Hairy black holes: Stability under odd-parity perturbations and existence of slowly rotating solutions

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We show that, independently of the scalar field potential and of specific asymptotic properties of the spacetime (asymptotically flat, de Sitter or anti–de Sitter), any static, spherically symmetric or planar, black hole solution of the Einstein theory minimally coupled to a real scalar field with a general potential is mode stable under linear odd-parity perturbations. To this end, we generalize the Regge-Wheeler equation for a generic self-interacting scalar field, and show that the potential of the relevant Schrödinger operator can be mapped, by the so-called S-deformation, to a semipositively defined potential. With these results at hand we study the existence of slowly rotating configurations. The frame dragging effect is compared with the corresponding effect in the case of a Kerr black hole.

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I. INTRODUCTION

In light of discoveries during the last decades, such as cosmic acceleration associated with "dark energy" or more definitive indications of the existence of "dark matter," Einstein's classical general relativity has appeared again "on the firing line," to use Cliff Will's metaphor from the October 1972 issue of *Physics Today*. Numerous modifications of Einstein's theory assume the existence of scalar field(s). Scalar fields are part of the inflationary paradigm and appear conspicuously in string theory and supergravity. The reality of a fundamental scalar field appears to be supported by the discovery of the Brout-Englert-Higgs boson. The interest in the interaction of scalar fields and gravity is, however, older than inflation and supergravity and can be traced back to the Brans-Dicke theory and the black hole no-hair theorems. The original, and well-known, no-hair theorem of Bekenstein states that a convex potential is incompatible with the existence of a static black hole in an asymptotically flat spacetime [1]. This is also supported by the fact that massive scalar fields develop a power-law singularity at the horizon when solved in the Kerr-Newman background [2]. Bekenstein's theorem was generalized and we understand now that the relevant requirement for an asymptotically flat black hole to exist in the presence of a regular, nontrivial, real scalar field profile is the existence of a negative region of the scalar field potential [3,4].

Interest has been revived in black holes in scalar-tensor theories and their perturbations: for solutions with a real minimally coupled scalar field, see [5]; for solutions in theories with nonminimal derivative coupling, see [6]; for stability, see [7]. In particular, a Japanese group studied black hole perturbations in a general gravitational theory with Lagrangian given by an arbitrary function of the Ricci scalar and the Chern-Simons pseudoinvariant [8,9]. Most recently, the authors associated with this group tackled a technically involved problem of perturbations of black holes in the most general scalar-tensor theory in which all field equations are of second order. Their Lagrangian includes, for example, the Brans-Dicke theory, $f(R)$ gravity, the nonminimal coupling to the Gauss-Bonnet term, etc. The odd-parity perturbations have been tackled first [10]; most recently, the even-parity sector was also analyzed [11]. The authors concentrate primarily on deriving perturbation equations from the second-order actions. In both cases they present general conditions, necessary but not sufficient, for the (gradient) stability of a static, spherically symmetric solution.

We remain in the framework of classical four-dimensional general relativity and consider perturbations and stability of static, spherically symmetric, planar and hyperbolic hairy black holes in asymptotically flat or asymptotically (anti–) de Sitter spacetimes in which a scalar field is minimally coupled to gravity (for a recent review, see [12]). Then we analyze their odd-parity perturbations following the general treatment of the "axial" perturbations of spherically symmetric (not necessarily vacuum) spacetimes by Chandrasekhar [13]. We prove the mode stability with respect to general perturbations in the odd-parity sector. It is worthwhile to remark that the situation is rather different from what happens in the hairy Yang-Mills black holes which were the first examples of hairy black holes discovered. In their case there are an infinite number of

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unstable odd-parity modes—see, e.g., the original work [14] and the review [15], and references therein.

