Black Holes: from Stars to Galaxies – across the Range of Masses
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Black holes and magnetic fields

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Abstract. Stationary axisymmetric magnetic fields are expelled from outer horizons of black holes as they become extremal. Extreme black holes exhibit Meissner effect also within exact Einstein–Maxwell theory and in string theories in higher dimensions. Since maximally rotating black holes are expected to be astrophysically most important, the expulsion of the magnetic flux from their horizons represents a potential threat to an electromagnetic mechanism launching the jets at the account of black-hole rotation.

Keywords. Black hole physics – magnetic fields – galaxies: jets

1. Introduction

The exact mechanism of formation of highly relativistic jets from galactic nuclei and microquasars remains unknown. Four ways by which a black hole or its accretion disk could power two opposite jets are indicated in figure 1 (taken from the *Czech* edition of Kip Thorne's popular book to indicate how, as compared with the last 1967 IAU meeting in Prague, black holes domesticated even in central Europe): (a) wind from the disk may blow a bubble in a spinning gas cloud and hot gas makes the orifices through which jets are shot out; (b) the surface of the puffed rotating disk forms funnels which collimate the wind; (c) magnetic field lines anchored in the disk are spinning due to disk's rotation and push plasma to form jets; (d) magnetic lines threading through the hole are forced to spin by the "rotating geometry" and push plasma outwards along the rotation axis.

The last way, the Blandford–Znajek mechanism, is considered to be the most relevant. The field brought into the innermost region and onto the black hole from the outside has clean field structure while the field in/around the disk is expected to be quite chaotic (Thorne et al 1986). The estimated power radiated out from the "load" regions farther away from the hole is

$$\Delta L_{\max} \simeq \left[10^{45}\frac{\mathrm{erg}}{\mathrm{sec}}\right]\left[\frac{a}{M}\right]^2\left[\frac{M}{10^9 M_\odot}\right]^2\left[\frac{B_n}{10^4 G}\right]^2. \qquad (1.1)$$

Here $a \equiv J/M$ is the hole's angular momentum per unit mass (velocity of light $c = 1$, gravitational constant $G = 1$), B_n is the normal magnetic field at the horizon. Hence, the highest power is achieved when the hole is "extreme", i.e., rotating with maximal angular momentum or approaching the $a = M$ limit, and B_n determining the magnetic flux across the horizon as high as possible.

2. Meissner effect

The main purpose of this contribution is to point out that these two aspects go one against the other: black holes approaching extremal states exhibit a "Meissner effect" – they expel external vacuum stationary (electro)magnetic fields.

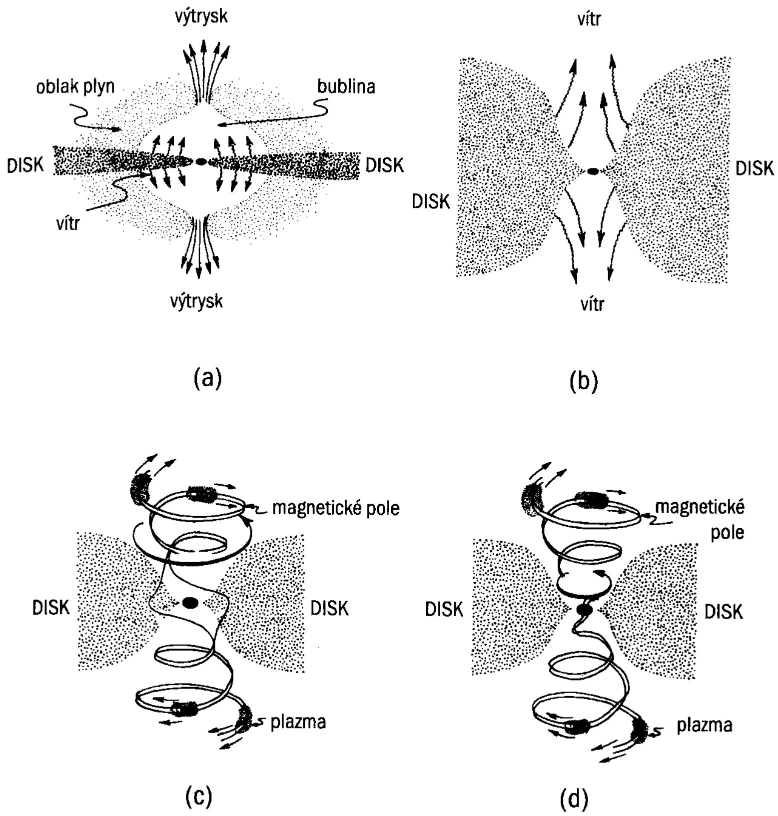

Figure 1. Four mechanisms of jet formation (drawing from "Černé díry a zborcený čas", the Czech edition of "Black Holes and Time Warps", by K. S. Thorne 2004).

