Nonlinear analysis of gravitational wave data

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regular czechLISA meeting - 11.03.2024

Institute of Physics at Silesian University in Opava

- Opava is (small, 60.000) city located in Upper Silesia (CZ)
- Institute of Physics ($\sim$ 60 people), but with **strong GR group**:

  Zdeněk Stuchlík, Roman Konoplya, Gabriel Török, Jorge Ovalle,...

physics.slu.cz unisfera.slu.cz whoo.slu.cz

Roman Konoplya

Black hole merger, ring down, quasi-normal modes, overtones

Martin Urbanec

Neutron star tidal deformabity, constraints on dense matter equation of state, compact binaries.

Radim Pánis

Nonlinear analysis, time series, observed data; New methods for chaos detection and data classification.

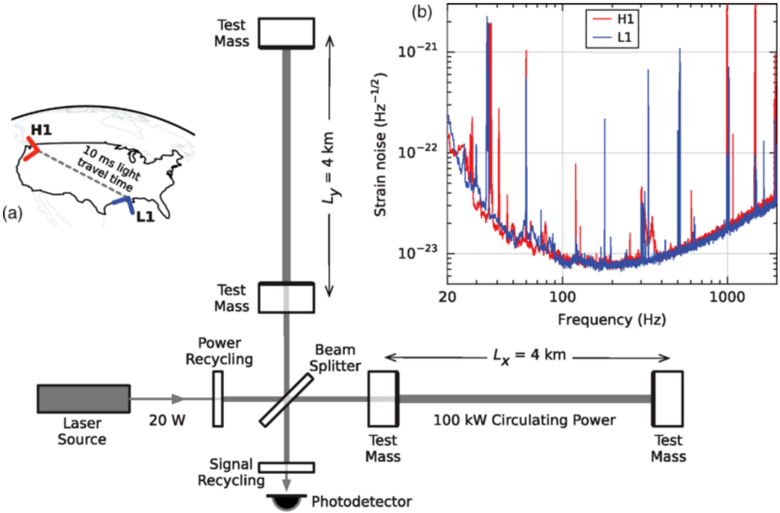
Arman Tursunov

Full GR radiation reaction, EM waves, Abraham–Lorentz force.

Marting Kološ

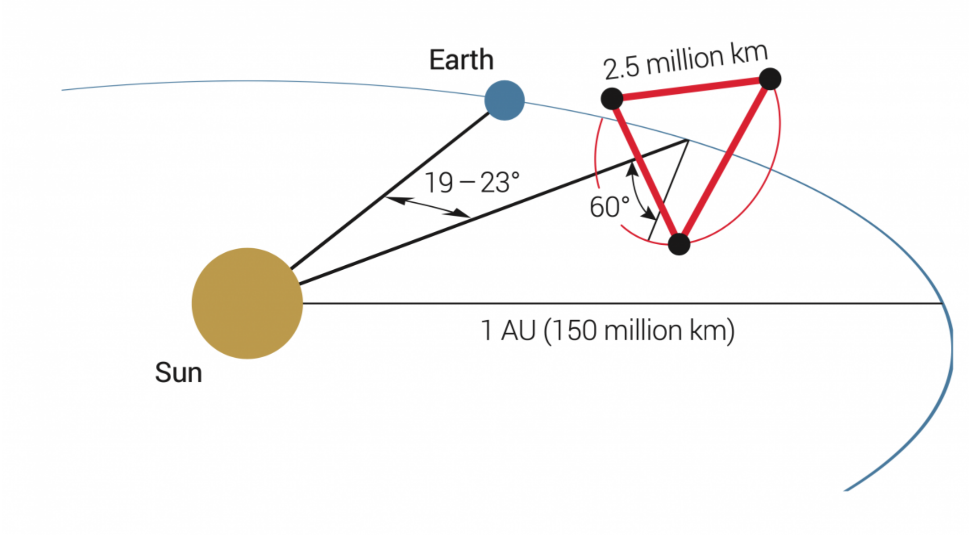
Collaboration with RP and AT.

