

Maxwellovy a Hertzovy Elektromagnetické vlny

T. Ledvinka

Ústav teoretické fyziky
MFF UK Praha



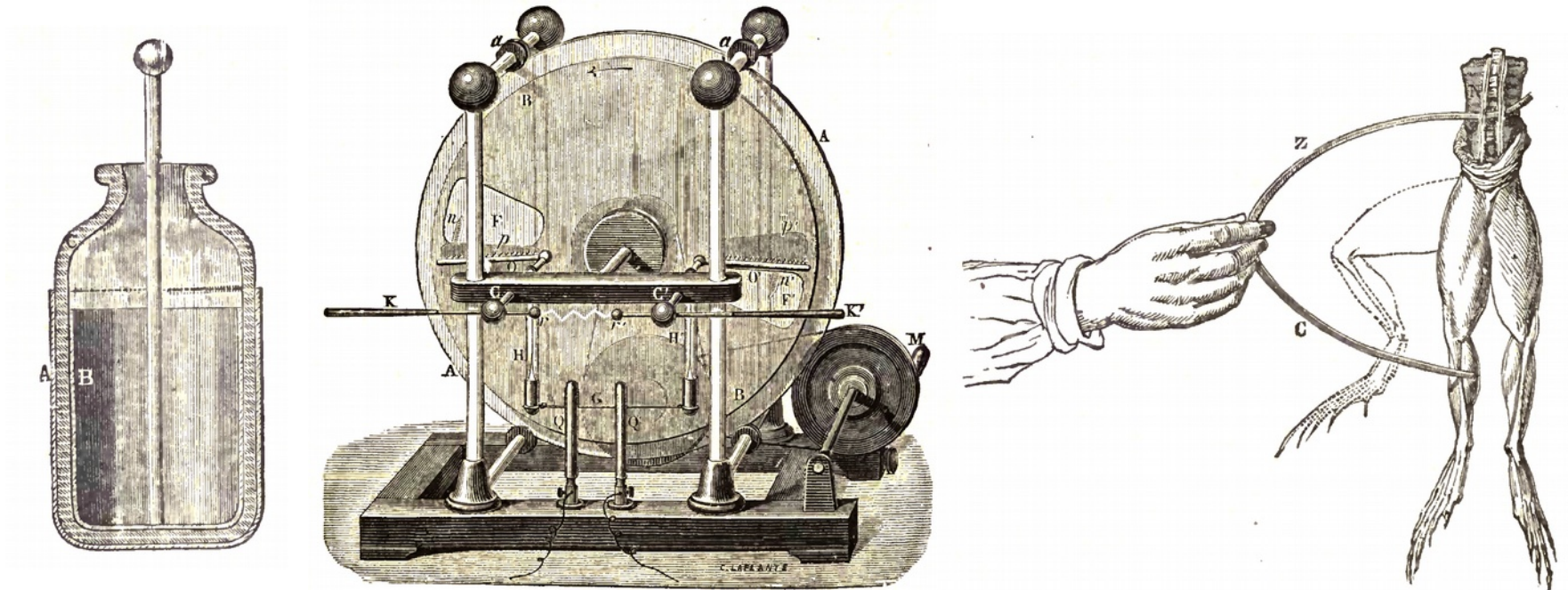
Z komiksu Csenge Kindli
(webgram.co/csenge.kindli)

Obsah

- Trocha teorie
(na co vlastně přišel J. C. Maxwell)
- První pokusy s vlnami
(za ně může H. Hertz)
- Jak vyrobit EM vlny
(na Zemi a ve vesmíru)
- Jak je zachytit
(o tom se ale zmíním jen mezi řádky)

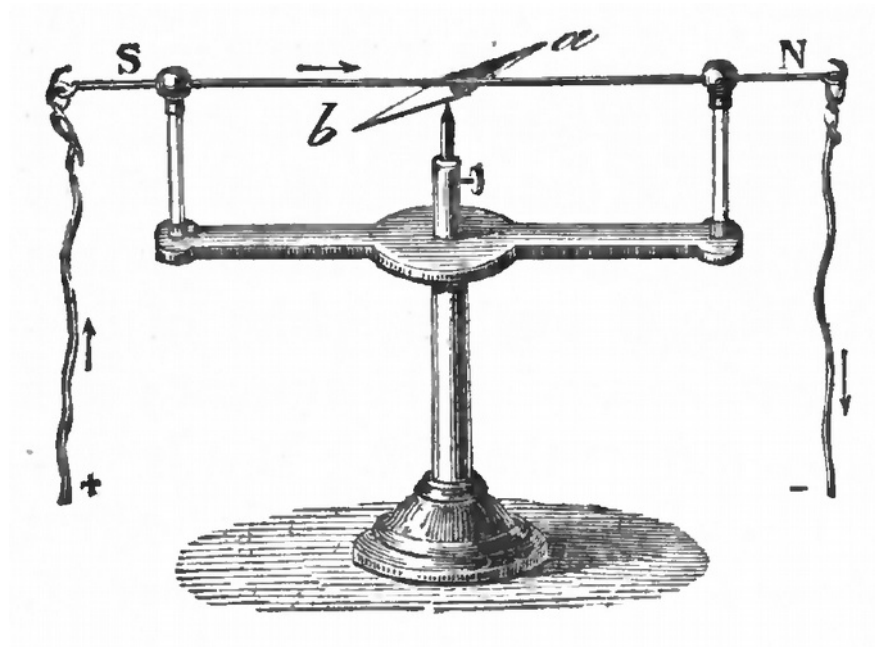
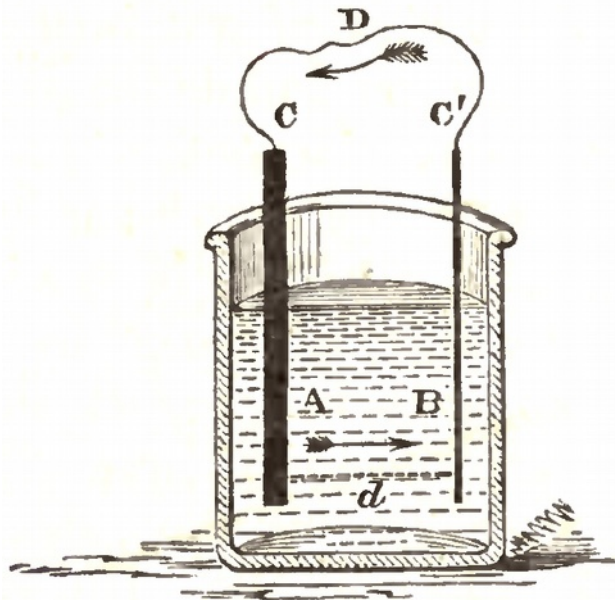
Elektrické a magnetické pole před objevem elektromagnetické indukce

- Náboje způsobují elektrické jevy (pole)
- Proudý způsobují magnetické jevy (pole)
- Nějak navzájem souvisí (Oersted)
- Mnoho zdrojů → superpozice (polí)