Very recently, Bhattacharya and Maeda [16] arrived at a conclusion, based on the slow-rotation approximation, that a Bocharova-Bronnikov-Melnikov-Bekenstein black hole [17,18] with a scalar field coupled conformally to gravity cannot rotate (for related discussions see also [19–21]). We provide examples where the rotation of a general class of hairy black holes is admissible. Although in the odd-parity case, the gravitational perturbations are fully decoupled from the scalar field perturbations, they can be strongly influenced by the background scalar field. The situation resembles general perturbations of charged (Reissner-Nordström) black holes where the background electric field influences gravitational perturbations (cf. [22–24]). The right-hand side of Einstein's equations is given in terms of the energy-momentum tensor of a (possibly strong) background scalar field and odd-parity perturbations of the metric. As a consequence, the metric component like $g_{\varphi t}$, which reflects the dragging of inertial frames outside a rotating black hole, depends on the character of the background scalar field. We illustrate this effect.

Rotating black holes and boson stars are known to exist when a complex scalar field is minimally coupled to Einstein theory [25]. However, much less is known when the scalar field is real. To construct the slowly rotating solutions it is necessary to specify the background. Thus, we use the static black hole family originally found in [26]. This family of solutions is the most general four-dimensional hairy black hole family with a single real scalar field and contains all other exact black holes available in the literature. The details can be found in [12].

The outline of this article is as follows. In the second section we present the proof that for minimally coupled scalar fields with arbitrary self-interaction, the spectrum of the generalized Regge-Wheeler equation is always positive. The proof is done in detail for spherical geometries and generalized to planar trasnversal geometries. The third section introduces the perturbative frame-dragging computation for the hairy black hole. We briefly describe the hairy black hole geometries before the frame draging effect is compared with the corresponding effect in the case of a Kerr black hole. Finally we present a discussion of our main results. Our conventions are such that the Riemann tensor, Ricci tesor and Ricci scalar of a D-sphere of radius 1 are $R^{\mu\nu}_{\cdot\cdot\lambda\alpha} = \delta^{\mu\nu}_{\lambda\alpha}$, $R^{\nu}_{\alpha} = \delta^{\nu}_{\alpha}(D-1)$ and $R = D(D-1)$, respectively. The metric signature is $(-,+,+,+)$ and we set $\kappa = 8\pi G$, $c = 1$.

II. ODD-PARITY PERTURBATIONS AND GENERALIZED REGGE-WHEELER EQUATION

Here we follow closely [13]. We shall consider a minimally coupled real scalar field with an arbitrary potential, $V(\phi)$. The field equations are

$$E_{\mu\nu} \equiv G_{\mu\nu} - \kappa T_{\mu\nu} = 0, \tag{1}$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right], \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor and a possibly non-vanishing cosmological constant can be included in $V(\phi)$. The perturbed metric reads

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)\left[\frac{dz^2}{(1-kz^2)} + (1-kz^2)(d\varphi + k_1 dt + k_2 dr + k_3 dz)^2\right], \tag{3}$$

where k_1, k_2 and k_3 are functions of (t, r, z). $A(r)$, $B(r)$ and $C(r)$ are the metric functions parameterizing the most general static background solution of a scalar-tensor theory. For asymptotically locally AdS solutions $k = \pm 1$or 0 [27]. Asymptotically flat or de Sitter solutions have $k = 1$. The scalar field is taken to be of the form

$$\phi = \phi_0(r) + \epsilon\Phi(t, r, z), \tag{4}$$

where ϕ_0 is the background field. The metric perturbations (k_1, k_2, k_3) are all taken to be first order in ϵ. Since any surface of constant (t, r) is of constant curvature, we consider only axisymmetric perturbations, without any loss of generality (for more details of the spherically symmetric case, see [13]). The Einstein field equations are truncated at first order in ϵ. This yields the vanishing of Φ. Indeed, using the notation introduced in Eq. (1) and the zeroth-order (background) equations, we find that

$$E^t_r = \epsilon\kappa\frac{d\phi_0}{dr}\frac{\partial_t\Phi}{A(r)} + O(\epsilon^2) = 0, \tag{5}$$

$$E^r_z = -\epsilon\kappa\frac{d\phi_0}{dr}\frac{\partial_z\Phi}{B(r)} + O(\epsilon^2) = 0, \tag{6}$$