Laser Interferometer Gravitational-Wave Observatory (LIGO)



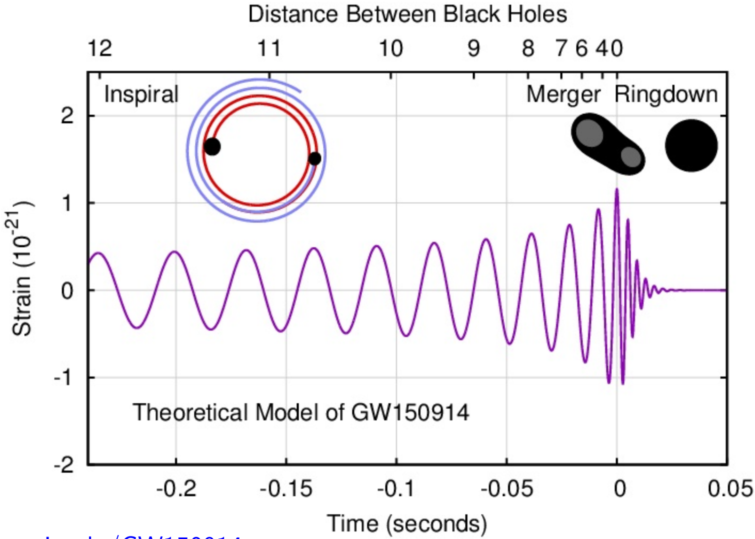
1st direct observation of gravitational waves: 2015 https://www.ligo.caltech.edu/

Laser Interferometer Space Antenna (LISA)



planned launch date: 2035   https://www.lisamission.org/

BH binary coalescence and typical gravitational wave signal



http://ccrg.rit.edu/GW150914

Gravitational Waveform Modeling

Effects of nonlinear dynamic; Can we detect deterministic chaos in GW signal?

A) constructing gravitational signal models

• G. Lukes-Gerakopoulos, O. Kopáček, Ondřej: *Recurrence analysis as a tool to study chaotic dynamics of extreme mass ratio inspiral in signal with noise*, International Journal of Modern Physics D, Volume 27, Issue 2, id. 1850010 (2018)

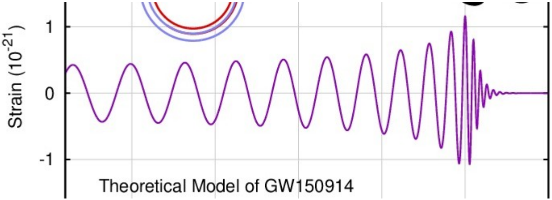
B) working with real measured data

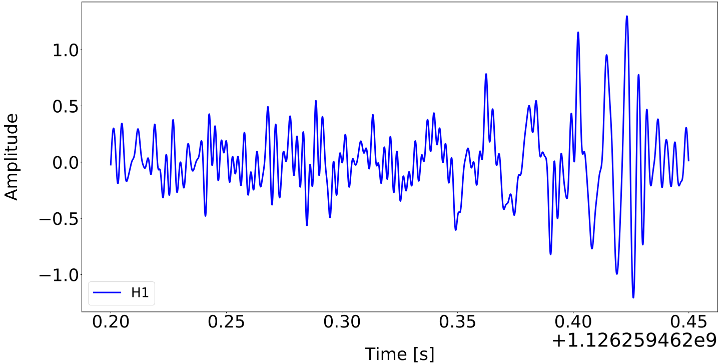
this talk; GW data from LIGO experiment

• Z.Fan, Q.Chen, G.Sun, N.Mastorakis, X.Zhuang: *Nonlinear analysis of gravitational wave signals based on recurrence quantification analysis*, MATEC Conference 210, 05011 (2018)

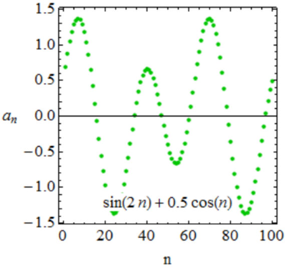
• Lenka Vozárová: *Nonlinear analysis of gravitational wave signal*, master thesis, Silesian University in Opava, supervisor Martin Kološ (2022)

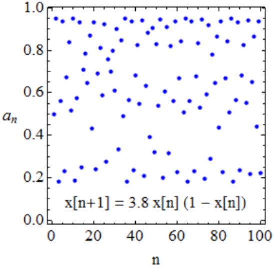
Smooth GW signal model vs. Messy real data (GWOSC.org)