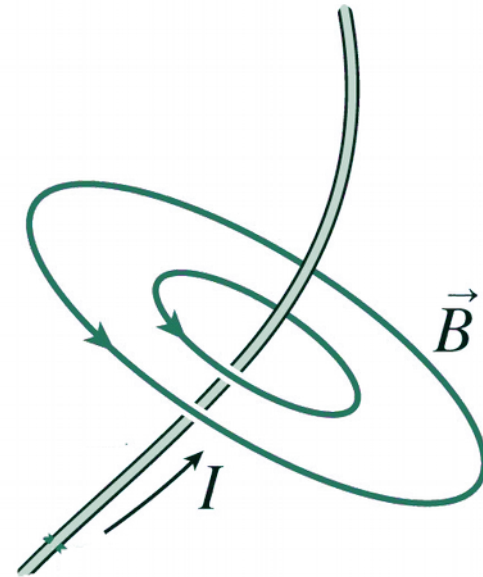
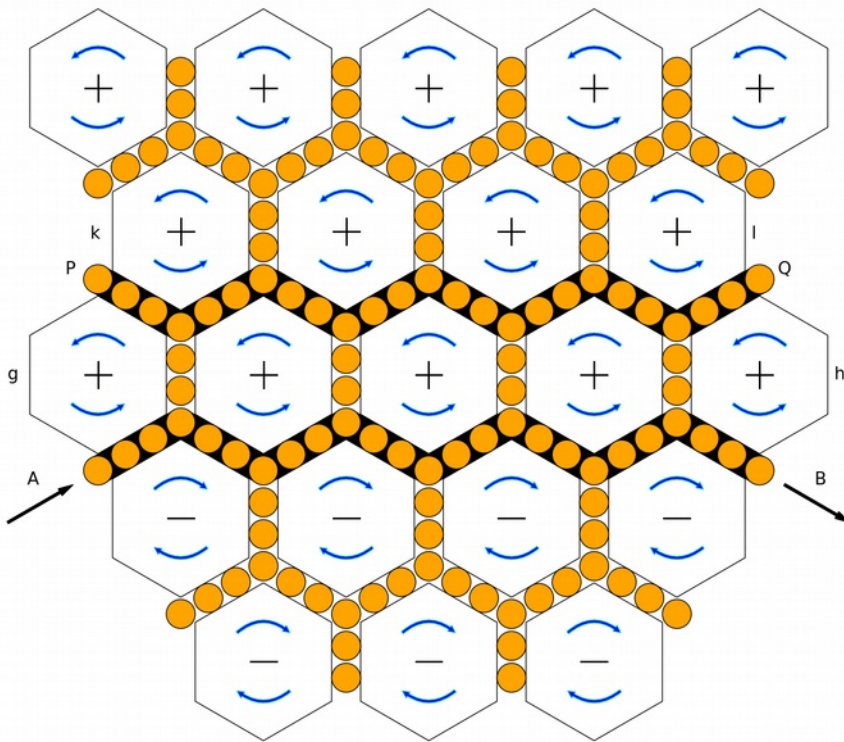


Elektrické a magnetické pole před objevem elektromagnetické indukce

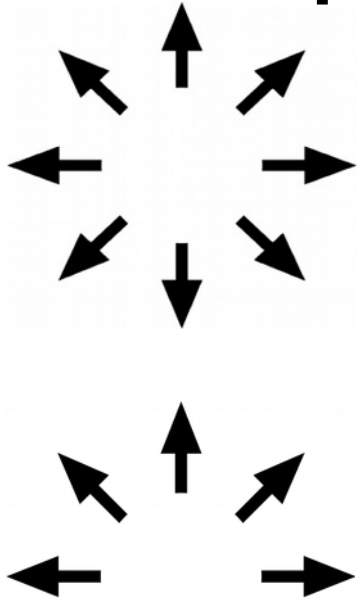
- Experimenty (... , *Cavendish*, Coulomb, Ampere, Faraday)
- Matematický popis (... , Laplace, Gauss, Ampere, ...)
- pod dojmem úspěchu Newtonovské mechaniky se hledá správná Teorie
(varování: moderní pojmy dále používané tou dobou teprve vznikaly)



Maxwellův mechanický model

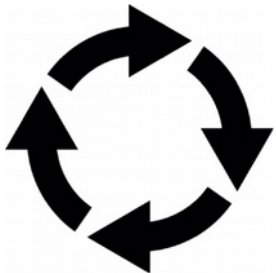


Tok a cirkulace vektorů



Tok vektorového pole

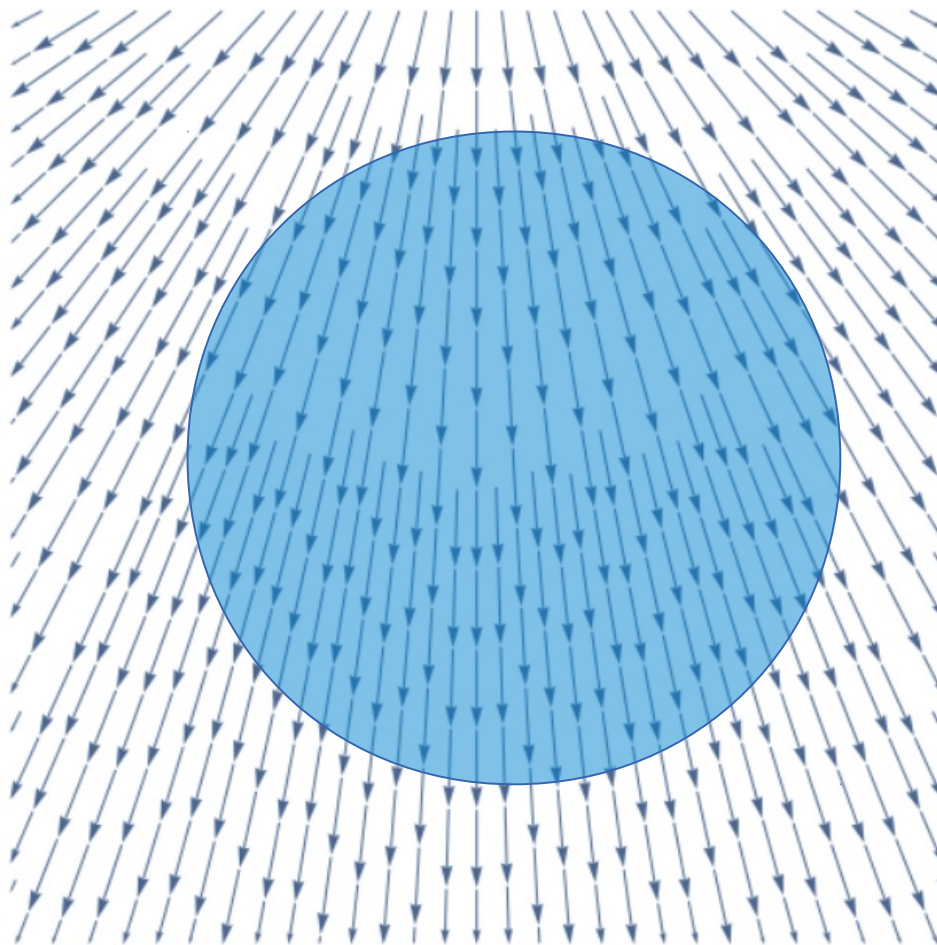
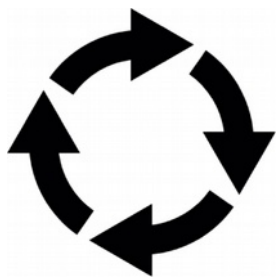
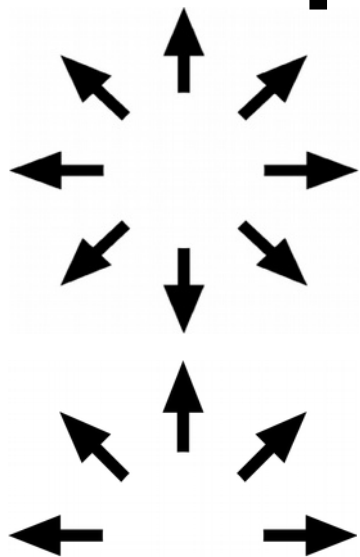
- počítá se přes **plochu**
- plocha **může** obklopovat nějaký objem
- důležitá jen složka vektorů **kolmá k ploše**
- součin plochy a průměrné kolmé složky