To see this effect in the simplest situation, consider the magnetic test field B_0 which is uniform at infinity and aligned with hole's rotation axis. Solution of Maxwell's equations on the background geometry of a rotating (Kerr) black hole with boundary condition of uniformity at infinity and finiteness at the horizon yields the field components; from these the lines of force are defined as lines tangent to the Lorentz force experienced by test magnetic/electric charges at rest with respect to locally non-rotating frames (preferred by the Kerr background field). The field lines are plotted in figure 2 for $a = 0.5M$ and in extreme case $a = M$. Notice that only weak expulsion occurs in the former case. There is a simple analytic formula for the flux across the hemisphere of the horizon: $\Phi = B_0 \pi r_+^2 (1 - a^4/r_+^4)$, where $r_+ = M + (M^2 - a^2)^{1/2}$ (King et al 1975; Bičák & Janiš 1985).

As a consequence of the coupling of magnetic field to frame-dragging effects of the Kerr geometry the electric field of a quadrupolar nature arises. Its field lines are shown in figure 3. Again the flux expulsion takes place. While even with $a = 0.95M$ it is still not very distinct, the expulsion becomes complete in the extreme case.

One can demonstrate that total flux expansion takes place for all axisymmetric stationary fields around a rotating black hole (Bičák & Janiš 1985; Bičák & Ledvinka 2000). In figure 4 the field lines of a current loop in the equatorial plane are shown.

The Meissner-type effect arises also for charged (Reissner–Nordström) black holes. Although extremely charged black holes ($e^2 = M^2$) are probably not important astrophysically they may be significant in fundamental physics (as very special supersymmetric BPS states mass of which does not get any quantum corrections). Since electromagnetic

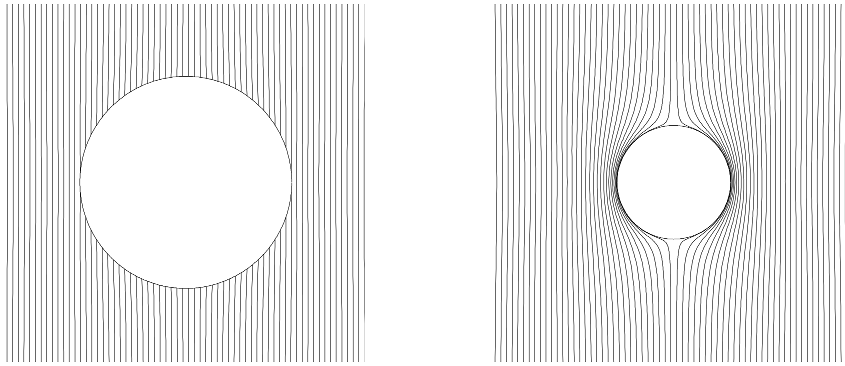

Figure 2. Field lines of the test magnetic field uniform at infinity and aligned with hole's rotation axis. Two cases with $a = 0.5M$ (left) and $a = M$ (right) are shown.

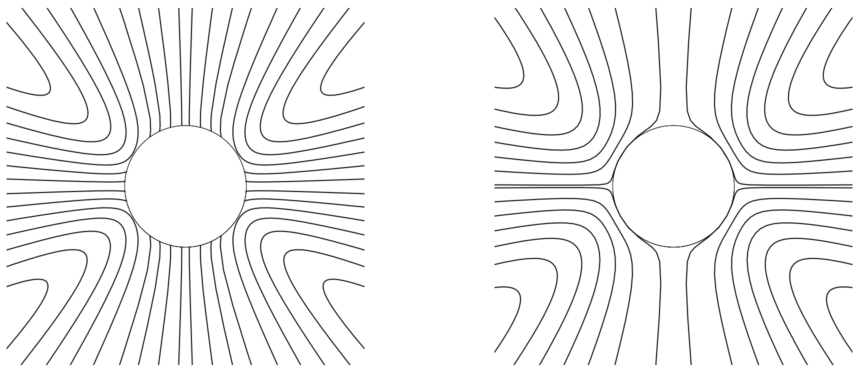

Figure 3. Field lines of the electric field induced by the "rotating geometry" of Kerr black hole in asymptotically uniform test magnetic field. $a = 0.95M$ (left), and $a = M$ (right).

perturbations are in general coupled to gravitational perturbations, the resulting formalism is involved. Nevertheless, one may construct explicit solutions, at least in stationary cases. From these the magnetic field lines follow as in the Kerr case. The magnetic field lines of a dipole far away from the hole look like in a flat space (Fig. 5a), however, when the dipole is close to the horizon, the expulsion in the extreme case is evident (Fig. 5b). Due to the coupling of perturbations closed field lines appear without any electric current inside; see Bičák & Dvořák (1980) for details.

