Relativistic disks as sources of Kerr-Newman fields

Tomáš Ledvinka, Martin Žofka and Jiří Bičák

Department of Theoretical Physics, Faculty of Mathematics and Physics,
Charles University, V Holešovičkách 2, 180 00 Prague 8, Czech Republic
ledvinka@mbox.troja.mff.cuni.cz, zofka@otokar.troja.mff.cuni.cz, bicak@mbox.troja.mff.cuni.cz

If an appropriate region of Kerr-Newman space-time is removed and suitable identifications are made, the resulting space-time can be interpreted as an infinitely thin disk producing the original electromagnetic and gravitational fields. We choose the shape of the regions removed in such a way that radial pressures in the disks vanish. Even the very inner parts such as ergoregions may exist around the disks. The surface energy-momentum tensor of the disks is checked to satisfy the weak and strong energy conditions. To emphasize the reality of these sources two models of the disks are presented: (i) rotating conductive charged rings which are supported against collapse by their internal pressure, (ii) two counter-rotating streams of charged particles moving along circular electro-geodesics. All these disk sources form a three-dimensional parameter space with specific electric charge Q/M, angular momentum a/M and the size of the excluded region being the parameters.

1 Introduction

Although much effort has been devoted to discovering exact solutions of Einstein's equations, there are only a few "physically acceptable" solutions available. What is in particular lacking are sources which would produce numerous known vacuum or electrovacuum metrics burdened by singularities.

Most vacuum **static** Weyl solutions can arise as the metrics of counter-rotating relativistic disks[1,2]. The method used to construct such sources resembles the method of images in electrostatics. Alternatively, it consists in first cutting out a portion of a stationary space-time "between" two hypersurfaces and then glueing suitably these hypersurfaces together. The jump in the normal derivatives of the resulting potentials induces a matter distribution in the disk which arises due to the identification. One has then to analyze whether the matter is physically acceptable.

Later the method has been used to construct physical disk sources of vacuum **stationary** space-times, in particular, of Kerr metrics[3,4].

A similar procedure can be used to construct disk sources of stationary electrovacuum space-times as, for example, of Kerr-Newman fields. The resulting disks then contain rotating matter and also electric currents which arise due to the jumps of the (normal components of) electromagnetic fields.

2 Kerr-Newman Fields

It is convenient to use the Weyl-Papapetrou (W-P) form of the line element

$$ds^2 = e^{-2\nu}\left[e^{2\zeta}(d\varrho^2 + dz^2) + \varrho^2 d\varphi^2\right] - e^{2\nu}(dt + A d\varphi)^2, \tag{1}$$

with the simple relation to Boyer-Lindquist (B-L) coordinates

$$z = (r - M)\cos\vartheta, \quad \varrho = \sqrt{r^2 - 2Mr + a^2 + Q^2}\sin\vartheta. \tag{2}$$

Regarding these relations, functions ν, ζ, A entering the metric can be found from the standard form of the K-N metric in B-L coordinates.

In the W-P coordinates we exclude the region $z \in [-b, b]$. Using then the formalism for thin shells[5] which can carry an electric charge[6], we find that the disk has no radial pressure and the surface stress-energy tensor and electric current can simply be expressed in terms of the derivatives of the metric and electromagnetic potentials at $z = b$ (indices a,b correspond to t,φ)

$$S_{ab} = \frac{\sqrt{g_{\varrho\varrho}}}{8\pi}\left(\frac{g_{ab}}{g_{\varrho\varrho}}\right)_{,z}, \tag{3}$$

$$J_a = -\frac{1}{2\pi} \quad (A_a)_{,z}. \tag{4}$$

The existence of dragging results in a non-diagonal stress-energy tensor. The following two models of the disk are considered. In the first model the disk is constructed from rings with internal pressure. The necessary condition is that S_{ab} can be diagonalized, i.e., one can find, at any radius, φ-isotropic observer (FIO) who sees only energy density and azimuthal pressure. In the charged case the rings are also charged and conduct a current to form the current density J_a.