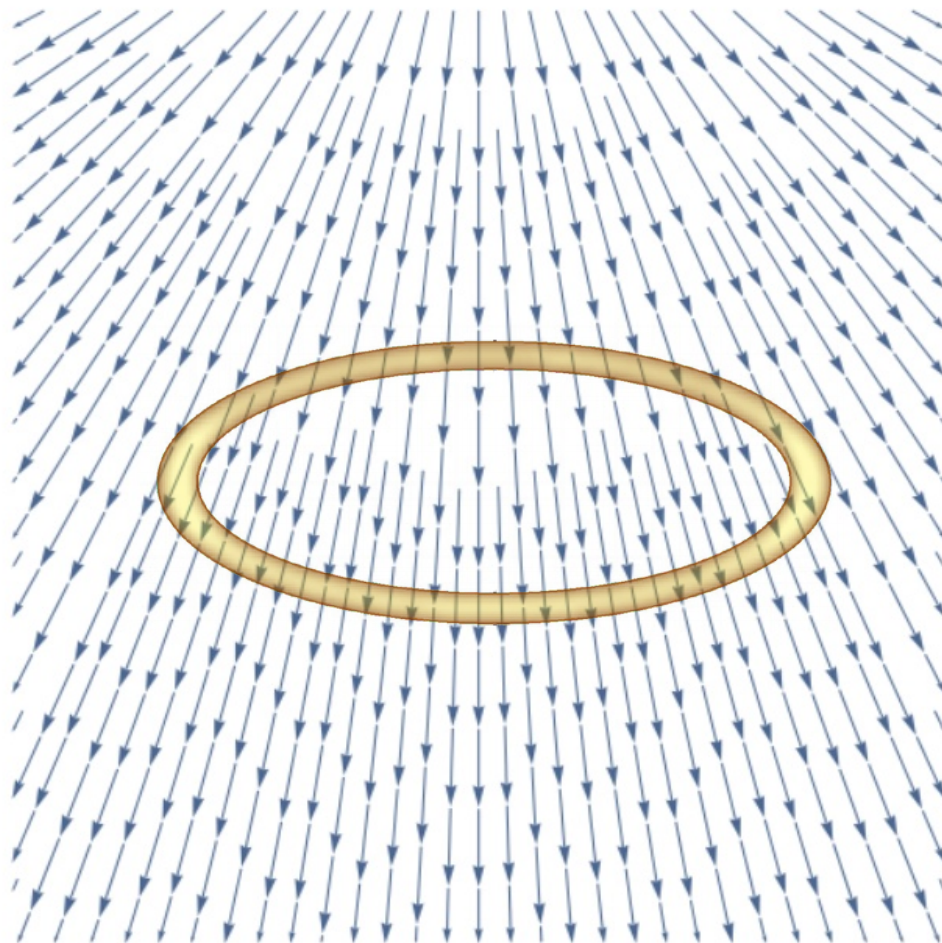
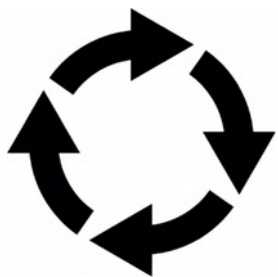
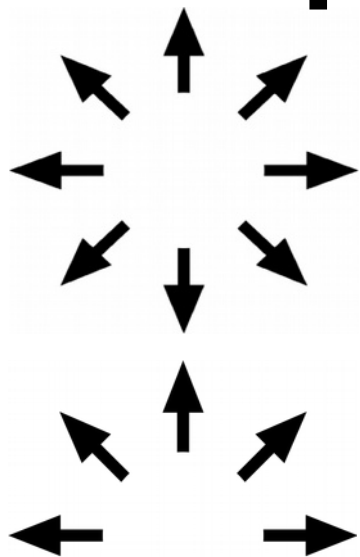
Cirkulace vektorového pole

- počítá se podél křivky
- důležitá je jen složka vektorů **podél křivky**
- součin délky a průměrné podélné složky

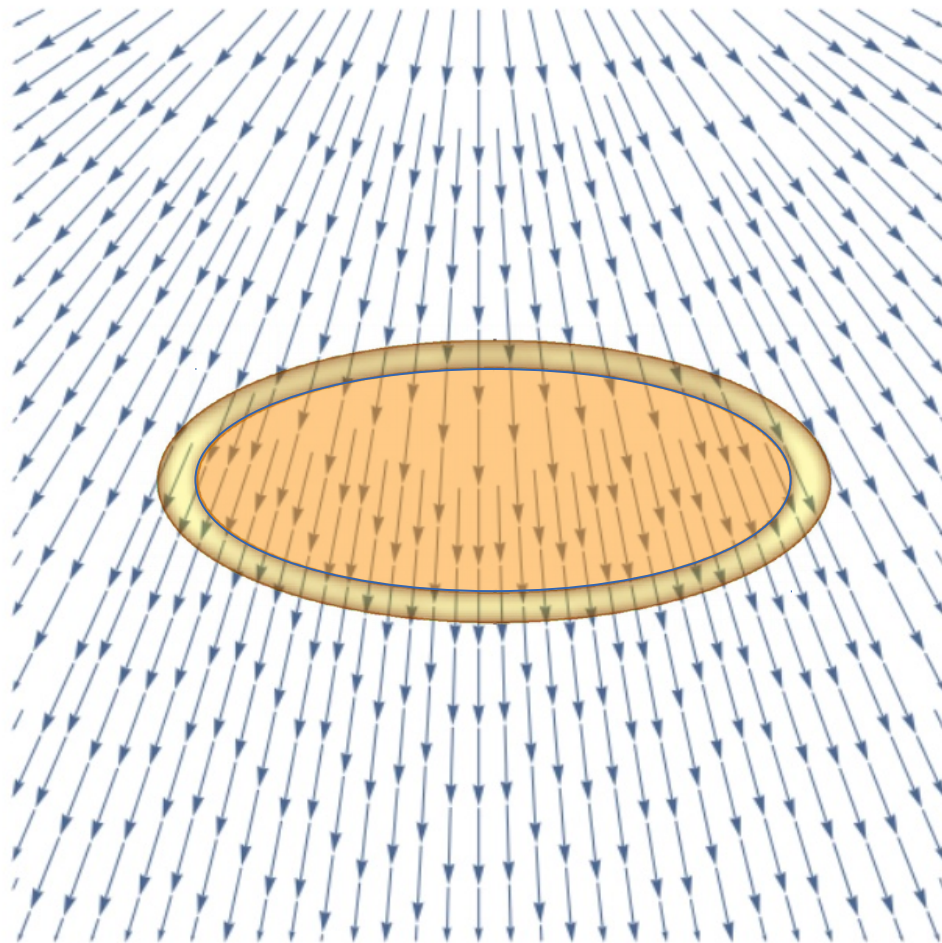
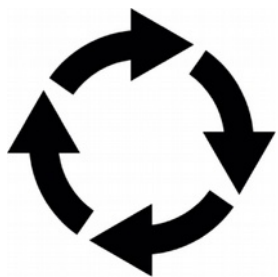
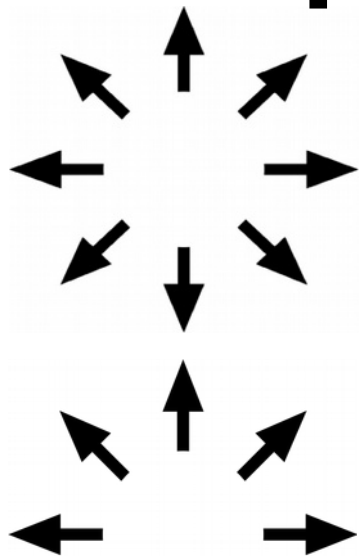
Tok a cirkulace vektorů