There exist exact models (exact solutions of the Einstein–Maxwell equations) representing in general rotating, charged black holes immersed in an axisymmetric magnetic field. The expulsion takes place also within this exact framework – see Bičák & Karas (1989), Karas & Vokrouhlický (1991), Karas & Budínová (2000).

Very recently the Meissner effect was demonstrated for extremal black-hole solutions in higher dimensions in string theory and Kaluza–Klein theory. The question of the flux expulsion from the horizons of extreme black holes in more general frameworks is not yet understood properly. The authors of Chamblin, Emparan & Gibbons (1998) "believe this to be a generic phenomenon for black holes in theories with more complicated field content, although a precise specification of the dynamical situations where this effect is present seems to be out of reach."

The flux expulsion does *not* occur for the fields which are not axisymmetric. In figure 6 the field lines are constructed for fields asymptotically uniform and perpendicular

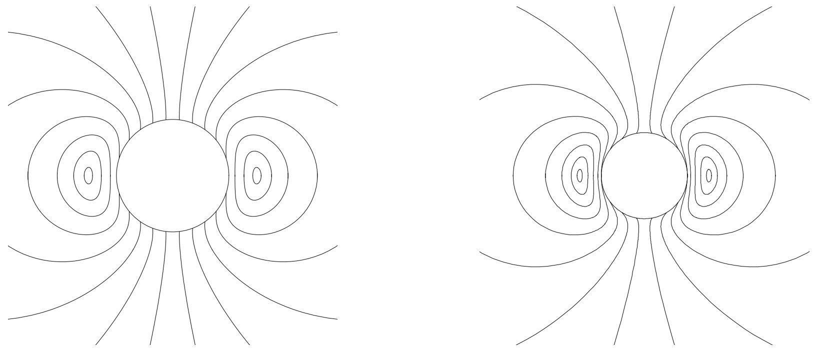

Figure 4. Field lines of the magnetic field of a current loop in the equatorial plane of Kerr black hole located at $r = 1.5\ r_+$. Two cases with $a = 0.9M$ (left) and $a = 0.995M$ (right) are shown.

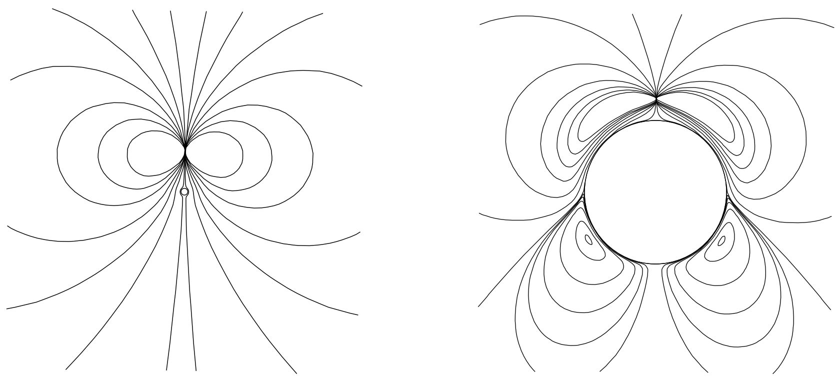

Figure 5. Field lines of the magnetic dipole placed far away from the extreme Reissner–Nordström black hole ($e = M$, left panel), and close to the hole (right panel).

to the axis of an extreme Kerr black hole. There is an angle $\delta_{\max} \sim -63°$ for which the flux of B_1 component across a hemisphere is maximal, $\Phi_{\max} \sim 2.25 B_1 \pi r_+^2$. This is the effect of the rotating geometry (for detailed description, see Bičák & Janiš 1985; Dovčiak et al 2000). In these "misaligned" situations, there is an angular momentum flux to infinity, so such conditions are not stationary.

Naturally, the shape of magnetic and electric field lines depends on the choice of observers, however, the whole discussion can be cast in equivalent and invariant form by employing surfaces of constant flux in which field lines reside. In this way, figure 7 demonstrates that the cross-section of the black hole for the capture of non-aligned magnetic fields indeed does not vanish as the hole rotation aproaches the extreme value.