$$E^t_t = \frac{\epsilon\kappa}{B(r)}\left(\frac{d\phi_0}{dr}\partial_r\Phi + BV_1\Phi\right) + O(\epsilon^2) = 0, \tag{7}$$

where V_1 arises from the expansion of the scalar field potential around the background configuration,

$$V_1 = \frac{dV}{d\phi}\bigg|_{\phi=\phi_0}. \qquad (8)$$

Equations (5)–(7) imply that $\partial_t\Phi = 0 = \partial_z\Phi$ and $\frac{d\phi_0}{dr}\partial_r\Phi = -BV_1\Phi$. This information simplifies the equation for E^r_r:

$$E^r_r = 2\epsilon\kappa V_1\Phi + O(\epsilon^2) = 0. \qquad (9)$$

Thus, it follows that $\Phi = 0$. Using the zeroth-order equations, it is possible to check that the remaining equations are satisfied up to linear order in ϵ if the following system of equations is satisfied:

$$\frac{\partial}{\partial r}\left[C\sqrt{\frac{A}{B}}(\partial_z k_2 - \partial_r k_3)\right] + \frac{\partial}{\partial t}\left[C\sqrt{\frac{B}{A}}(\partial_t k_3 - \partial_z k_1)\right] = 0, \qquad (10)$$

$$\frac{\partial}{\partial z}\left[\frac{A}{C}(1-kz^2)^2(\partial_z k_2 - \partial_r k_3)\right] + \frac{\partial}{\partial t}[(1-kz^2)(\partial_r k_1 - \partial_t k_2)] = 0, \qquad (11)$$

$$\frac{\partial}{\partial z}\left[C\sqrt{\frac{B}{A}}(1-kz^2)^2(\partial_z k_1 - \partial_t k_3)\right] + \frac{\partial}{\partial r}\left[(1-kz^2)\frac{C^2}{\sqrt{AB}}(\partial_r k_1 - \partial_t k_2)\right] = 0. \qquad (12)$$

Introducing the variable $Q = CA^{1/2}B^{-1/2}(1-kz^2)^2 \times (\partial_z k_2 - \partial_r k_3)$, Eqs. (10)–(11) yield

$$\frac{A^{1/2}}{CB^{1/2}(1-kz^2)^2}\frac{\partial Q}{\partial r} = -\partial_t^2 k_3 + \partial_t\partial_z k_1, \qquad (13)$$

$$\frac{\sqrt{AB}}{C^2}\frac{1}{(1-kz^2)}\frac{\partial Q}{\partial z} = -\partial_t\partial_r k_1 + \partial_t^2 k_2. \qquad (14)$$

The combination $\partial_r(13) + \partial_z(14)$ can be written in terms of Q,

$$\frac{C^2}{\sqrt{AB}}\frac{\partial}{\partial r}\left[\frac{A^{1/2}}{CB^{1/2}}\frac{\partial Q}{\partial r}\right] + (1-kz^2)^2\frac{\partial}{\partial z}\left[\frac{1}{(1-kz^2)}\frac{\partial Q}{\partial z}\right] = \frac{C}{A}\partial_t^2 Q. \qquad (15)$$

This equation can be solved by separation of variables. Writing $Q = q(r,t)D(z)$, we obtain

$$\frac{C^2}{\sqrt{AB}}\frac{\partial}{\partial r}\left[\frac{A^{1/2}}{CB^{1/2}}\frac{\partial q}{\partial r}\right] - \lambda q = \frac{C}{A}\partial_t^2 q, \qquad (16)$$

$$(1-kz^2)^2\frac{\partial}{\partial z}\left[\frac{1}{(1-kz^2)}\frac{\partial D}{\partial z}\right] = -\lambda D. \qquad (17)$$