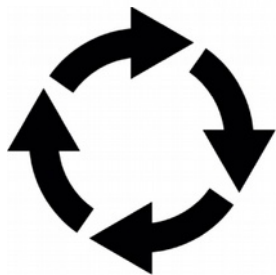
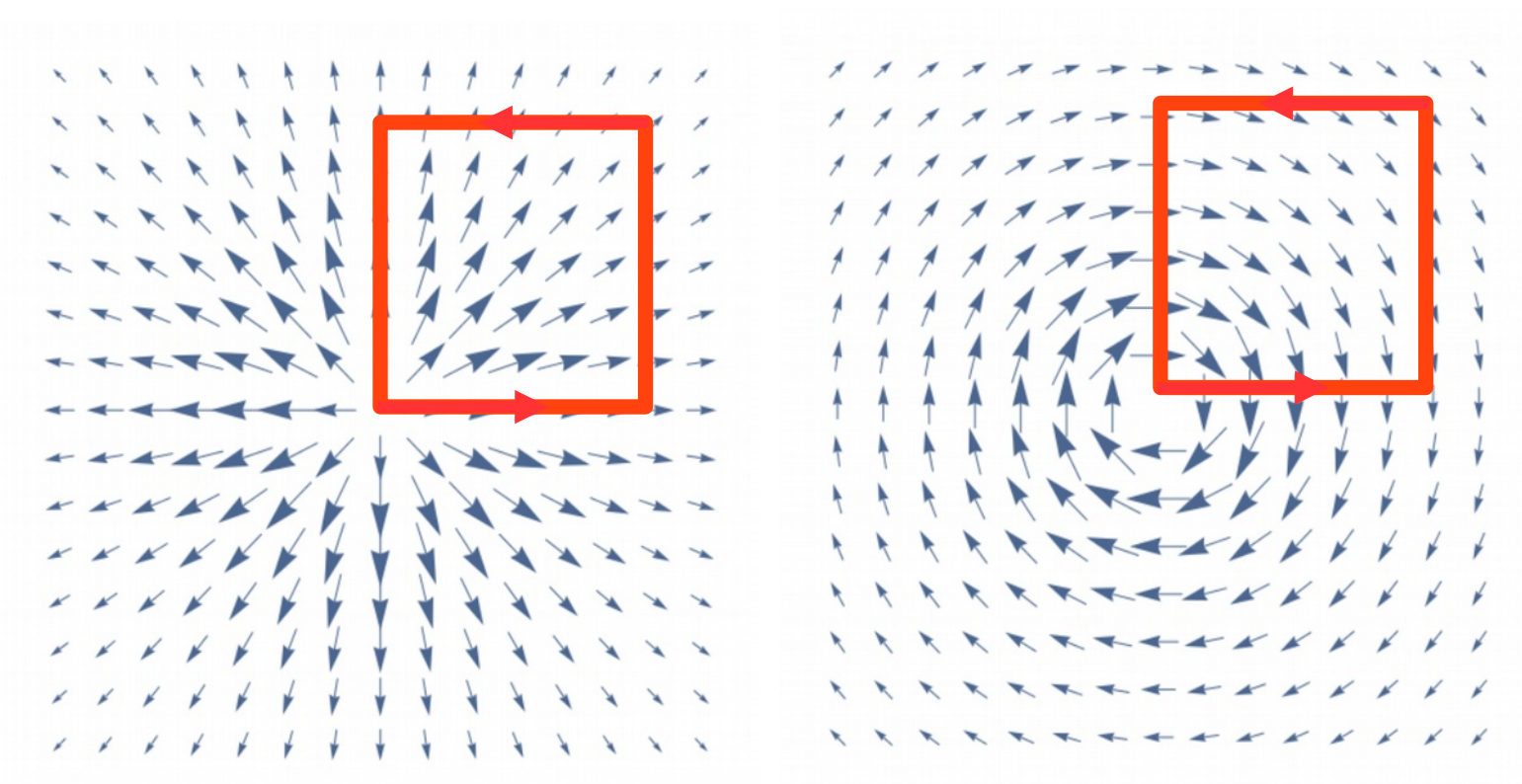
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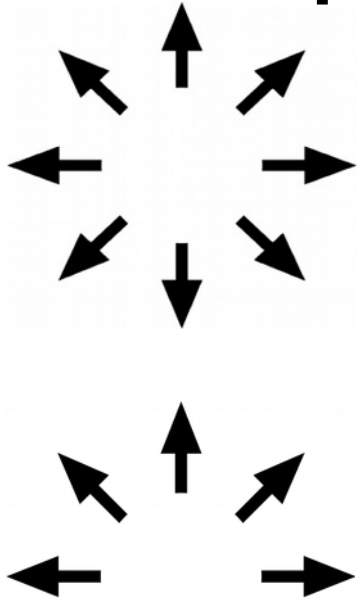


