COULD THE KERR GEOMETRY CONTRIBUTE TO THE COLLIMATION OF COSMIC JETS?*

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ABSTRACT

The latitudinal motion of free test particles outwards from central regions in the Kerr geometry is investigated. As compared with particles with parabolic energies, particles with hyperbolic energies escape closer to the rotation axis, while slower particles tend to fall towards the equatorial plane. The effect is not large in the case of black holes, but it is significant for naked singularities. In a region still closer to the center, the repulsive character of the field near the rotation axis and near the disk spanning the ring singularity gives particles outward accelerations and collimates the particles along the rotation axis.

1. Introduction

Although there are no direct observational grounds available, it is generally believed that cosmic jets emerging from quasars and active galactic nuclei are initiated on scales of order of the size of a central engine. As experts in the field emphasize, "the major unsolved problem is to understand how and where jets are collimated"[1]. In this paper we wish, in this context, to examine a simple question which does not appear to have been tackled yet: could not the gravitational field of the central source, i.e. — as is generally supposed — of the Kerr geometry itself, possibly help in (pre)collimating particles ejected from the vicinity of the source by some process? We discuss this question by studying trajectories, in particular the latitudinal and radial motion, of free test particles moving outwards in Kerr geometry from given positions and under given initial conditions (see Fig. 1). We thus

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deal with idealized situations in which other physical processes are not considered, only the gravity is taken into account.

2. Results

Just few results of our numerical calculations can be given here (Figs. 2–4). The forms of the trajectories depend on the coordinates in which they are plotted. Two coordinate systems are used: the Boyer-Lindquist (BL) coordinates (t, r, θ, φ) and the Kerr–Schild (KS) coordinates $(\tilde{t}, x, y, z)$. BL coordinates are geometrically natural coordinates in the Kerr spacetime; in these coordinates the geodesic equations separate; and they go over into the Schwarzschild coordinates if the angular momentum parameter a of the Kerr metric vanishes. However, when the mass parameter $M = 0$, the BL coordinates are oblate spheroidal coordinates in flat spacetime. The whole disk $r = 0$, $\theta \neq \pi/2$ which is spanning the ring singularity $r = 0$, $\theta = \pi/2$, is mapped into the origin in the plots with $r \sin\theta$, $r\cos\theta$ as axes. The disk is well described just in the KS coordinates by $z = 0$, $x^2 + y^2 < a^2$; and the KS coordinates become standard Cartesian coordinates when $M = 0$ and spacetime is flat. In Fig. 4 $\varrho_f = (x^2+y^2)^{1/2}$, $r_f = (\varrho_f^2 + z^2)^{1/2}$, $\cos\theta_f = z/r_f$. In the case of a black hole, particles ejected radially (along $\theta =$ const and with zero angular momentum) with hyperbolic energies escape — in comparison with particles with lower energies — closer towards the rotation axis, but the effect is not very large (Fig. 2). In the case of naked singularities, the effect is much more pronounced (Fig. 3). This, primarily, is due to the fact that particles may strongly be influenced by the field in the regions close to the disk and then move towards more distant regions. It is known[2] that some repulsive effects occur close to the disk. The repulsive character of the Kerr field above the disk is clearly demonstrated in Fig. 4. The trajectories constructed there also indicate how the field in this region can collimate them considerably. Particles with parabolic energies ejected from a point situated above the disk (at about an

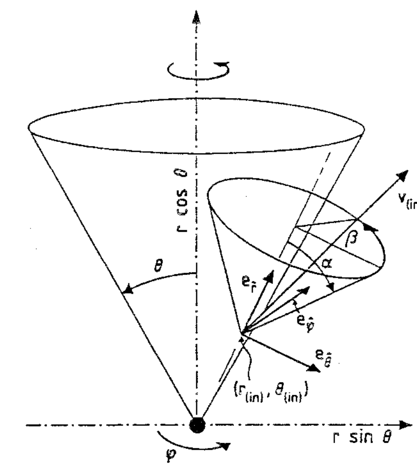
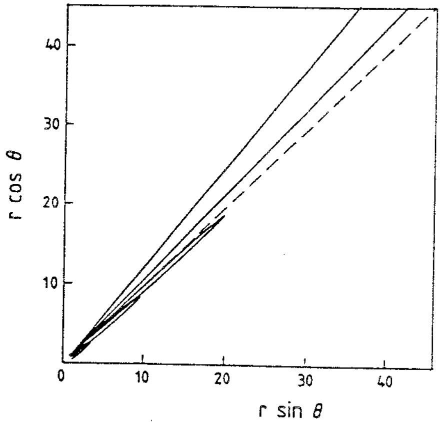


Figure 1. The particle is ejected from a point $(r_{(\mathrm{in})}, \theta_{(\mathrm{in})})$. The initial velocity is represented by its magnitude $v_{(\mathrm{in})}$, by local latitude α and azimuth β, as measured with respect to the orthonormal basis of the locally non-rotating frame.



