INTERPLAY BETWEEN FORCES IN KERR-NEWMAN FIELD

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We discuss dynamical properties of generally non-Keplerian equatorial circular orbits and zero-angular-momentum spherical polar orbits around Kerr-Newman black holes. By considering charged test particles, the thrust is represented by the Lorentz force. Below photon orbits, the acceleration of a given type of orbits depends on their orbital angular velocity in a counter-intuitive manner. We interpret this result in terms of suitably defined forces of classical type.

1 Introduction

In the last decade much interest has been devoted to *rotospheres* – the strong-gravity regions where dynamics of angular motion is related to kinematical parameters of the orbits in a counter-intuitive manner. The effect can be seen in its "purest" form in the Schwarzschild field: below the radius $r = 3M$ of the circular photon orbit, for instance, an increase of the angular velocity ω of a test particle, forced to move on a given circular (non-Keplerian) orbit, requires an increase of the thrust in the *outward* direction. It has been shown recently [1] that also in general stationary axisymmetric spacetimes the rotospheres are bounded by photon orbits. In this case, however, the "rotosphere effects" mix with dragging and the picture is no longer so simple (e.g., it is different for co-rotating and counter-rotating particles).

The appearance of rotospheres has served as a challenge to "explain" the counter-intuitive effects in a generally valid and covariant, but also simple and intuitive pseudo-classical language. Here we only refer to the most recent papers [2,3,4,1] for survey and thorough lists of literature.

The aim of the present contribution is to demonstrate the effect on charged test particles in the field of the Kerr-Newman black hole. We leave the particles' thrust to be fully represented by the electromagnetic Lorentz force which in fact is the only fundamental macroscopic force available. This force, however, is itself dependent on velocity and the question arises of how this dependence interplays with the particle's anomalous gravitational and inertial properties in the closest vicinity of the central source. We shall show that near black holes the particles orbiting faster typically need *larger* "repulsive" specific charge to keep them in the orbit, i.e. the behaviour opposite to what one (correctly) expects far from the centre. Here only main results are given, for a full treatment, see [5]. We use Boyer-Lindquist coordinates (t, r, θ, ϕ) and geometrized units ($c = G = 1$), Greek indices running 0-3.

2 The rotosphere effect seen on simple spherical orbits

The rotosphere effects are connected with tangential motion around a central source, so it is natural to consider orbits which are purely angular ("spherical") in some sense. In studying the dependence of their characteristics on the angular velocity, one can hardly expect to obtain simple results for a *general* spherical orbit, along which 4-acceleration and other quantities change – both in modulus and in direction – even without a change of ω. Thus, usually equatorial ($\theta = 90°$) circular ($r = \text{const}$) orbits are analysed, along which particles move purely in azimuthal direction with constant angular velocity $\omega = d\phi/dt$. These are the only orbits whose 4-acceleration a^μ points, for any r and ω, in the same spatial direction and, for any fixed r, only its magnitude varies with ω. The radial component of the 4-acceleration reads

$$a^r = \frac{\Delta}{r^3}\,\frac{(Mr - Q^2)\,(1 - a\omega)^2 - r^4\omega^2}{r^2 - (2Mr - Q^2)\,(1 - a\omega)^2 - (r^2 + a^2)\,r^2\omega^2} \tag{1}$$

and the specific charge which is required to ensure the corresponding thrust is

$$\frac{e}{m} = \frac{1}{Q(1 - a\omega)}\,\frac{(Mr - Q^2)\,(1 - a\omega)^2 - r^4\omega^2}{[r^2 - (2Mr - Q^2)\,(1 - a\omega)^2 - (r^2 + a^2)\,r^2\omega^2]^{1/2}}\,, \tag{2}$$

m being the particle's rest mass, M, Q and a parameters (mass, electric charge and specific angular momentum) of the black hole, and $\Delta = r^2 - 2Mr + Q^2 + a^2$.

At the lower limit $\omega_{\rm min}$ of permitted ω's, the dependence of a^r on ω at certain fixed r is "intuitive" above the radius of the counter-rotating (outer) photon orbit $r_{\rm ph-}$, whereas it is "counter-intuitive" below $r_{\rm ph-}$: for any fixed $r > r_{\rm ph-}$, a^r goes to $-\infty$ for $\omega \to \omega_{\rm min}$, i.e. the particle needs greater and greater inward thrust as its velocity approaches the velocity of light; for $r < r_{\rm ph-}$, on the other hand, a^r goes to $+\infty$ for $\omega \to \omega_{\rm min}$, i.e. the particle needs greater and greater *outward* thrust as its velocity approaches that of light. At the upper limit $\omega_{\rm max}$ of permitted ω's, we find an analogous behaviour – with co-rotating (inner) photon orbit $r_{\rm ph+}$ now dividing the "intuitive" and "counter-intuitive" regions. The nature of the curves of $e(\omega; r)$ is given by the interplay between the velocity dependence of a^r and that of the Lorentz force. At the limiting values of ω, however, the rotosphere effect appears clearly and the behaviour of e/m is similar to that of a^r. Figures are given in [5].