Cirkulace – aneb chodíme ve větru

vlevo – co na začátku ušetříme, na konci zase vydáme

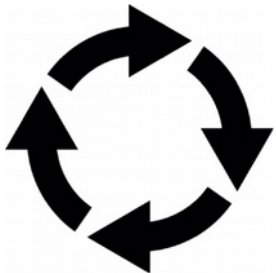
vpravo – celou dobu půjdeme proti větru

Tok a cirkulace vektorů



Tok vektorového pole

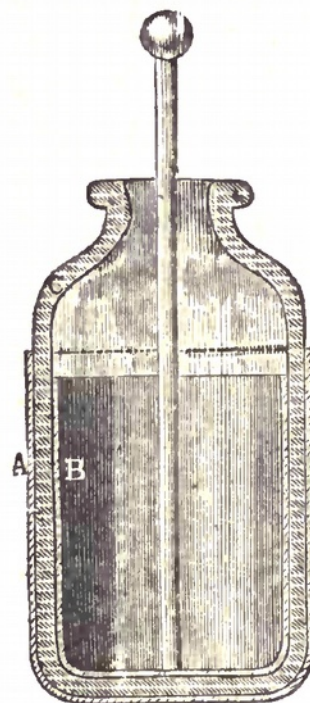
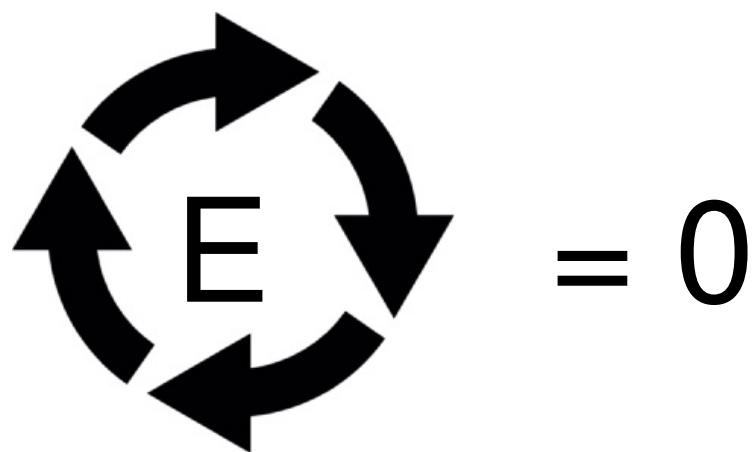
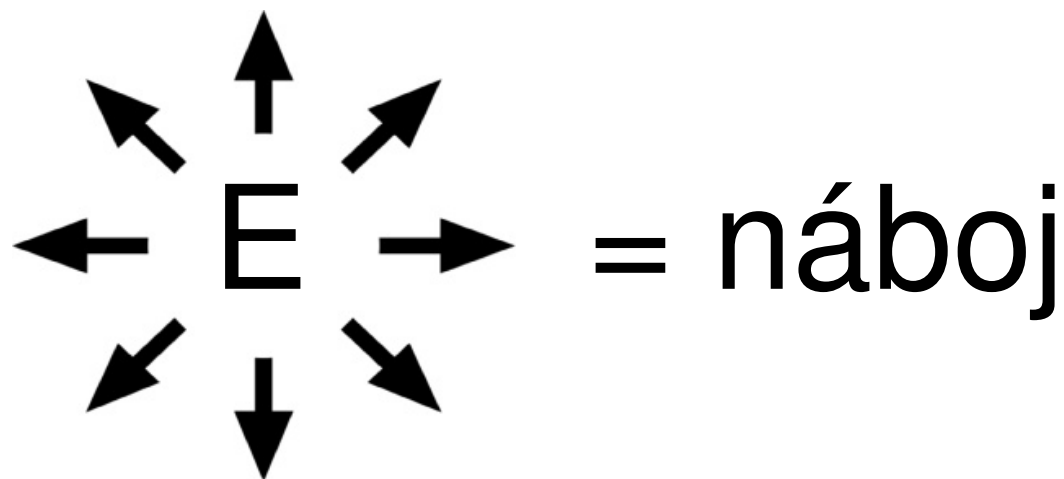
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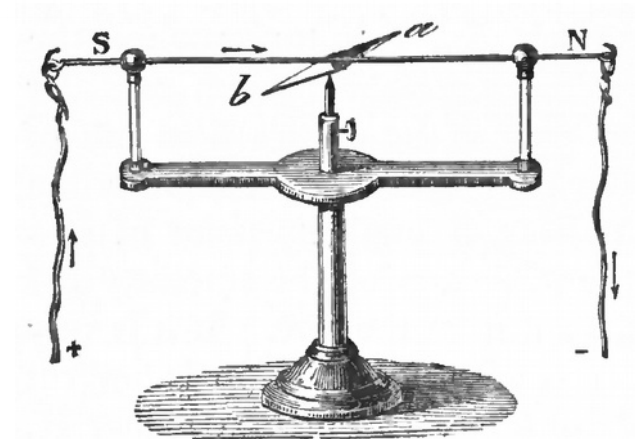
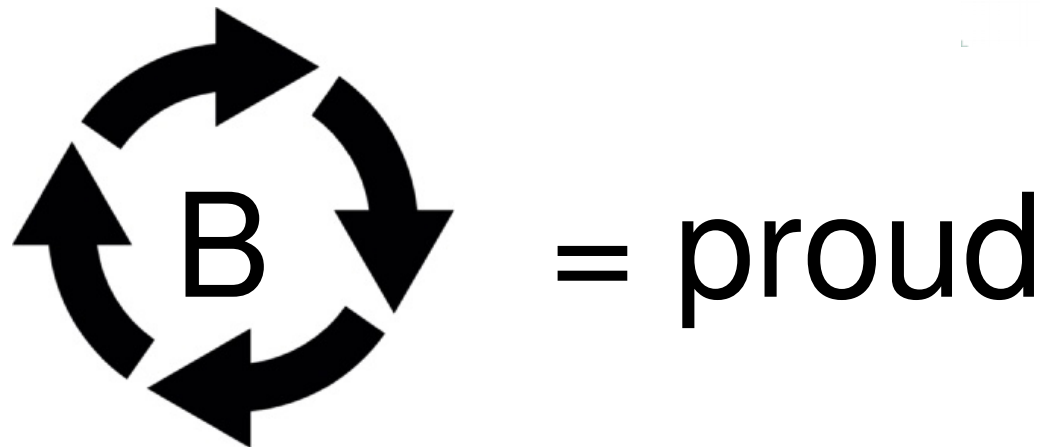
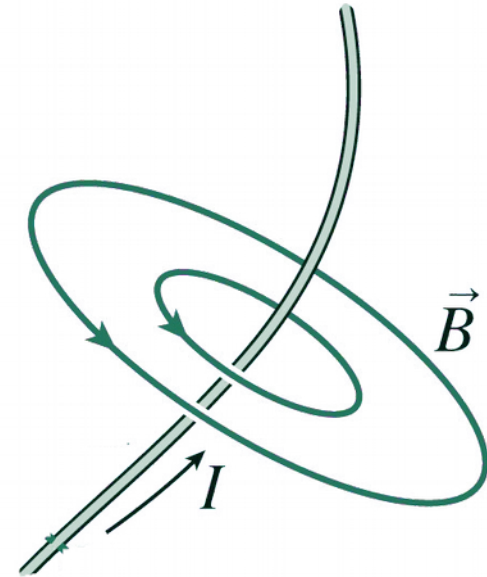
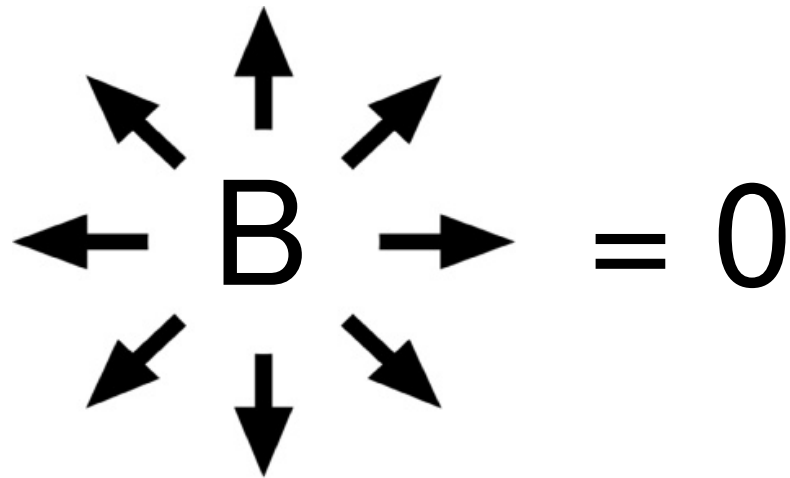
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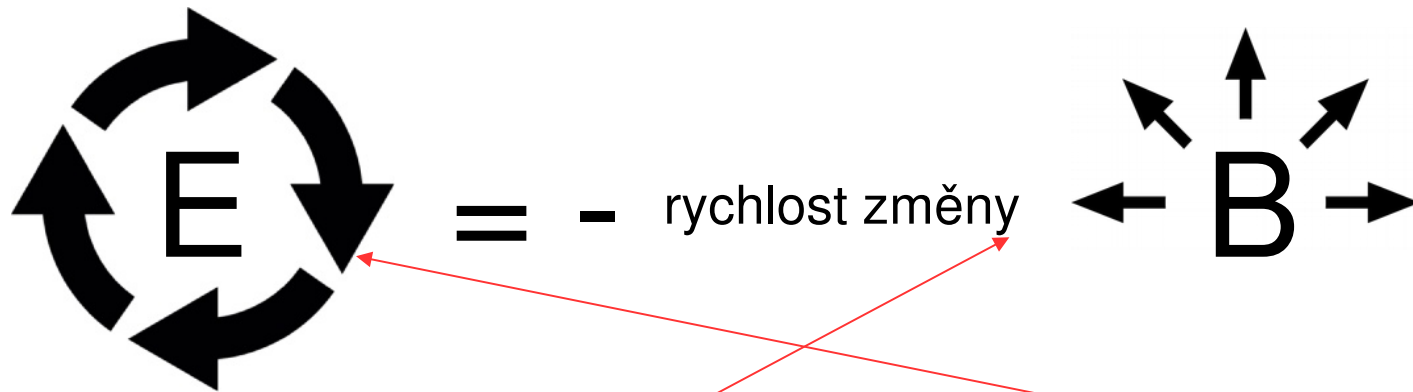
Rovnice elektrostatického pole