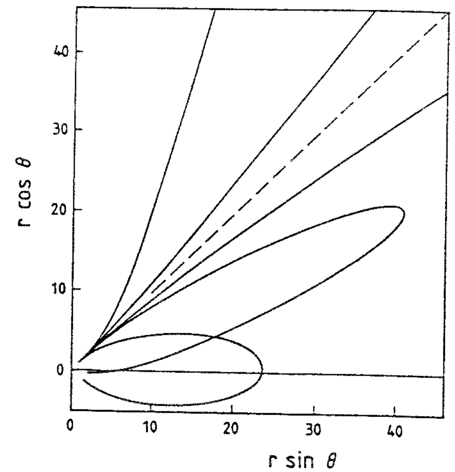


Figure 2. Particles ejected radially ($\alpha = 0°$) at $r_{(\text{in})} = 1.15$, $\theta_{(\text{in})} = 45°$, from a black hole with $a = 0.990$. Initial velocities (from "bottom to top"): $v_{(\text{in})} = 0.99920, 0.99947, 0.99952, 0.99965, 0.99999$; specific energies at infinity: $\varepsilon = 0.745383, 0.915706, 0.962207, 1.126811, 6.661199$. (The dimensionless $r = r^*/(G^*M^*/c^{*2})$, $a = a^*/(G^*M^*/c^*)$, $v = v^*/c^*$, $\varepsilon = E^*/m^*c^{*2}$, E^*, m^* and v^* — the energy at infinity, the rest mass and the velocity of the particle; the asterisk denotes standard units.) The trajectories of the particles with hyperbolic energies are bent away from the straight line with $\theta = 45°$ (the dashed line) towards the rotation axis. The BL coordinates are plotted along the axes.

Figure 3. Particles ejected radially ($\alpha = 0°$) at $r_{(\text{in})} = 1.1$, $\theta_{(\text{in})} = 45°$, from a Kerr naked singularity with $a = 7$. Local initial velocities: $v_{(\text{in})} = 0, 0.2, 0.247, 0.34, 0.9999$. Specific energies at infinity: $\varepsilon = 0.95805, 0.977805, 0.988684, 1.018741, 214.227$.

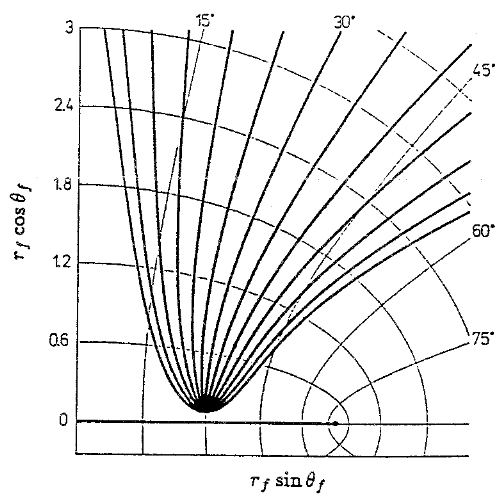


Figure 4. The set of particles with parabolic energies, ejected above the Kerr disk ($a = 2$), with initial velocity $v_{(\text{in})} = 0.25566$ from the LNRF at $r_{(\text{in})} = 0.1$, $\theta_{(\text{in})} = 30°$ in all directions spanning 180° locally: going "from right to left" $\alpha = 90°, 75°, \ldots, 15°, 0°$ (with $\beta = 0°$), $15°, 30°, \ldots, 75°, 90°$ (with $\beta = 180°$). The Kerr-Schild coordinates are plotted along the axes. Ellipsoids $r =$ const. and hyperboloids $\theta =$ const. of the Boyer-Lindquist coordinates are also indicated.

equal coordinate distance from its center and its rim) into all directions spanning locally 180°, are collimated into an angle of about 90° at infinity. Moreover, particles spanning locally 90° — those directed "to the left" towards the rotation axis — are collimated to an angle of only about 15° at infinity, as can be better seen when the trajectories are numerically followed up to $r \approx 200$. Here, indeed, the collimation is a coordinate-independent phenomenon.

3. Concluding Remarks

Our calculations indicate that even if cosmic jets originate close to a rapidly rotating black hole, the influence of the Kerr geometry will not significantly disturb collimation effects due to other causes. An unorthodox conclusion would be that the effects of the Kerr naked singularity to collimate (and accelerate) particles close to the disk spanning the singularity suggest that such type of objects could account for an initial collimation of cosmic jets. Despite the results indicating that not all collapses with rotation lead to the formation of a Kerr black hole[3], and that naked singularities may form in general relativity[4], such a suggestion is, needless to say, speculative. Using the formulation of Charlton and Clarke[5], "the development of numerical work ... should soon provide a complete answer to the question of whether there is a physically significant range of cases in which gravitational collapse leads to a ring singularity, as opposed to a black hole."

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