frac{Q_\omega^0(\omega)}{\nu}}_{\text{0PA}} + \underbrace{Q_\omega^1(\omega)}_{\text{1PA}} + \underbrace{Q_\omega^2(\omega)\nu}_{\text{2PA}} + O(\nu^2)$$

fitting the coefficients
for a set of mass ratios
at fixed values of the frequency

THE COEFFICIENTS Q_ω^0, Q_ω^1, Q_ω^2



IMPROVING TEOBRESUMS

- The Q_ω analysis indicates that as the mass ratio increases, the dominant contributions are given by Q_ω^0 and Q_ω^1
- Q_ω^0 : depends on the 1st order self-force (1SF) flux
- Q_ω^1 : depends the 1SF and 2SF fluxes and on the 1SF contribution to the orbital potential
- Hence for higher mass ratios we have to improve TEOBResumS both in the conservative and in the dissipative sector of the model
- … and we will turn off all NR calibration

GSF-INFORMED EOB POTENTIALS

$$A(u;\nu) = 1 - 2u + \nu a_{1\mathrm{SF}}(u) + \mathcal{O}(\nu^2)$$
$$\bar{D}(u;\nu) = \frac{1}{AB} = 1 + \nu \bar{d}_{1\mathrm{SF}}(u) + \mathcal{O}(\nu^2)$$
$$Q(u;\nu) = \nu q_{1\mathrm{SF}}(u) p_r^4$$

EOB orbital potentials

Expressions for a_{1SF} , $\bar{d}_{1SF}$, q_{1SF} at 8.5PN order
+ suitable factorization & Padé-resummation
+ fit on the numerical GSF data of
Akcay & van de Meent, arXiv:1512.03392v2
… but singularity at the light-ring!

RADIATION REACTION (FLUX AT INFINITY)

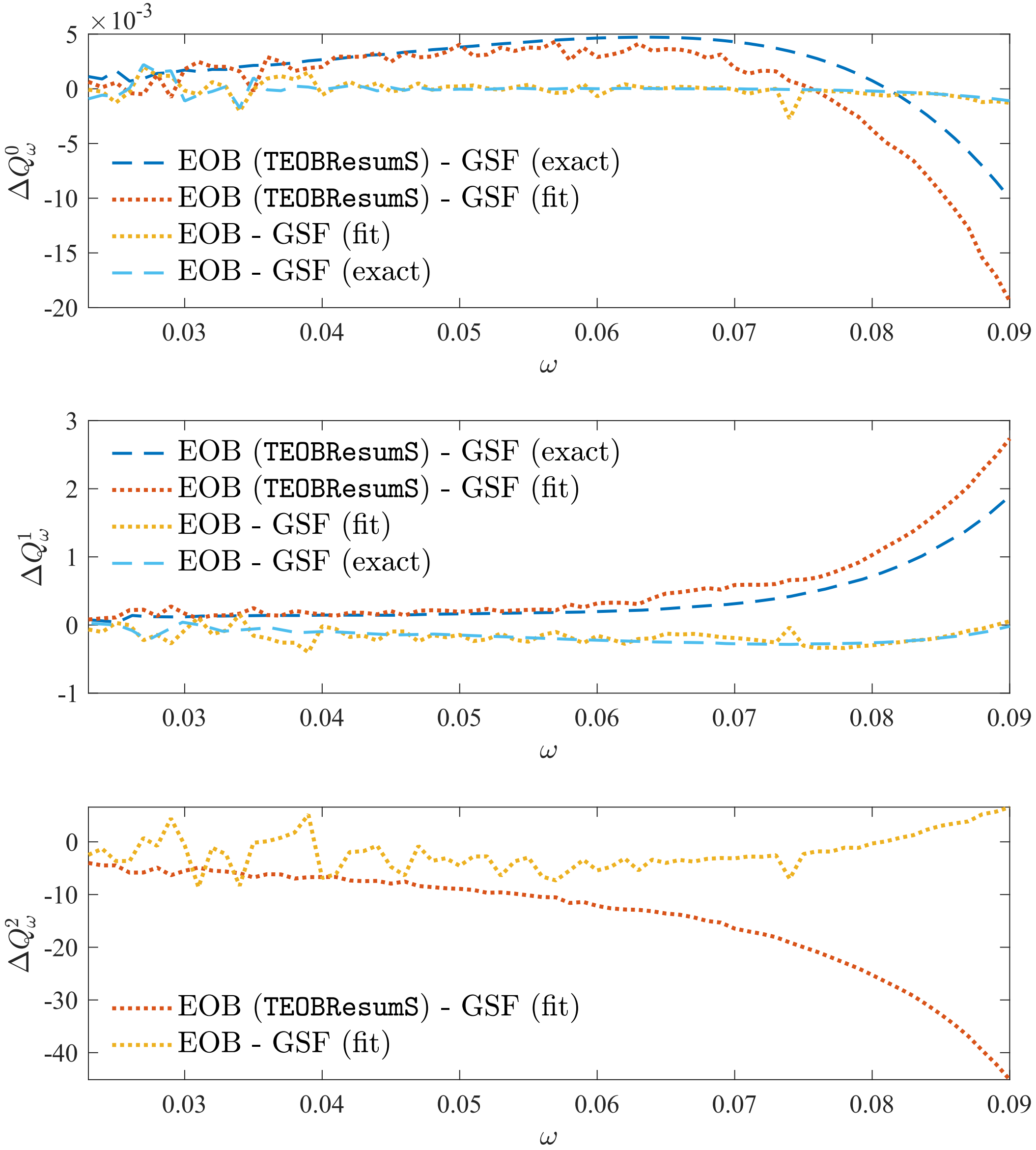
- Flux multipoles are factorized into different contributions, among which the residual amplitude corrections:

$$\rho_{\ell m} = \underbrace{1 + c_1 x + c_2 x^2 + \ldots}_{\text{PN series}} \qquad x = \Omega^{2/3}$$

orbital frequency

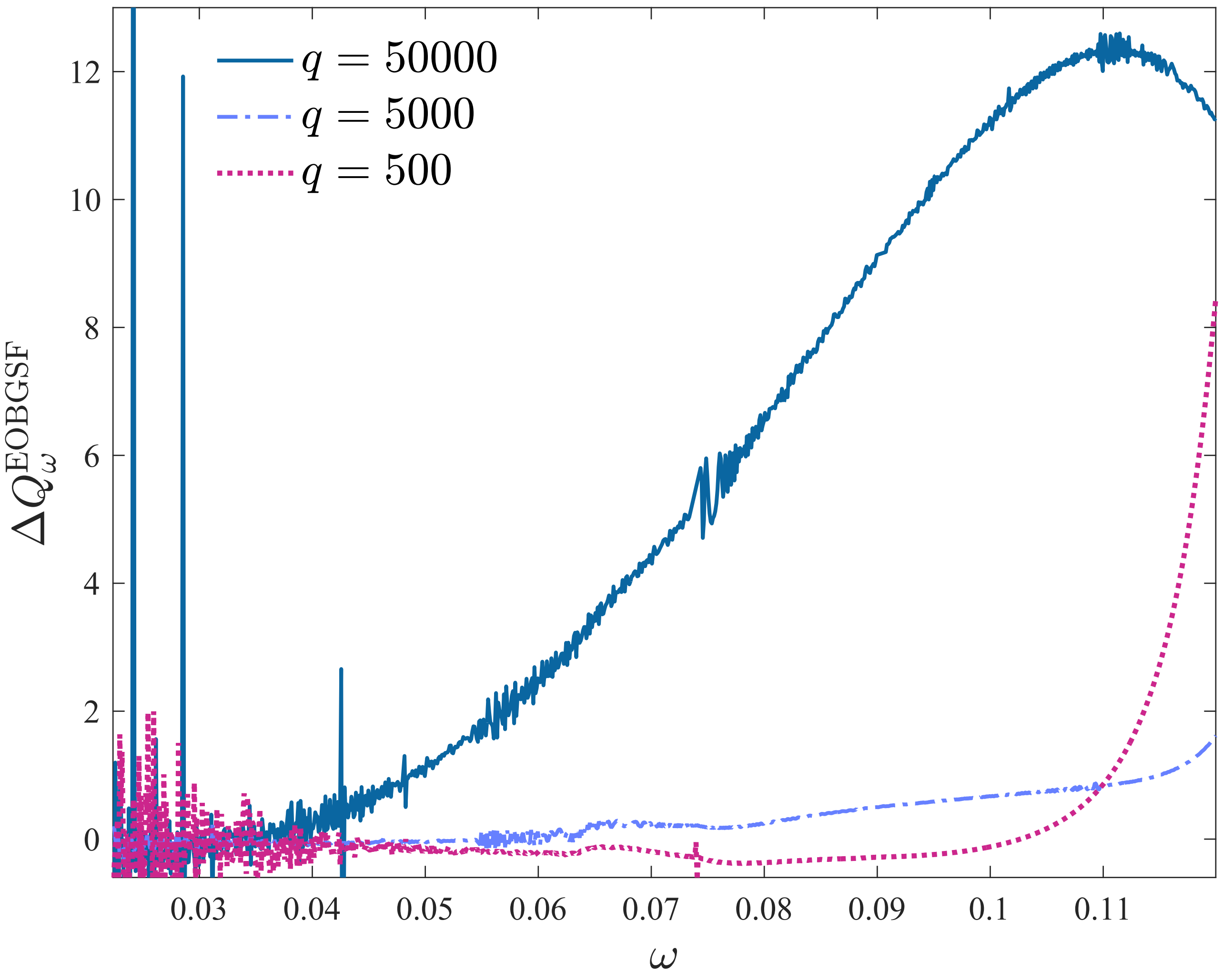
- The standard TEOBResumS has Padé-resummed 6PN expressions
- We hybridize 22PN results (Fujita 2012) with the known ν-dependence for every ℓm multipole (e.g. $c_1(\nu), c_2(\nu), c_3(\nu)$) up to $\ell = 8$