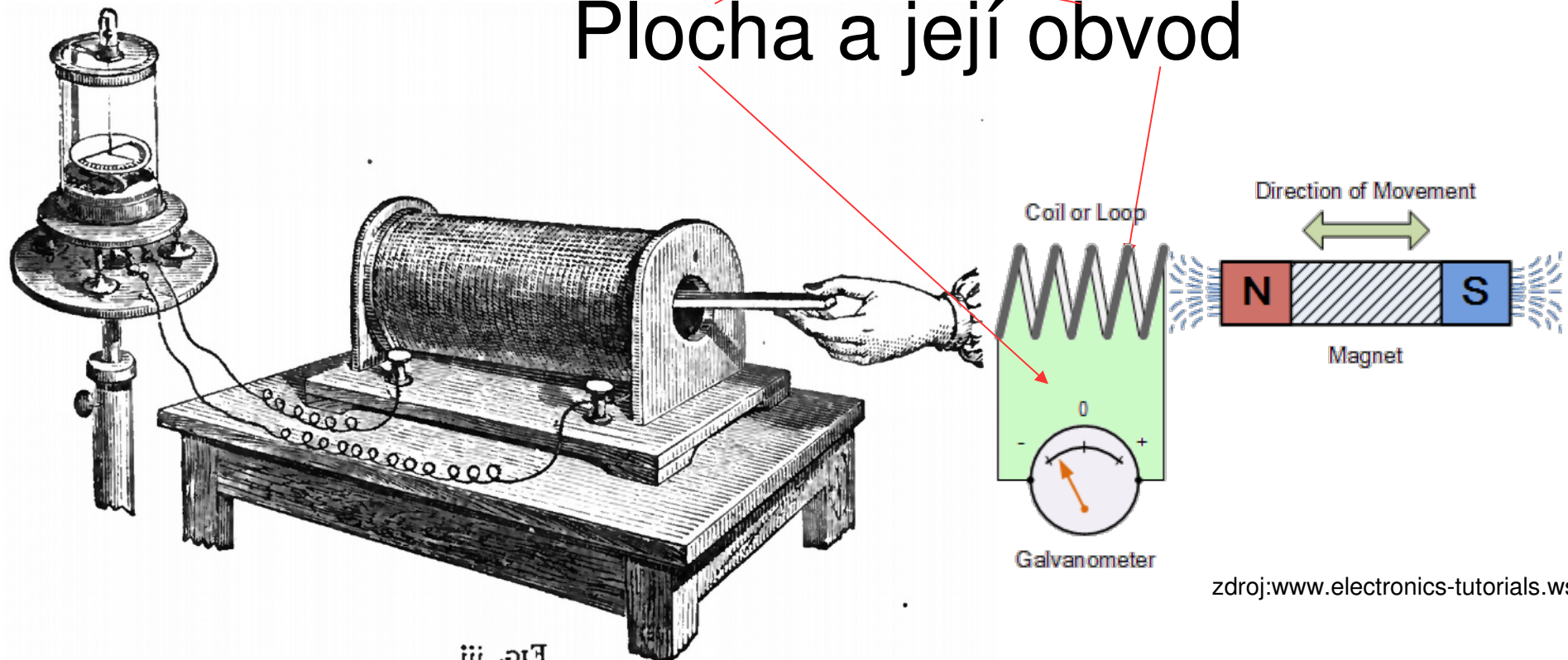
Rovnice magnetického pole



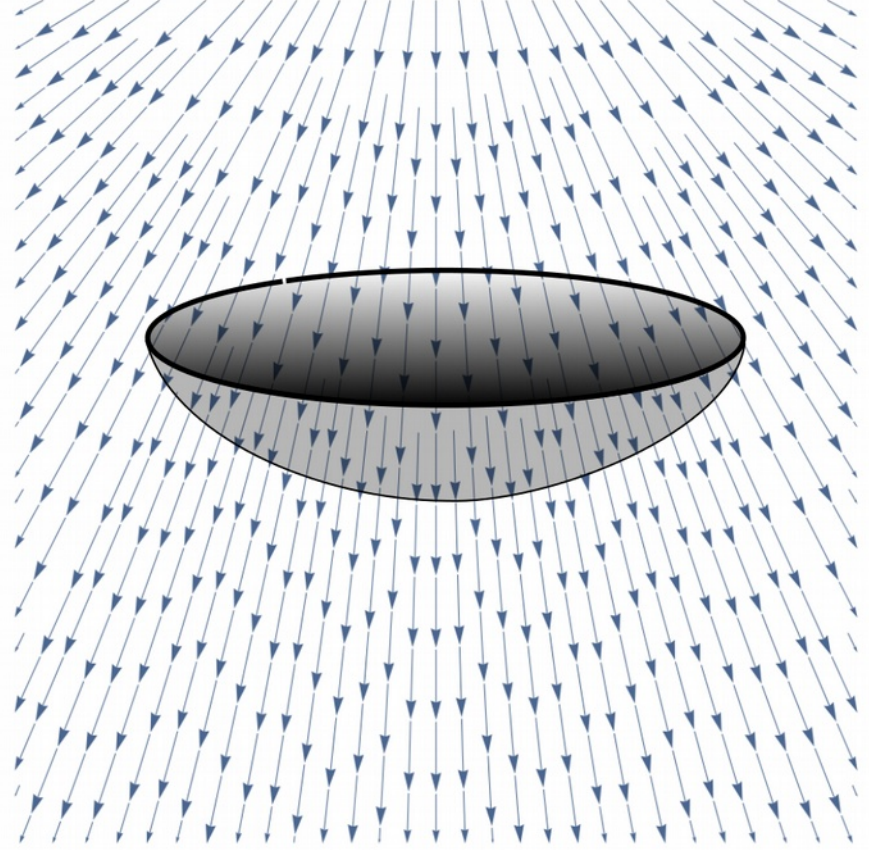
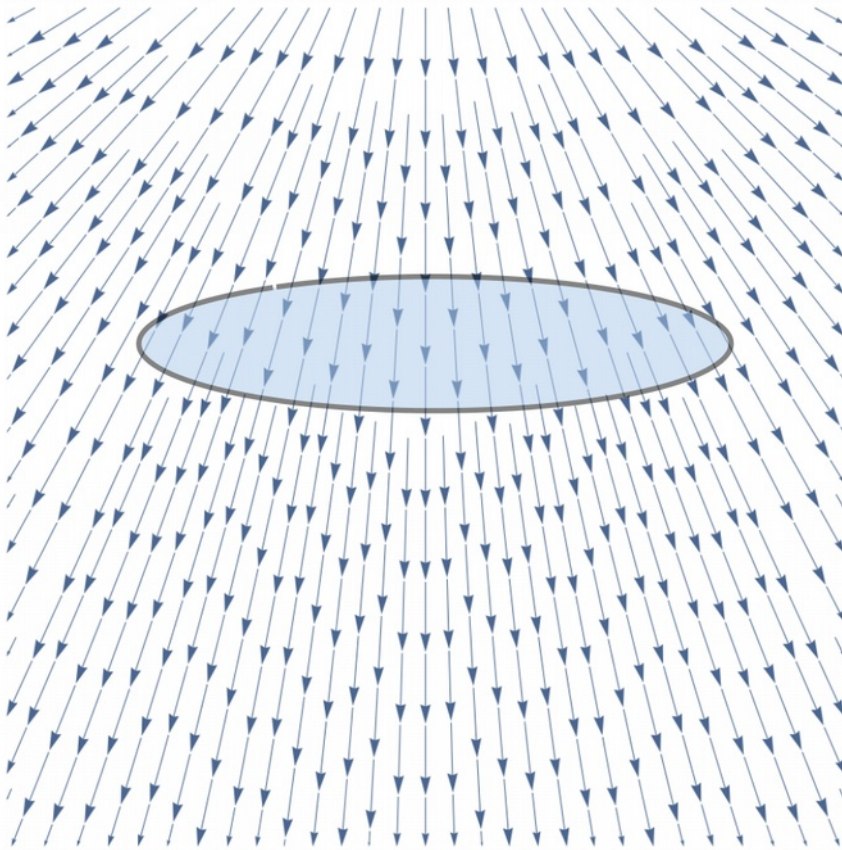
Elektromagnetická indukce



Plocha a její obvod

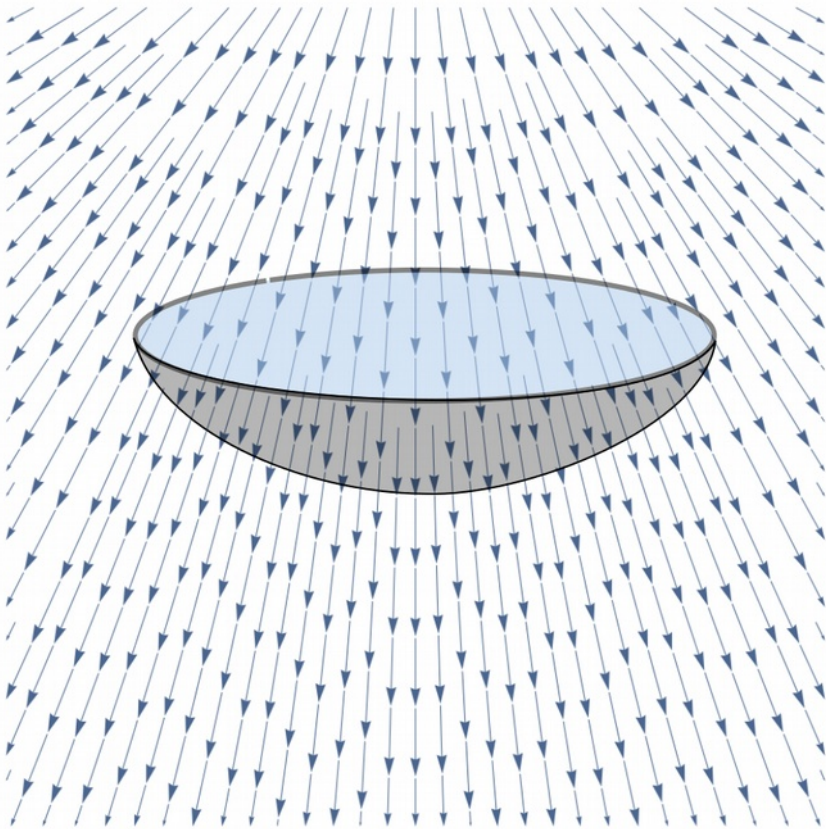


Jakou plochu ?



- Kupodivu na tom nezáleží !