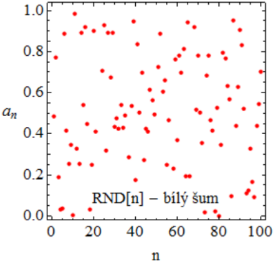




Data: regular — deterministic chaos — noise

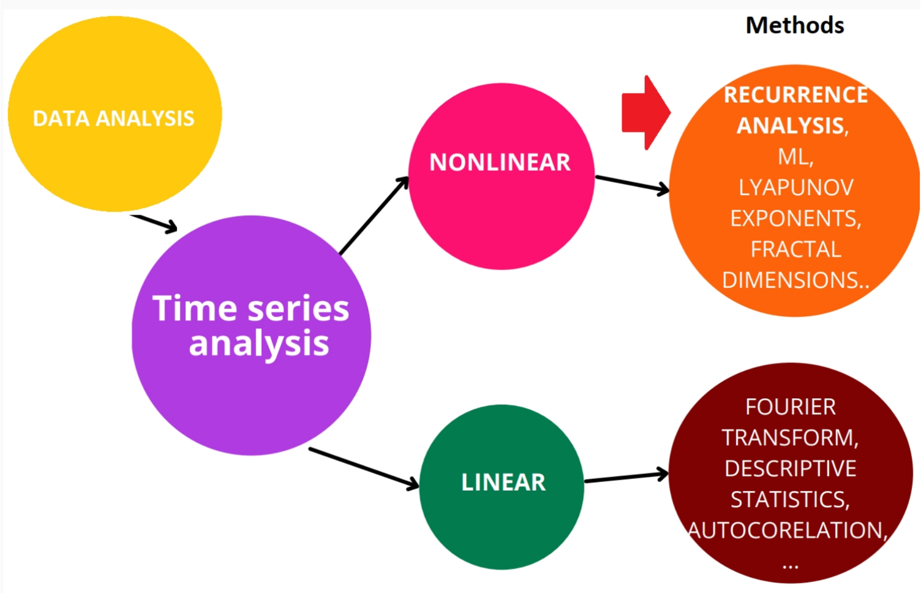






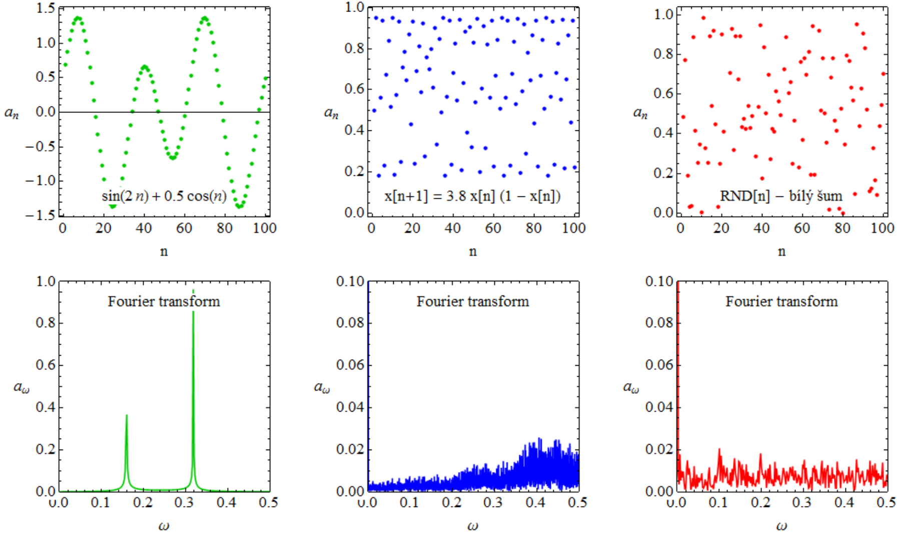
- **regular** - there is law (formula) for every point, results are nice - you can predict future points, results easily to distinguishable from det. chaos or random
- **deterministic chaos** - there is law, but the results are complicated
- **noise** - there is no law; data are random numbers with no mutual connection

- Real data will be combinations (signal+noise) or (chaos+noise).
- Are there methods for chaos detection? **deterministic chaos** × **noise**.


Methods
DATA ANALYSIS
Time series analysis
NONLINEAR
LINEAR
RECURRENCE ANALYSIS, ML, LYAPUNOV EXPONENTS, FRACTAL DIMENSIONS..
FOURIER TRANSFORM, DESCRIPTIVE STATISTICS, AUTOCORELATION, ...