Another simple type of orbits are those which are spherical ($r = \text{const}$), have zero angular momentum with respect to the axis – thus "co-rotate with the geometry" with the azimuthal angular velocity $\omega = -g_{t\phi}/g_{\phi\phi}$, and have some latitudinal angular velocity $\Omega = d\theta/dt$. If $|\Omega|$ is constant, the radial acceleration is

$$a^r = \frac{(r^2 + a^2)^2[M\,(2r^2 - \Sigma) - rQ^2] - r\,(2Mr - Q^2)^2 a^2 \sin^2\theta - r\mathcal{A}^2\Omega^2}{\Sigma^2\mathcal{A}\,(1 - \mathcal{A}\Omega^2/\Delta)}\,, \tag{3}$$

where $\Sigma = r^2 + a^2\cos^2\theta$ and $\mathcal{A} = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta$. It exhibits a reversal in the dependence on $|\Omega|$ of the same nature as was found above at $a^r(\omega)$ for circular orbits: far from the source, the dependence of a^r on $|\Omega|$ is intuitive, i.e. $\partial a^r/\partial|\Omega| < 0$, but it becomes counter-intuitive (i.e. $\partial a^r/\partial|\Omega| > 0$) below the radius $r_{\rm ph}$ of the photon spherical polar orbit, given by the equation $r\,(\Delta + Q^2) - M\,(r^2 - a^2) = 0$. The sign of $\partial a^r/\partial|\Omega|$ does not depend on θ, so the boundary of the region where $\partial a^r/\partial|\Omega|$ is positive (i.e. counter-intuitive) is given by $r = \text{const} = r_{\rm ph}$. Since there is no dragging in the θ-direction, the curves of $a^r(\Omega; r)$ are symmetric about $\Omega = 0$.

For the charge e which can keep the particle on a given zero-angular-momentum spherical polar trajectory, different components of the equation of motion imply different *functions* $e(\theta)$: a given charge can provide the prescribed acceleration only

at some specific θ. For example, at the axis of symmetry, $\theta = 0°$ or $180°$, we find $a^t = a^\theta = a^\phi = 0$ and the required e/m to be given unambiguously by

$$\frac{e}{m} = \frac{\sqrt{\mathcal{A}}}{Q(r^2 - a^2)}\,\frac{M\,(r^2 - a^2) - rQ^2 - r\,(r^2 + a^2)^2\Omega^2}{\sqrt{\Sigma(\Delta - \mathcal{A}\Omega^2)}}\,. \tag{4}$$

3 Interpretation in terms of forces

The results described above can be well understood in terms of quantities measured by the zero-angular-momentum observers (ZAMOs) with respect to their local orthonormal frames (locally non-rotating frames, LNRFs). According to the definition of forces given for a motion in a general spacetime in [6], we can rewrite the equation of motion in the 3-vector "classical" form as a balance between the gravitational, the dragging, the Coriolis, the centrifugal, the "tangent-inertial-resistance", and the Lorentz forces, $m(\vec{a}_{\rm g} + \vec{a}_{\rm d} + \vec{a}_{\rm C} + \vec{a}_{\rm cf} + \vec{a}_{\rm ti}) = \vec{F}_{\rm L}$ (the arrows denote the spatial parts of the corresponding contravariant 4-vectors). The Lorentz force has the usual form $\vec{F}_{\rm L} = \hat{\gamma} e\,(\vec{E} + \vec{v} \times \vec{B})$, $\vec{v}$ denoting the particle's relative velocity measured by ZAMO, $\hat{\gamma} = (1 - \hat{v}^2)^{-1/2}$, and $\vec{E}$ and $\vec{B}$ are the electric and the magnetic fields felt by ZAMO. The terms on the left-hand side also have simple, "classical" forms. For circular orbits (not necessarily equatorial), $\vec{a}_{\rm g} = \hat{\gamma}^2 \vec{a}_{\rm ZAMO}$, $\vec{a}_{\rm d} = \vec{a}_{\rm C} = \hat{\gamma}^2\,\vec{\Omega}_{\rm LNRF} \times \vec{v}$, $\vec{a}_{\rm cf} = \hat{\gamma}^2\hat{v}^2\vec{n}$, $\vec{a}_{\rm ti} = \vec{0}$. For zero-angular-momentum spherical polar orbits, $\vec{a}_{\rm g}$ and $\vec{a}_{\rm cf}$ are again given as above, while $\vec{a}_{\rm d} = -\vec{a}_{\rm C} = -(u^t_{\rm ZAMO})^2\hat{\gamma}^2\,\vec{\Omega}_{\rm LNRF} \times \vec{v}$, $\vec{a}_{\rm ti} = \hat{\gamma}^3\,(\vec{v}_0 + \hat{v}\vec{u}_{\rm ZAMO})\,d\hat{v}/d\tau$. In the above expressions, $u^\mu_{\rm ZAMO}$ is the 4-velocity and $\vec{a}_{\rm ZAMO}$ the acceleration of the ZAMO, $\vec{\Omega}_{\rm LNRF}$ the angular velocity of the LNRF relative to the (Fermi-Walker transported) gyroscopes carried by the ZAMO, and $\vec{v}_0 = \vec{v}/\hat{v}$. The interior normal to the projection of the trajectory into the local ZAMO's 3-space, $\vec{n}$, is given by $n^i = \Gamma^i{}_{\phi\phi}/g_{\phi\phi}$ in the case of a circular orbit, and by $n^i = \Gamma^i{}_{\theta\theta}/g_{\theta\theta}$ in the case of a zero-angular-momentum spherical polar orbit.

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