THE COEFFICIENTS Q_ω^0, Q_ω^1, Q_ω^2



Improved agreement in all the three functions. Also in Q_ω^2 due to having the EOB potentials only up to 1SF order now (no higher-order-in-ν corrections)

TOWARDS EXTREME-MASS-RATIO INSPIRALS



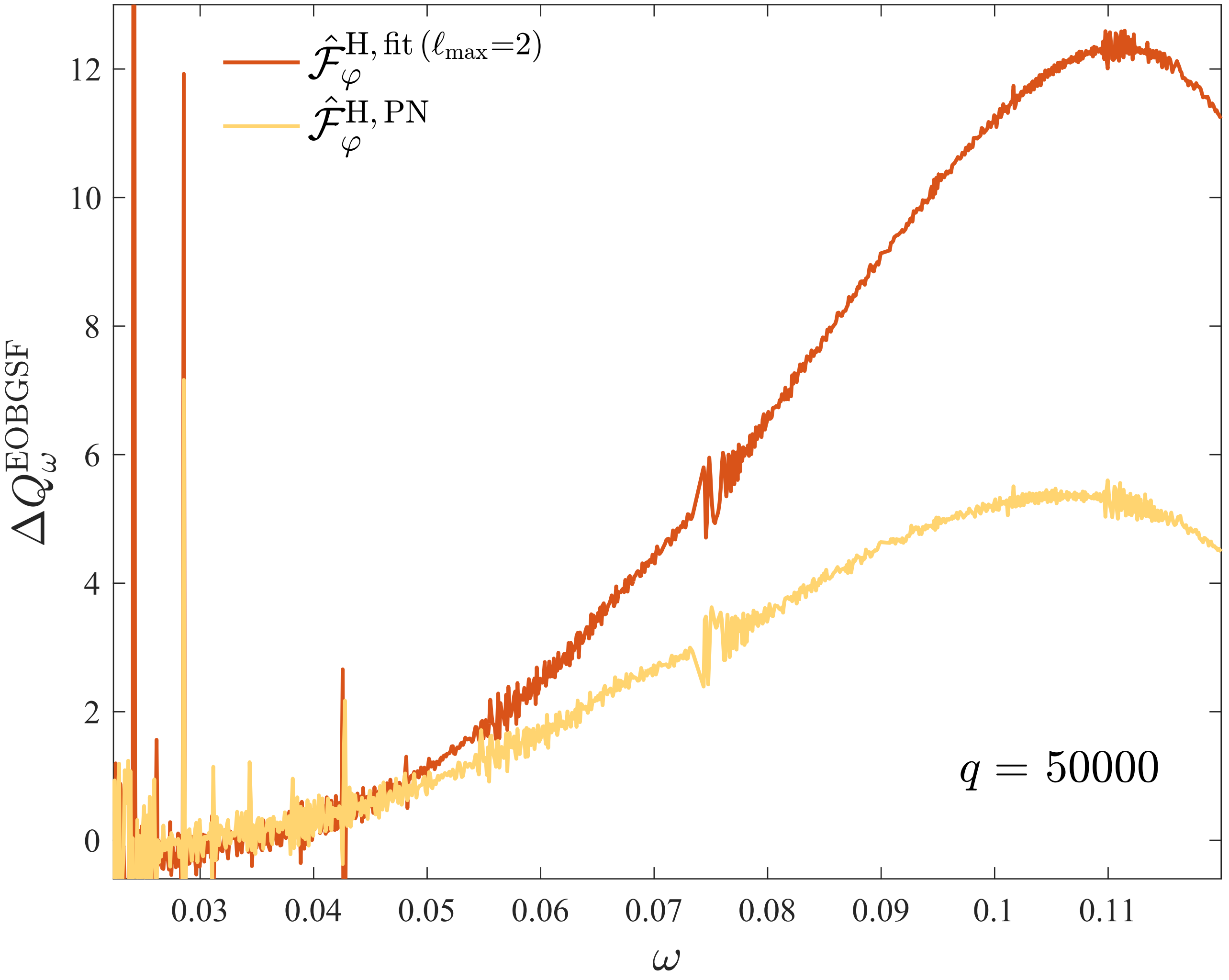
Integrated phase differences:

q = 500 $\Delta\phi \sim 0.07$

q = 5000 $\Delta\phi \sim 0.27$

q = 50 000 $\Delta\phi \sim 5.88$

THE HORIZON FLUX CONTRIBUTION

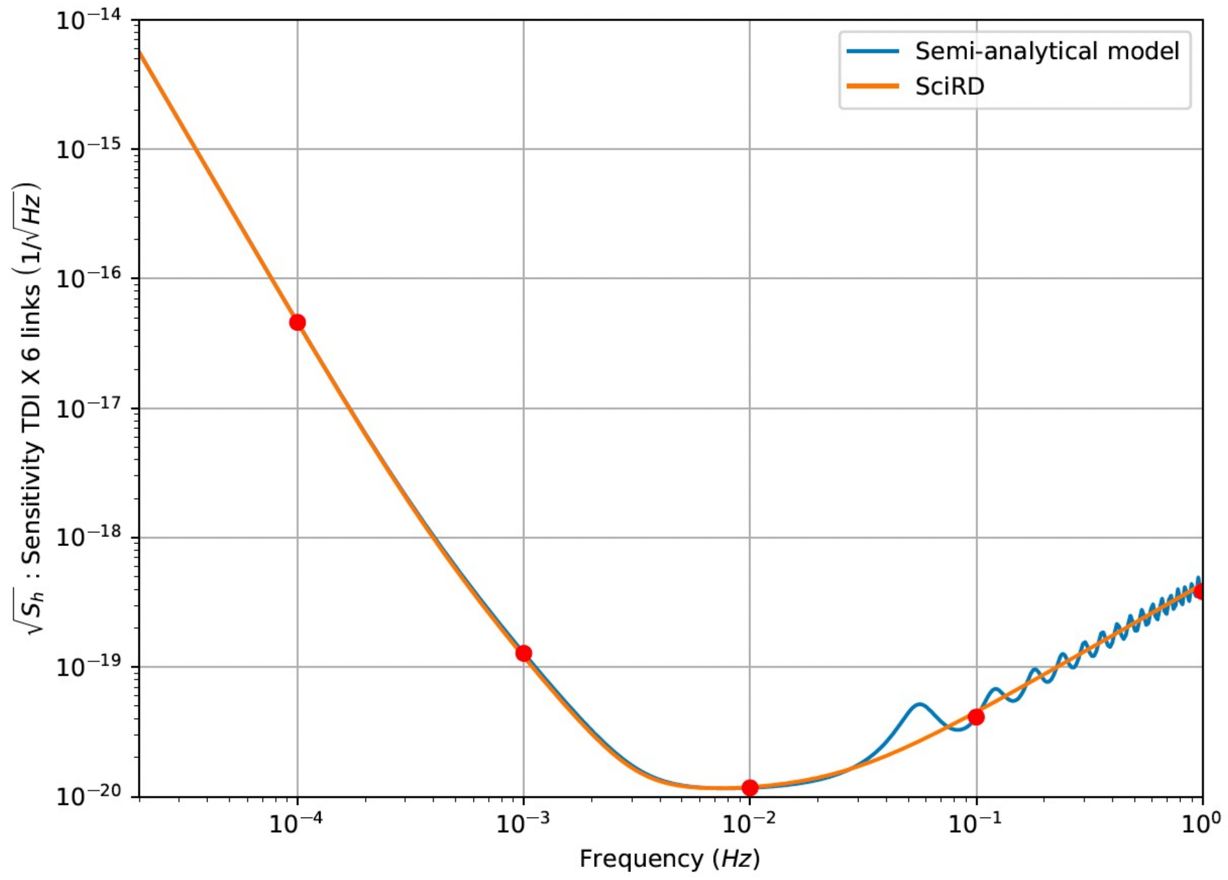


- To increase the agreement in $Q_\omega{}^0$: improving the horizon flux

- Integrated phase differences:

  Standard: $\Delta\phi \sim 5.88$
  Improved: $\Delta\phi \sim 2.94$

TALKING ABOUT LISA

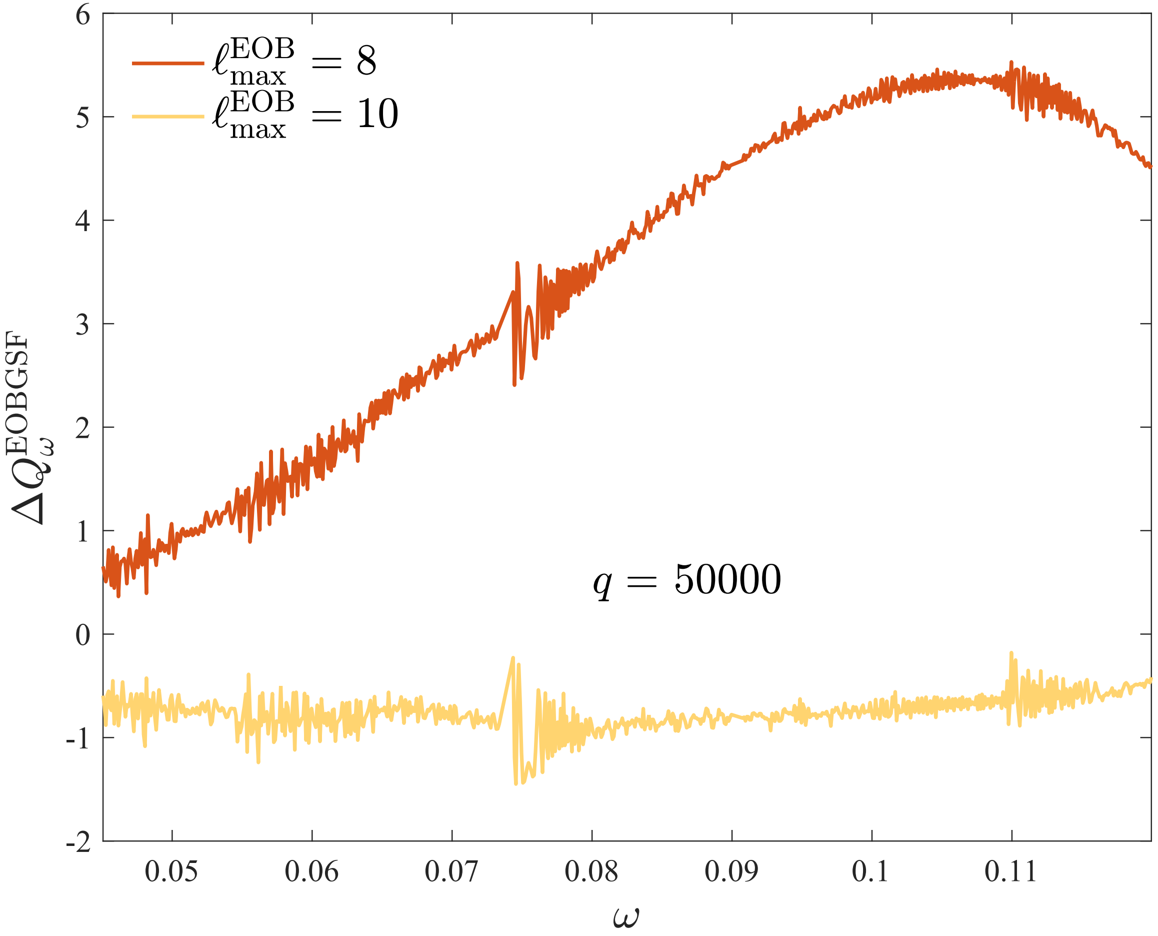


from Babak et al 2021, arXiv:2108.01167

So far we gained insight from the theorical point of view… but we should take into account:

- Frequency band where LISA will be sensitive
- Involved masses !