Invariance magnetického toku



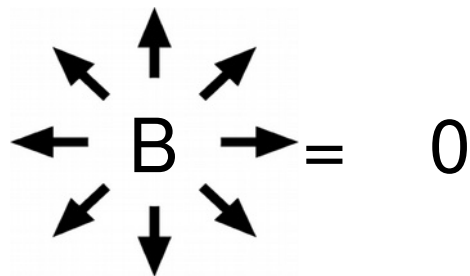
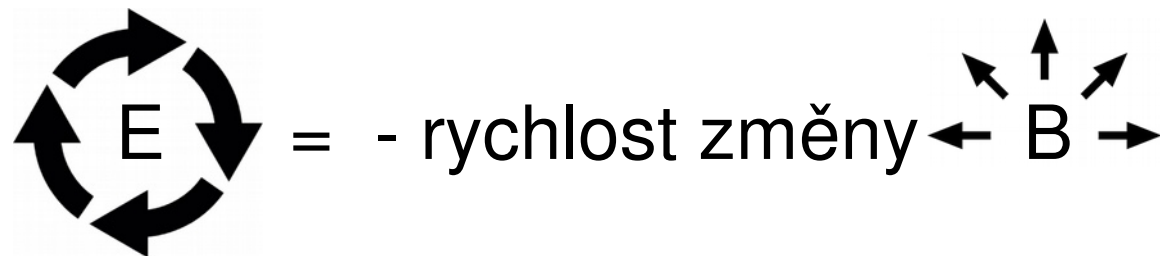
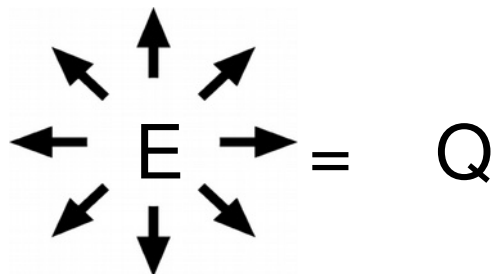
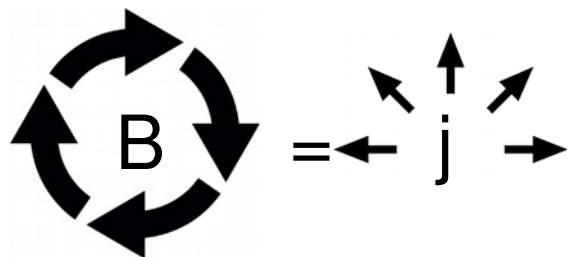
- Obě plochy sdílejí společný okraj
- Vymezují tedy objem mezi nimi
- Magnetické pole ovšem splňuje

