

Critical-Curve Topologies of Triple Gravitational Lenses

Kamil Daněk and David Heyrovský

Abstract An extrasolar analog of the Sun–Jupiter–Saturn system has been discovered recently by detecting its gravitational microlensing action on the flux from a background star [1]. More generally, however, gravitational lensing by a system of three bodies has not yet been satisfactorily analyzed theoretically. Correct interpretation of microlensing light curves requires an understanding of the geometry of the underlying lens caustic and critical curves. These curves correspond to source positions and image positions, respectively, with infinite point-source-flux amplification. Following the pioneering Ertl and Schneider analysis of the parameter dependence of binary lensing [2], we extend their approach to special cases of the triple lens. While the binary lens is characterized by two parameters, three more parameters are needed to describe the triple lens. We present here an example of a three-dimensional cut through the five-dimensional parameter space, identifying the boundaries of regions with different critical-curve topology.

1 Lens Equation and Critical-Curve Topology

An n -point-mass gravitational lens deflects light from a background source and forms multiple images, which can be found by solving the lens equation

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \sum_{i=1}^n \frac{4GM_i}{c^2} \frac{D_{LS}}{D_L D_S} \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_i}{|\boldsymbol{\theta} - \boldsymbol{\theta}_i|^2}, \quad (1)$$

where $\boldsymbol{\beta}$ is the angular source position, $\boldsymbol{\theta}$ angular image position, $\boldsymbol{\theta}_i$ angular lens positions, M_i lens masses, G gravitational constant, c speed of light, distances D_L

K. Daněk (✉) · D. Heyrovský
Institute of Theoretical Physics, Faculty of Mathematics and Physics,
Charles University in Prague, Prague, Czech Republic
e-mail: Kamil.Danek@gmail.com

from observer to lens, D_S observer to source, and D_{LS} lens to source. We can express all angular positions in units of the Einstein radius

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}} \quad (2)$$

corresponding to total mass M , and instead of vectors we use complex notation introduced by [3], so that $\boldsymbol{\beta} \rightarrow \zeta$, $\boldsymbol{\theta} \rightarrow z$, $\boldsymbol{\theta}_i \rightarrow z_i$.

For $n = 3$ with relative masses $\mu_i = M_i/M$ we then get the triple lens equation in the form

$$\zeta = z - \frac{\mu_1}{\bar{z} - \bar{z}_1} - \frac{\mu_2}{\bar{z} - \bar{z}_2} - \frac{\mu_3}{\bar{z} - \bar{z}_3}. \quad (3)$$

Its critical curve parameterized by phase $\varphi \in [0, \pi)$ is given by

$$\frac{\mu_1}{(z - z_1)^2} + \frac{\mu_2}{(z - z_2)^2} + \frac{\mu_3}{(z - z_3)^2} = e^{-2i\varphi}, \quad (4)$$

and the caustic $\zeta(z(\varphi))$ is obtained by plugging critical-curve points $z(\varphi)$ into (3). The critical curve generally consists of several separate loops that may merge (or split) only when the critical curve passes through a saddle point of the lens-equation Jacobian. These points can be found by solving

$$\frac{\mu_1}{(z - z_1)^3} + \frac{\mu_2}{(z - z_2)^3} + \frac{\mu_3}{(z - z_3)^3} = 0. \quad (5)$$

Lens parameters corresponding to these topology changes can be obtained using the Sylvester matrix method to find the conditions for the existence of common roots of polynomials obtained from (4) and (5).

For the binary lens it is possible to obtain the conditions in analytical form, but for the general triple lens the equations are prohibitively intricate. However, for various simple two-parameter triple-lens models the conditions for topology change can be found in the form of polynomial equations in terms of the lens parameters. Several such examples can be found in [4]. We also found an algorithm for obtaining the conditions numerically for three-parameter triple-lens models. We demonstrate here the results obtained for the general equal-mass triple lens.