In the second model the disks are described as two counter-rotating streams of freely moving charged particles. The fact that in the static case their velocities must be identical and opposite, allows one to find $V_\pm = \pm\sqrt{S^\varphi_\varphi/S^t_t}$. When dragging is present, the stream velocities $U^a_\pm$ must be determined from the geodesic (or the electro-geodesic) equation

$$\frac{1}{2}\epsilon_\pm g_{ab,(\varrho)} U^a_\pm U^b_\pm = -\sigma_\pm F_{(\varrho)c} U^c_\pm. \tag{5}$$

Although S^{ab} has three different components, the Einstein and contracted Gauss-Codazzi equations guarantee that the following decompositions can be made

$$S^{ab} = \epsilon_+ U^a_+ U^b_+ + \epsilon_- U^a_- U^b_-, \tag{6}$$

$$J^\alpha = \sigma_+ U^a_+ + \sigma_- U^a_-. \tag{7}$$

3 Properties of the disks

The central mass densities and the central electric charge densities of the disks can be expressed in analytic forms

$$\epsilon_c = \frac{M}{2\pi}\frac{(b+M)^2 - a^2 - Q^2(1 + b/M)}{[(b+M)^2 + a^2]^{3/2}[b^2 + a^2 - M^2 + Q^2]^{1/2}}, \tag{8}$$

$$\sigma_c = \frac{Q}{2\pi}\frac{(b+M)^2 - a^2}{[(b+M)^2 + a^2]^2}\sqrt{\frac{(b+M)^2 + a^2}{b^2 + a^2 + Q^2 - M^2}}. \tag{9}$$

Other physical properties of the disks can be best exhibited graphically. Here we limit ourselves to present just one example corresponding to the disk producing the Kerr-Newman gravitational and electromagnetic fields with $M = 1$, $a = 0.4$ and $Q = 0.1$. In the figures the disks with $b/M = 0.912, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5$ are considered. The disk with $b/M = 0.912$ cannot be constructed from the counter-rotating streams, even though it satisfies both the weak and strong energy conditions. The disks become highly relativistic in central regions but they all have "classical" properties at large R (proper circumferential radius). The mass and charge densities (as measured by FIOs) decrease rapidly with R. The disks have infinite extensions, their total mass, charge, and angular momentum are finite. They should have properties common with finite relativistic disks. And they represent the only physical sources of Kerr-Newman fields available today.

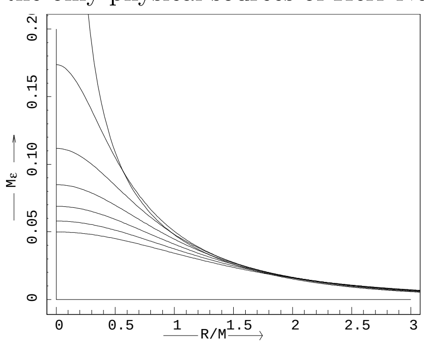

Figure 1. Radial distribution of mass density as seen by FIO

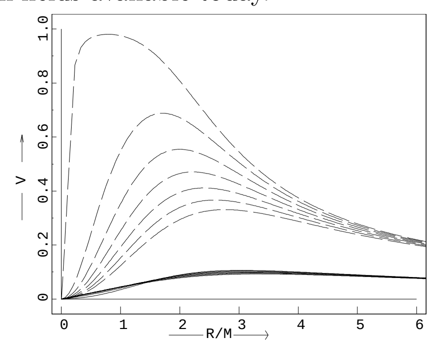

Figure 2. Velocity of FIO with respect to LNRF and the ratio of azimuthal pressure to mass density (dashed)

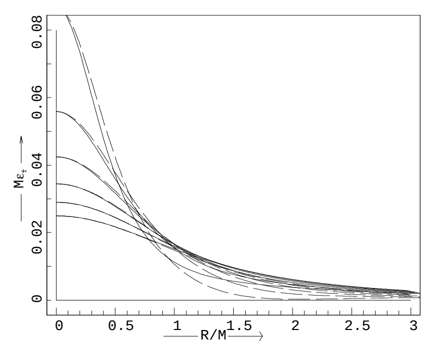

Figure 3. Distribution of mass density in both streams ϵ_+ and ϵ_- (dashed)

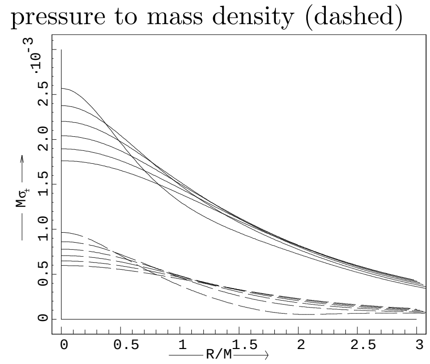

Figure 4. Distribution of charge density in both streams σ_+ and σ_- (dashed)

References

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