Fourier transform acting on sequence $a_n = \{a_1, a_2, \dots\}$

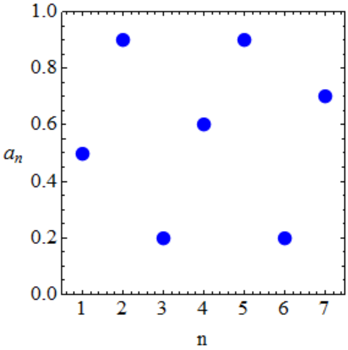
regular — **deterministic chaos** — **noise**

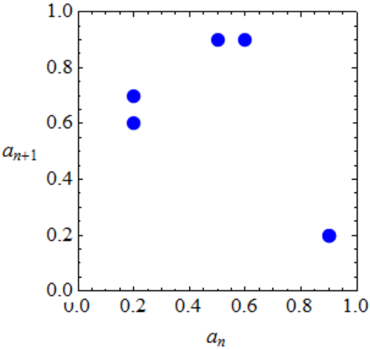


Phase space reconstruction for $a_n = \{a_1, a_2, \ldots\}$

From number sequence $a_n : \mathbb{N} \to \mathbb{R} \quad a_n = \{a_1, a_2, a_3, a_4, \ldots\}$ we will create doubles (n-tuple) $A_{\mathrm{RFP}} = \{(a_n, a_{n+1})\} = \{(a_1, a_2), (a_2, a_3), \ldots\}$, and plot them

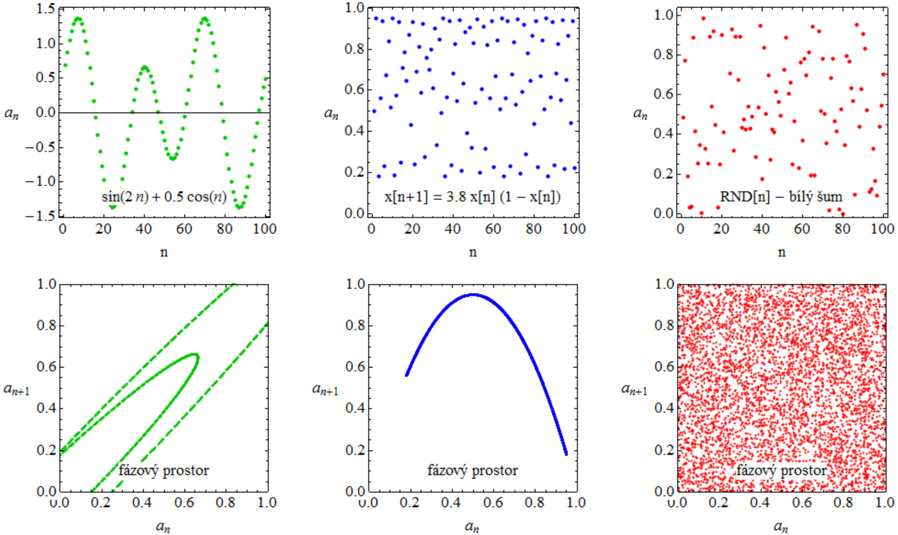
$$\begin{aligned} a_n &= \{0.5, 0.9, 0.2, 0.6, 0.9, 0.2, 0.7\}, \\ A_{\mathrm{RPS}} &= \{(0.5, 0.9), (0.9, 0.2), (0.2, 0.6), (0.6, 0.9), (0.9, 0.2), (0.2, 0.7)\} \end{aligned}$$



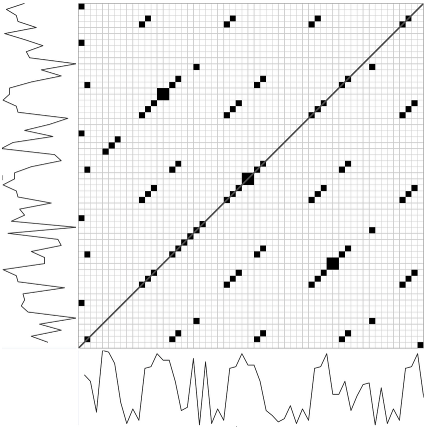


Phase space reconstruction for $a_n = \{a_1, a_2, \ldots\}$

regular — **deterministic chaos** — **noise**



What is Recurrence Quantification Analysis (RQA) ?