Let us now concentrate on the case with $k=1$; we shall briefly comment on the other cases later. Setting $z = \cos\theta$ in Eq. (17) allows to identify $C^{-3/2}_{l+2}(\theta) = D(z)$ with a Gegenbauer polynomial with $\lambda = (l-1)(l+2)$ where $l \geq 1$ holds.[1] The master variable in this case is $\Psi(r^*,t) = q(r,t)C^{-1/2}$ where $\frac{\partial}{\partial r} = \frac{B^{1/2}}{A^{1/2}}\frac{\partial}{\partial r^*}$. Inserting all this information in Eq. (16) yields the master equation

$$\frac{\partial^2\Psi}{\partial r^{*2}} + \left(\frac{1}{2C}\frac{d^2C}{dr^{*2}} - \frac{3}{4C^2}\left(\frac{dC}{dr^*}\right)^2 - \lambda\frac{A}{C}\right)\Psi = \partial_t^2\Psi. \qquad (18)$$

The mode stability can be studied using the Fourier decomposition of the master variable, $\Psi = \int\Psi_\omega e^{i\omega t}dt$, which yields

$$\mathcal{H}\Psi_\omega \equiv -\frac{d^2\Psi_\omega}{dr^{*2}} + \left(\lambda\frac{A}{C} + \frac{3}{4C^2}\left(\frac{dC}{dr^*}\right)^2 - \frac{1}{2C}\frac{d^2C}{dr^{*2}}\right)\Psi_\omega = \omega^2\Psi_\omega. \qquad (19)$$

The scalar field perturbation vanishes; however, Eq. (19) depends on the background scalar field through its influence on the background metric. In vacuum, $A = 1 - 2m/r$, $C = r^2$, and Eq. (19) becomes the Regge-Wheeler equation. The operator $\mathcal{H}$ is not manifestly positive, however, its spectrum is positively defined as follows from[2]

$$\int dr^*(\Psi_\omega)^*\mathcal{H}\Psi_\omega = \int dr^*[|D\Psi_\omega|^2 + V_S|\Psi_\omega|^2] - (\Psi_\omega D\Psi_\omega)|_{\text{Boundary}}, \qquad (20)$$

where $D = \frac{d}{dr^*} + S$ and

$$V_S = \lambda\frac{A}{C} + \frac{3}{4C^2}\left(\frac{dC}{dr^*}\right)^2 - \frac{1}{2C}\frac{d^2C}{dr^{*2}} + \frac{dS}{dr^*} - S^2. \qquad (21)$$

Choosing $S = \frac{1}{2C}\frac{dC}{dr^*}$, we find

$$V_S = \lambda\frac{A}{C}. \qquad (22)$$

Therefore $l \geq 1 \Rightarrow \lambda \geq 0 \Rightarrow V_S \geq 0$ whenever $A > 0$, namely in any static region of the spacetime. From Eqs. (20)–(22) it follows that all the spherically symmetric

[1] That $l \geq 1$ follows from the definition of the Gegenbauer polynomials in terms of the Legendre polynomials: $C^{-3/2}_{l+2}(\theta) = \sin^3\theta\frac{d}{d\theta}\frac{1}{\sin\theta}\frac{dP_l(\theta)}{d\theta}$.

[2] For a more detailed discussion, see [28].

four-dimensional hairy configurations are mode stable under odd-parity perturbations. To reach this conclusion it is necessary that

$$(\Psi_\omega D\Psi_\omega)|_{\mathrm{Boundary}} = 0, \tag{23}$$

which requires that the perturbation vanishes at the horizon. This is not a very strong requirement, as follows from [29]; linear stability under the boundary condition $\Psi = 0$ at the horizon implies stability under boundary conditions with Ψ taking a finite value at the horizon.

Note that k goes into the perturbation equation (19) only through λ. When $k = 0$ the requirement that the perturbations are everywhere well defined is satisfied only if $\lambda > 0$ which implies $V_S \geq 0$. The equation for the angular part is just the equation for the harmonic oscillator with frequency $\sqrt{\lambda}$. When $k = -1$, further analysis is required.