3. Recent progress, open problems

Can these important properties of electromagnetic fields in neighborhood of black holes be relevant in astrophysical conditions? Recently remarkable progress in axisymmetric simulations of rotating black holes surrounded by magnetized plasma based on general-relativistic MHD has been achieved by various authors, in particular by C. Gammie, S. Komissarov, J. McKinney, J. Krolik, D. Uzdensky, H. Kim, A. Aliev and others (see the E-print archive for references). Such simulations could bring an answer although some new analytic insight may also be required. For example, McKinney & Gammie (2004) remark "we see no sign of expulsion of flux from the horizon... It is possible that we have

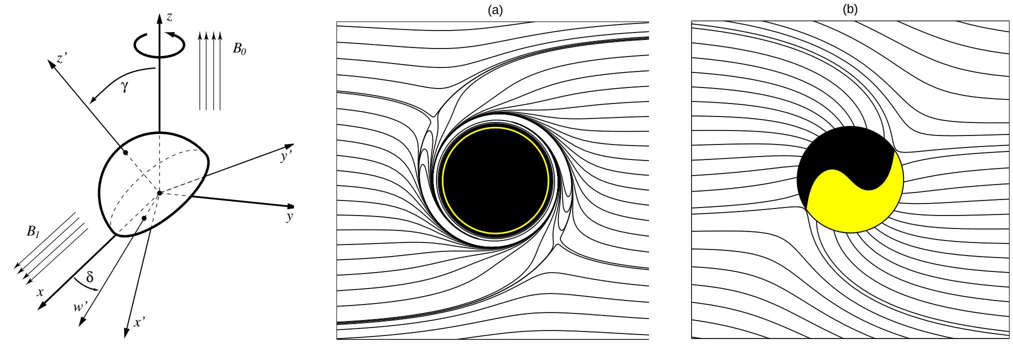

Figure 6. Lines of the magnetic field which is asymptotically uniform and perpendicular to the rotation axis. The equatorial plane is shown as viewed from top, i.e. along the rotation axis, (a) in the frame of zero angular momentum observers orbiting at constant radius; (b) in the frame of freely falling observers. In the panel (b), two regions of ingoing/outgoing lines are distinguished by different levels of shading of the horizon (the hole rotates counter-clockwise, $a = M$).

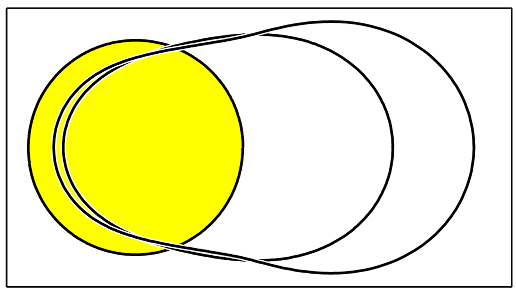

Figure 7. The cross-sectional area for the capture of magnetic field lines (asymptotically uniform magnetic field perpendicular to the rotation axis). The three curves correspond to different values of the black-hole angular momentum: $a = 0$ (the black hole cross-section is a perfect circle and its projection coincides with the black-hole horizon of radius $2M$, indicated here by shading), $a = 0.95M$, and $a = M$ (the most deformed shape refers to the maximally rotating case). Non-vanishing cross-sectional area for the extreme case demonstrates that the Meissner-type expulsion of the magnetic field does not operate on non-axisymmetric fields. The hole's rotation axis is vertical and the magnetic field lines are pointing "towards us" (see Dovčiak et al 2000 for details).

not gone close enough to $a/M = 1$." They have $a/M = 0.938$ and, indeed, in view of simplest (though just vacuum) situations illustrated in figures 2 and 3 this value may still be far from 1 to see the effect.

An interesting issue arises in connection with the "black-hole membrane paradigm" of Thorne et al (1986). For an extreme black hole the proper distance from any $r > r_+$ to the horizon is logarithmically infinite, so one might tend to explain the vanishing of magnetic flux by this fact. However, although the flux across a "stretched horizon" (at a finite distance) is non-vanishing even in the extreme case, it depends on where the stretched horizon is located. It turns out, (see section 4 of Bičák & Ledvinka 2000) that for any $\epsilon > 0$ one can find such a stretched horizon that the flux is less then ϵ. This suggests the following question: does the power in the Blandford–Znajek model arise from regions with "relatively large $\sqrt{-g_{00}}$" in near extreme cases?

Another potential obstacle for Blandford–Znajek mechanism to operate efficiently is a low value of magnetic field brought onto the black hole from an accretion disk in realistic situations. Most recently Reynolds, Garofalo & Begelman (2006) obtained an encouraging result on trapping of magnetic flux by the plunge region of a black hole accretion disk. Their analysis is so far limited to slowly rotating black holes; it does not use general relativity; and it depends crucially on the chosen boundary conditions. It is not clear of how large is the disk region from which flux can be dragged inwards.

The main open issue can be stated simply: "Do extremely rotating black holes produce relativistic jets?" A compelling answer may be out of reach for some time yet.

Acknowledgements

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