Let's have trajectory in phase space or time series $\{x_t\}$; $t \in T$; $i, j = 1, \ldots, N$
Recurrence plot is matrix (with 0,1 only)

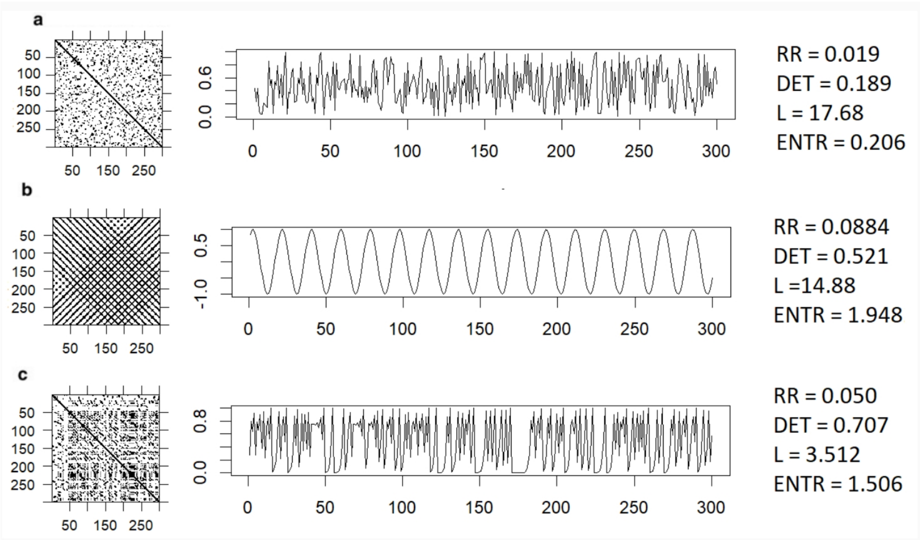
$$R_{ij} = H(\epsilon - \|x_i - x_j\|), \qquad (1)$$

where $H : \mathbb{R} \to \{0, 1\}$ is the Heaviside step function and ϵ is tolerance.

Quantitative analysis (RQA tools):
RR - The recurrence rate, measures density of recurrence points. RR reflects the chance that some state of the system will recur. **DET**, **L**, **ENTR**,...

RQA quantifies the number and duration of recurrences of a dynamical system in its phase space - detection of deterministic chaotic behavior in complex apparently random data.

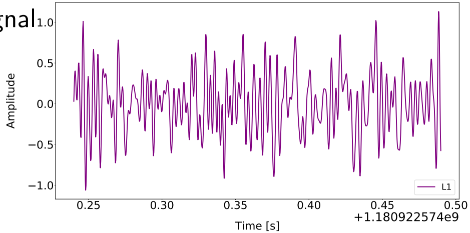
Recurrence Quantification Analysis

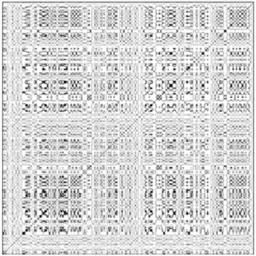


a) homogeneous (noise) b) periodic (oscillations) c) chaotic (logistic map)

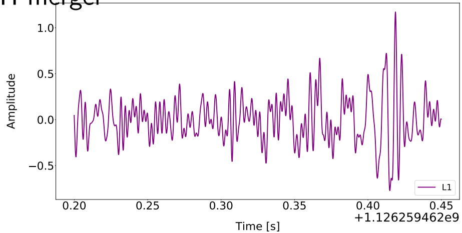
Recurrence plots for GW **Open** Science Center data (LIGO)

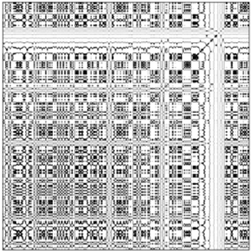
a) no signal





b) BH-BH merger





https://gwosc.org/

Recurrence plots for GW **Open** Science Center data (LIGO)

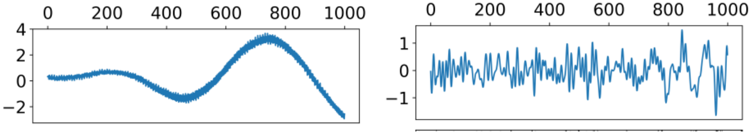
Gravitational Wave Open Science Center https://gwosc.org/

Discover Gravitational-Wave Observatory Data, Tutorials, and Software Tools.