III. SLOWLY ROTATING HAIRY BLACK HOLES

Here we want to establish the existence of slowly rotating hairy black holes. To this end we shall consider only stationary perturbations with $k_2 = k_3 = 0$ and $k_1 = \omega(r)$. In this case Eq. (12) yields

$$\omega = -c_1 \int \frac{\sqrt{AB}}{C^2} dr + c_2, \tag{24}$$

where c_1 and c_2 are two integration constants. To warm up, let us consider now the case of the Schwarzschild black hole. We have $\sqrt{AB} = 1$ and $C = r^2$, so it follows that

$$\omega = \frac{c_1}{3r^3} + c_2. \tag{25}$$

Hence, choosing $c_2 = 0$ and $c_1 = 3Ma$, we find that the perturbed metric is the Schwarzschild metric plus the perturbation $g_{t\varphi} = \frac{Ma(1-z^2)}{r}$ which, when terms proportional to a^2 are neglected, coincides exactly with the Kerr metric in Boyer-Lindquist coordinates.

Now let us consider the hairy black hole family [12,26]. The following configurations are exact background solutions of the Einstein equations[3] (1):

$$ds^2 = \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2 dx^2}{f(x)} + \frac{dz^2}{1-kz^2} + (1-kz^2)d\varphi^2\right], \tag{26}$$

$$\Omega(x) = \frac{\nu^2 x^{\nu-1}}{\eta^2 (x^\nu - 1)^2}, \tag{27}$$

$$f(x) = \frac{x^{2-\nu}(x^\nu - 1)^2 \eta^2 k}{\nu^2} + \left(\frac{1}{\nu^2 - 4} - \frac{x^2}{\nu^2}\left(1 + \frac{x^{-\nu}}{\nu - 2} - \frac{x^\nu}{\nu + 2}\right)\right)\alpha + \frac{1}{l^2}, \tag{28}$$

with energy-momentum tensor given by Eq. (2), scalar field potential and background scalar field by

$$V(\phi) = \frac{\alpha}{\kappa\nu^2}\left[\frac{\nu-1}{\nu+2}\sinh(\phi l_\nu(\nu+1)) - \frac{\nu+1}{\nu-2}\sinh(\phi l_\nu(\nu-1)) + 4\frac{\nu^2-1}{\nu^2-4}\sinh(\phi l_\nu)\right] - \frac{(\nu^2-4)}{2\kappa l^2 \nu^2}\left[\frac{\nu-1}{\nu+2}\exp(-\phi l_\nu(\nu+1)) + \frac{\nu+1}{\nu-2}\exp(\phi l_\nu(\nu-1)) + 4\frac{\nu^2-1}{\nu^2-4}\exp(-\phi l_\nu)\right], \tag{29}$$

and

$$\phi_0 = l_\nu^{-1} \ln x, \tag{30}$$

where parameter η is the unique integration constant; it arises in a nonstandard form which allows us to write the solution in terms of a dimensionless radial coordinate x and $l_\nu^{-1} = \sqrt{\frac{\nu^2-1}{2\kappa}}$.The metric and the potential are invariant under the change $\nu \to -\nu$, therefore, it is possible to take $\nu \geq 1$. Furthermore, it should be noted that the asymptotic region is at $x = 1$ which can be seen from the pole of order two in $\Omega(x)$. There are two solutions, depending on whether the scalar field is negative, $x \in (0, 1)$, or positive, $x \in (1, \infty)$, which allows us to cover all the values of the scalar field potential. The metric is regular for any value of $x \neq 0$ and $x \neq \infty$ as can be seen from the introduction of advanced and retarded coordinates, $u_\pm = t \mp \int \frac{\eta}{f(x)} dx$. The scalar field and the geometries are singular at $x = 0$ and $x = \infty$ but these singularities can be covered by event horizons. The metric reduces to the Schwarzschild-(A)dS solution in Schwarzschild-Droste coordinates when we set $\nu = 1$ and $x = 1 \pm 1/(\eta r)$. The mass of the spherically symmetric solution, computed with the Hamiltonian method [30], yields

[3] For a single real scalar field, the scalar field equation is a consequence of the Einstein equations through the conservation of the energy-momentum tensor.