2 Equal-Mass Triple Lens

In this model we treat the “size” and “shape” of the triangular lens configuration separately. The size is expressed by the perimeter p , while the shape is determined

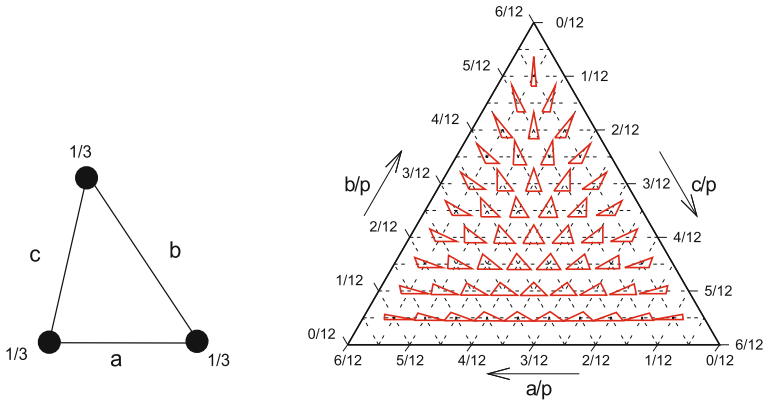


Fig. 1 *Left panel* Sketch of triple lens with equal-mass lenses at the vertices. *Right panel* Relative side-length ternary plot showing the shapes of triangles corresponding to the grid-point at their centroid

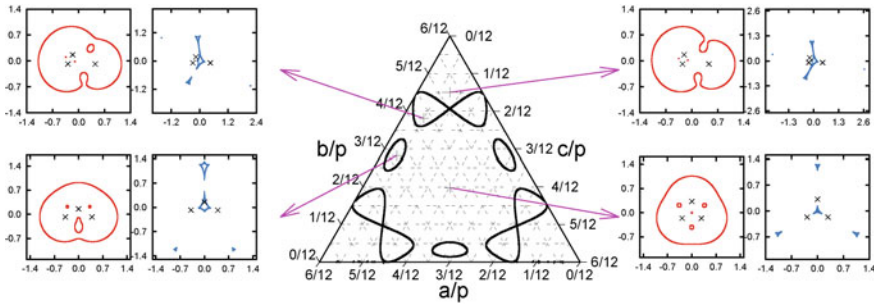


Fig. 2 *Center* Parameter space cut for perimeter $p = 1.68$ divided according to critical-curve topology. *Left and Right* Examples of critical curves (red) and caustics (blue) from each region, with lens positions marked by black crosses

by the position in the ternary plot shown in Fig. 1. The full parameter space is represented by a sequence of vertically stacked ternary plots with increasing p .

For illustration we show the $p = 1.68$ cut in Fig. 2. Here the parameter space is divided into 13 regions, but because of symmetry there are only four types of regions with three critical-curve topologies. In Fig. 3 we present a sequence of ternary plots with increasing p . There are 39 disjoint regions in the 3-D parameter space that can be grouped into 12 types using the symmetry of the problem. The equal-mass triple lens permits nine different critical-curve topologies, sketched at the left side of Fig. 3.

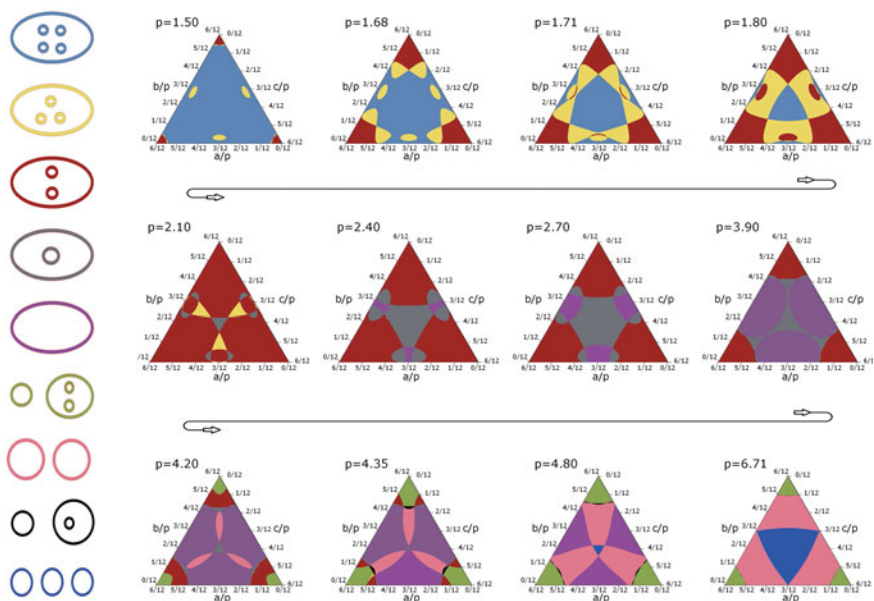


Fig. 3 Constant perimeter cuts and critical-curve topologies: Ternary plots for values of p increasing from *top left* to *bottom right*. Colors correspond to critical-curve topologies sketched in the *left* column

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