Google Colab, Jupyter Notebook (Python)

For 83 compact object mergers observed by LIGO (2022):

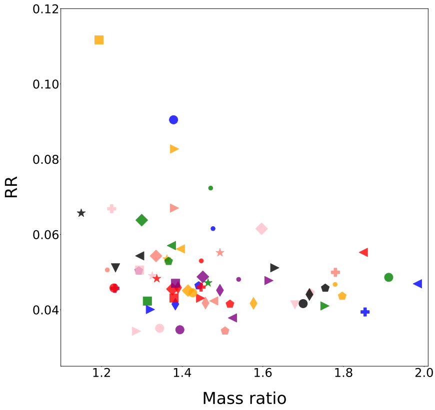
- time of the event; data download
- filtering the known noise

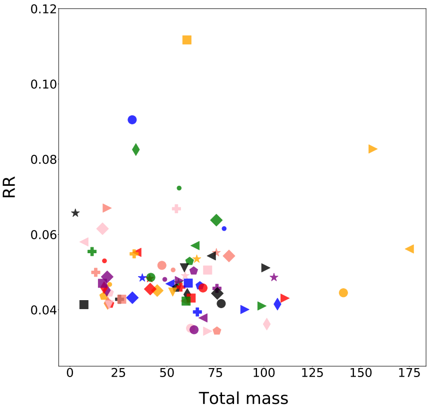


- time series for every event - Recurrence Quantification Analysis

• Lenka Vozárová: *Nonlinear analysis of gravitational wave signal*, master thesis, Silesian University in Opava, supervisor Martin Kološ (2022)

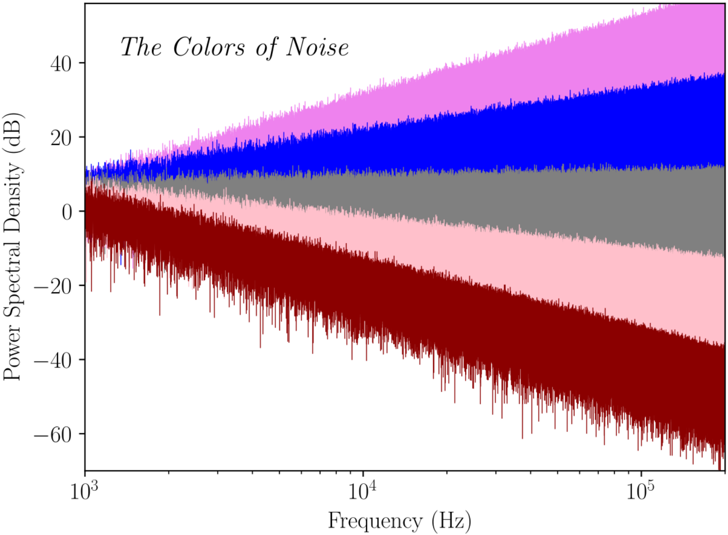
Is nonlinearity related to mass? (LIGO GW data, 83 mergers)





for 83 compact object mergers Recurrence Quantification Analysis has been performed: Parameters measuring chaos in data are given as function of total object mass or mass ratio - each mark represent one GW event (BH-BH,NS-NS).

Differentiate chaos from noise: white $\neq$ chaos $\|$ red $\sim$ chaos



- violet, purple
- blue
- white, Random[ ]
- gray
- pink
- red, Brownian

https://en.wikipedia.org/wiki/Colors_of_noise

What Opava can do within LISA collaboration - future plans

- it is tricky to directly detect nonlinearity in real data (noise!); - resonances(?)
- RQA method upgrade:
  • R.Pánis; K.Adámek, N.Marwan: *Averaged recurrence quantification analysis: Method omitting the recurrence threshold choice*, EPJ Spec.Topics,232,1 (2023)
- RQA time complexity is $\mathcal{O}(N^2)$ - problem for long data
  new AccRQA code: GPU accelerated calculation of RQA metrics using CUDA
  https://github.com/KAdamek/AccRQA
- GW waveform modeling: quasinormal modes (Roman Konoplya), neutron stars merger (Martin Urbanec), full GR radiation reaction (Arman Tursunov),...

Thank you for your attention.

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