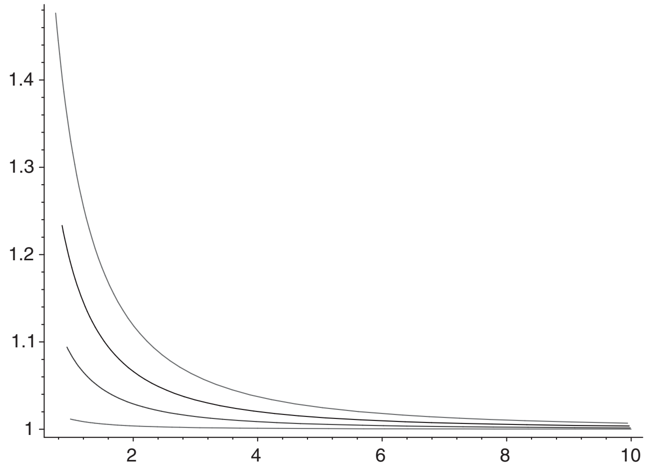

FIG. 1. The ratio $\omega/\omega_{\nu=1}$ versus the square root of the areal function, $\sqrt{\Omega(x)}$, for different values of ν. The plots are for $\nu = 1.2, \nu = 2.1, \nu = 3$ and $\nu = 4$ (from down up).

$$M = \pm\frac{\alpha + 3\eta^2}{6\eta^3 G}, \tag{31}$$

where the $\pm$ depends on whether one is considering the branch where $x > 1$ or $x < 1$.

In analogy with the Kerr solution, the slowly rotating hairy black hole is a deformation of the static one plus $g_{t\varphi} = \omega_\nu(1 - z^2)\Omega(x)$. The metric component $g_{t\varphi}$ determines the frame dragging potential (see e.g. [31] Ex. 3.4). We find that

$$\omega_\nu = \bar{c}_1 \frac{x^{2-\nu}}{\nu^2(\nu^2 - 4)}((\nu - 2)x^{2\nu} + (4 - \nu^2)x^\nu - 2 - \nu) + \bar{c}_2; \tag{32}$$

requiring that $\omega_\nu(x = 1) = 0$ fixes ω up to an overall multiplicative constant,

$$\omega_\nu = \bar{c}_1\left(\frac{x^{2-\nu}}{\nu^2(\nu^2 - 4)}((\nu - 2)x^{2\nu} + (4 - \nu^2)x^\nu - 2 - \nu) + \frac{1}{\nu^2 - 4}\right). \tag{33}$$

To measure the deviation of the dragging effects from those from the slowly rotating Kerr solution we plot the ratio $\omega/\omega_{\nu=1}$ versus the square root of the areal function $\sqrt{\Omega(x)}$. The integration constant η has units of inverse length squared. Hence the x axis is measured in units of η^{-1} (e.g. km, parsec, etc.). In Figure 1 it can be seen that there is a smooth departure from the Kerr frame dragging as both coincide when ν approaches 1 or asymptotically for large $\sqrt{\Omega(x)}$. It should be noticed that the departure from Kerr dragging can be important and that the horizon can be located at any point in the graph. Indeed, the location of the horizon is defined by the equation $f(x_+) = 0$, which has a solution for any x_+ by adjusting the value of α in expression (28).

IV. CONCLUSIONS

In this paper we addressed the issue of odd-mode stability in a rather general class of scalar-tensor theories. We proved that for a minimally coupled real scalar field, independently of its self-interaction and of the asymptotic properties of the spacetime (asymptotically flat, de Sitter or anti– de Sitter), any static black hole solution is mode stable under these perturbations. The situation is such that the scalar field only contributes through the backreaction of the background solution and the dynamics is dictated by the linearized Einstein equations. This is in contrast of what happens with the spherically symmetric mode where the only propagating mode is the scalar one [7]. The linearized Einstein-scalar field equations allowed us to study the existence of slowly rotating hairy black hole solutions. Using the hairy black hole family [12,26], we have shown that there is no obstruction for the existence of rotating hairy black holes and that they can have a behavior that strongly departs from the Kerr solution. Indeed, it would be very interesting to study the even modes outside the spherically symmetric regime, namely when the scalar and the tensor perturbations interact nontrivially. We leave this question open to further research.

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