- Mission duration

… AND THINGS GET EVEN BETTER



- Adding ℓ = 9, 10 to the infinity flux
- Shorter frequency interval:

$\omega = [0.045, 0.12]$ $f = [0.003, 0.007]\ (Hz)$

if $m_2 = 10M_\odot$

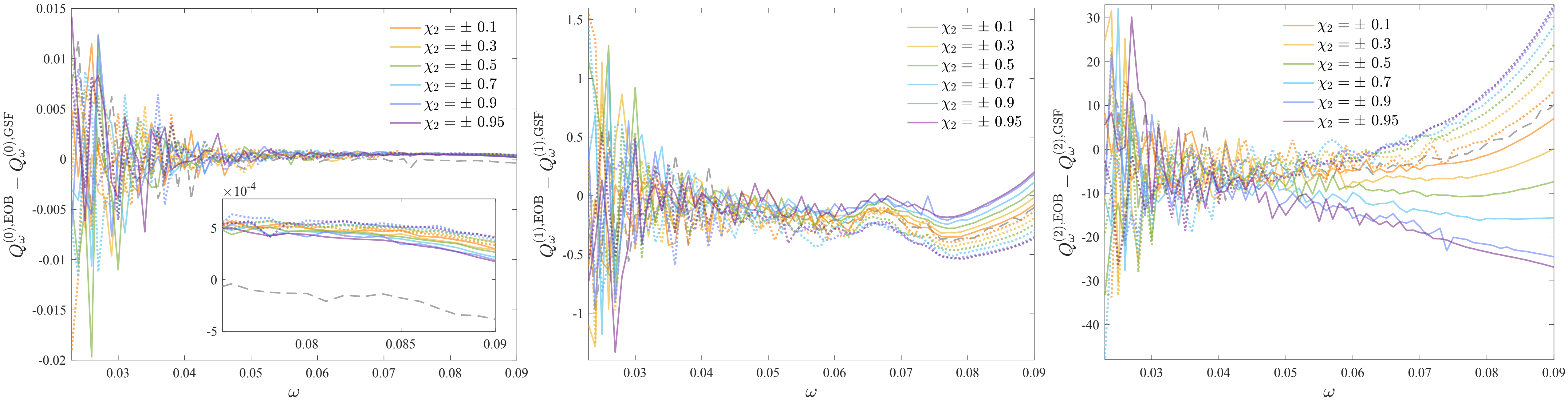
- Corresponding to ~1.2 years of EOB evolution, ~1.5 × 10^5 cycles
- Integrated phase differences:

Standard: $\Delta\phi \sim 2.99$
Improved: $\Delta\phi \sim -0.74$

SOME TECHNICAL DETAILS

- All the results shown here are obtained with the private MATLAB implementation of the code (quite slow)
- We have a public C implementation, that also exploits the post-adiabatic evolution (in EOB sense, see arXiv:1805.03891v2) during the inspiral. The GSF-informed potentials have been already implemented in the eccentric branch of the code, the new flux not yet!
- Other improvements:
  - different integration algorithm for the ODEs (maybe symplectic?)
  - eventually turning to machine learning for speed-up
  (e.g. see arXiv:2210.15684v2 for a frequency domain surrogate model for BNS based on TEOBResumS)

WORK IN PROGRESS: SPINNING SECONDARY



comparing $Q_\omega^{0,1,2}$ for different values of the secondary spin

WHY DO WE NEED BENCHMARKS?

- Not always for "calibration", but frequently just to make the right analytical choice:

χ_2	$\Delta\phi^{\rm EOBGSF}_{\rm DJS_{3.5PN}}$	$\Delta\phi^{\rm EOBGSF}_{\rm DJS_{4.5PN}}$	$\Delta\phi^{\rm EOBGSF}_{\rm antiDJS_{3.5PN}}$
0.5	−0.1037	−0.1025	−0.0886
−0.5	0.0477	0.047	0.0338
0.9	−0.1434	−0.1419	−0.0955
−0.9	0.1137	0.1116	0.0648

TABLE I. EOB − GSF phase difference evaluated via time-domain phasing for $q = 500$, either using the inverse resummation of G_{S_*} at 3.5PN or 4.5PN or its anti-DJS representation (Rettegno et al paper).

… and even different possibilities for the EOB spin-orbit sector could be explored

GENERIC DYNAMICS

- **Eccentricity**: could be switched on easily, currently exploring choices for the radiation reaction: arXiv:2104.10559v4, arXiv:2207.14002v1, arXiv:2305.19336v1 checked with respect to Teukolsky/RWZ solutions
- **Precession**: current version of TEOBResumS computes the evolution in the 'co-precessing' frame and then twists the waveform. Only spherical, and not tested for large mass ratios yet… Another possibility: Balmelli-Damour Hamiltonian (arXiv:1509.08135v1) for a real precessing evolution, but missing radiation reaction

BEYOND GR: SCALAR-TENSOR EOB

- Ongoing work in computing EOB quantities within massless scalar-tensor (ST) theories: see arXiv:2211.15580v2, arXiv:2301.01070, arXiv:2304.09052v1
  So far only conservative part of the dynamics: local-in-time and non-local-in-time ST corrections to the EOB potentials at 3PN + computation of the scattering angle

AND FINALLY… ENVIRONMENTAL EFFECTS

EOB is super flexible, so…

- Accretion (thin) disks: can be included in the flux
- Gravitating contribution: goes into the potentials
- Could also include a NS secondary (BNS and BHNS EOB models already working for comparable-mass binaries)

DISCUSSION AT THE 1ST TRIESTE MEETING ON GRAVITATIONAL WAVES (LAST WEEK IN TRIESTE, SISSA)

- Gravitating contribution not expected to be as relevant as the flux contribution from the disk (enters the dynamics at higher order, see arXiv:1404.7149v2)
- Mostly should include all the available spinning particle info (parallel work)
- As for accretion, could compare to FastEMRIWaveform package with/without inclusion of the effect (arXiv:2207.10086v2) or also to augmented analytical kludge

CONCLUSIONS

- By having a benchmark at large mass ratios we are able to make the necessary modifications to TEOBResumS so as to make it useful for (quasi-circular nonspinning) EMRIs
- Still to do:
  - improve the code and its speed
  - add several features
  - various type of resonances will play a role!