Erratum: New form of all black holes of type D with a cosmological constant [Phys. Rev. D 107, 084034 (2023)]

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The metric given by Eqs. (47)–(51) in the paper is correct, namely

$$\mathrm{d}s^2 = \frac{1}{\Omega^2}\left(-\frac{Q}{\rho^2}\left[\mathrm{d}t - \left(a\sin^2\theta + 4l\sin^2\frac{1}{2}\theta\right)\mathrm{d}\varphi\right]^2 + \frac{\rho^2}{Q}\mathrm{d}r^2 + \frac{\rho^2}{P}\mathrm{d}\theta^2 + \frac{P}{\rho^2}\sin^2\theta[a\mathrm{d}t - (r^2+(a+l)^2)\mathrm{d}\varphi]^2\right), \tag{1}$$

where

$$\Omega = 1 - \frac{\alpha a}{a^2+l^2}r(l+a\cos\theta), \qquad \rho^2 = r^2 + (l+a\cos\theta)^2, \tag{2}$$

$$P = 1 - 2\left(\frac{\alpha a}{a^2+l^2}m - \frac{\Lambda}{3}l\right)(l+a\cos\theta) + \left(\frac{\alpha^2 a^2}{(a^2+l^2)^2}(a^2-l^2+e^2+g^2) + \frac{\Lambda}{3}\right)(l+a\cos\theta)^2, \tag{3}$$

$$Q = \left[r^2 - 2mr + (a^2-l^2+e^2+g^2)\right]\left(1+\alpha a\frac{a-l}{a^2+l^2}r\right)\left(1-\alpha a\frac{a+l}{a^2+l^2}r\right) - \frac{\Lambda}{3}r^2\left[r^2 + 2\alpha a l\frac{a^2-l^2}{a^2+l^2}r + (a^2+3l^2)\right], \tag{4}$$

with the parameters $m, a, l, e, g, \alpha, \Lambda$ denoting mass, Kerr-like rotation, NUT parameter, electric charge, magnetic charge, acceleration, and cosmological constant, respectively.

However, the corresponding Maxwell electromagnetic field given by Eqs. (100)–(102) is not fully general because it does not properly involve the magnetic charge g. (The same mistake appears also in our previous work [1] on black holes with $\Lambda = 0$.) The correct version of the 1-form potential $\mathbf{A} = A_a \mathrm{d}x^a$ reads as

$$\mathbf{A} = -\frac{er + g(l+a\cos\theta)}{r^2+(l+a\cos\theta)^2}\mathrm{d}t + \frac{(er+gl)(a\sin^2\theta + 4l\sin^2\frac{1}{2}\theta) + g(r^2+(a+l)^2)\cos\theta}{r^2+(l+a\cos\theta)^2}\mathrm{d}\varphi. \tag{5}$$

This implies nonzero components of $\mathbf{F} = \mathrm{d}\mathbf{A} = \frac{1}{2}F_{ab}\mathrm{d}x^a \wedge \mathrm{d}x^b$ (that is, $F_{ab} \equiv A_{b,a} - A_{a,b}$):

$$\begin{aligned} F_{rt} &= \rho^{-4}[e(r^2-(l+a\cos\theta)^2) + 2gr(l+a\cos\theta)], \\ F_{\varphi\theta} &= \rho^{-4}[g(r^2-(l+a\cos\theta)^2) - 2er(l+a\cos\theta)](r^2+(a+l)^2)\sin\theta, \\ F_{\varphi r} &= \left(a\sin^2\theta + 4l\sin^2\frac{1}{2}\theta\right)F_{rt}, \\ F_{\theta t} &= \frac{a}{r^2+(a+l)^2}F_{\varphi\theta}. \end{aligned} \tag{6}$$

With respect to the null tetrad introduced by Eq. (85) in the paper, the Newman–Penrose scalars $\Phi_0 \equiv F_{ab} k^a m^b$, $\Phi_2 \equiv F_{ab} \bar{m}^a l^b$, $\Phi_1 \equiv \frac{1}{2} F_{ab}(k^a l^b + \bar{m}^a m^b)$ are $\Phi_0 = 0 = \Phi_2$ and

$$\Phi_1 = \frac{\frac{1}{2}(e + \mathrm{i}g)\Omega^2}{(r + \mathrm{i}(l + a\cos\theta))^2}. \tag{7}$$

It follows that $\Phi_{11} = 2\Phi_1 \bar{\Phi}_1$ gives

$$\Phi_{11} = \frac{1}{2}(e^2 + g^2)\frac{\Omega^4}{\rho^4}, \tag{8}$$

in agreement with Eq. (87) of the paper [notice also the typo below Eq. (102) of the paper where we wrongly put $\Phi_1 \bar{\Phi}_1 = 2\Phi_{11}$ instead of $\Phi_{11} = 2\Phi_1 \bar{\Phi}_1$].

The correct version of the electromagnetic field potential (5) was first presented in paper [2] by Astorino and Boldi; see Eq. (3.31) and a more detailed explanation in Appendix A therein. The explicit relation to (5) is obtained by a straightforward identification, $p = g$, $x = \cos\theta$, $\mathcal{R}^2 = \rho^2$, $\Delta_r = Q$, and $\Delta_x = P\sin^2\theta$. Also (6) agrees with (A.8)–(A.11) in [2] when the opposite convention $F_{ab} \to -F_{ab}$ is taken into account.

Finally, let us mention that in the particular case of the vanishing NUT parameter (for $l = 0$) the expression (5) reduces to

$$\mathbf{A} = -\frac{er + ga\cos\theta}{r^2 + a^2\cos^2\theta}\mathrm{d}t + \frac{ear\sin^2\theta + g(r^2 + a^2)\cos\theta}{r^2 + a^2\cos^2\theta}\mathrm{d}\varphi. \tag{9}$$

This agrees with the corresponding (unnumbered) equation in Sec. II of [3] valid for accelerating Kerr–Newman black holes in (anti-)de Sitter space-time. For nonrotating charged black holes ($a = 0 = l$) we get $\mathbf{A} = -(e/r)\mathrm{d}t + g\cos\theta\mathrm{d}\varphi$.

[1] J. Podolský and A. Vrátný, New improved form of black holes of type D, Phys. Rev. D **104**, 084078 (2021).
[2] M. Astorino and G. Boldi, Plebanski-Demianski goes NUTs (to remove the Misner string), J. High Energy Phys. 08 (2023) 085.
[3] J. Podolský and J. B. Griffiths, Accelerating Kerr–Newman black holes in (anti-)de Sitter space-time, Phys. Rev. D **73**, 044018 (2006).