$$\vec{B} = 0$$

- Co šedivou plochou vyteče
to modrou přiteče

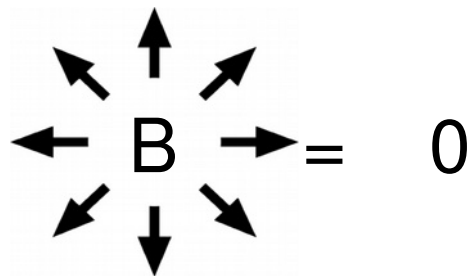
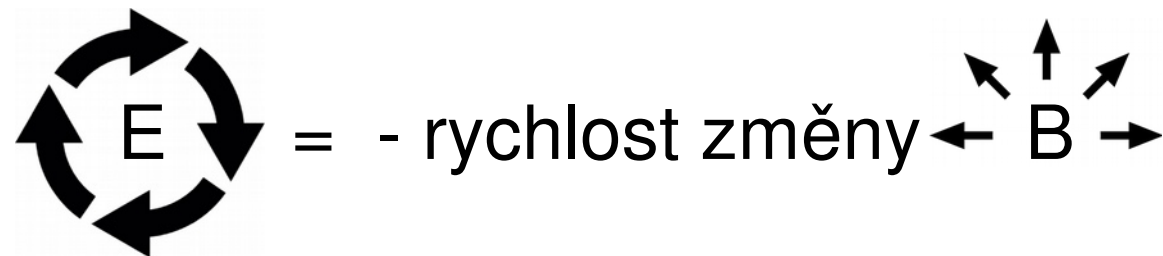
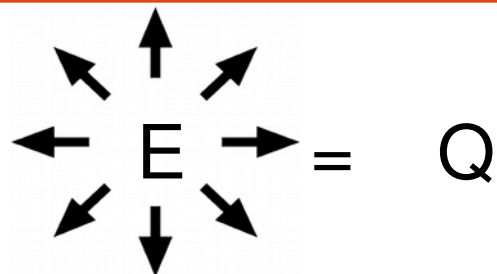
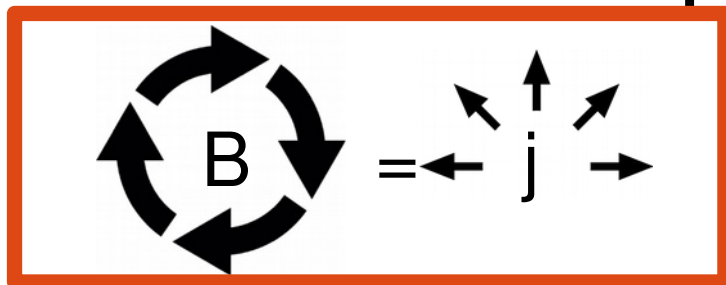
Na volbě plochy smyčkou nesejde !

Před Maxwelllem:

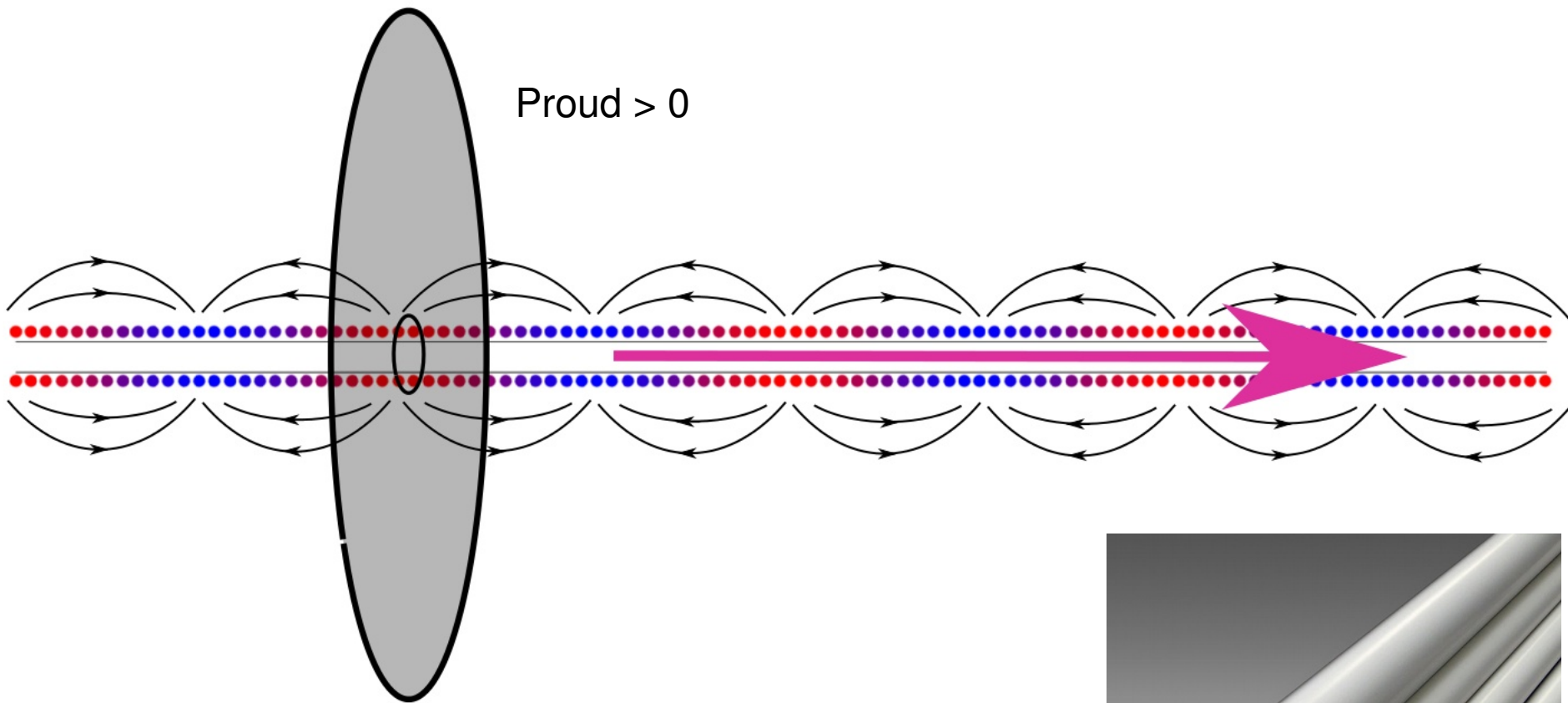


!! Bez nábojů a proudů není E a B

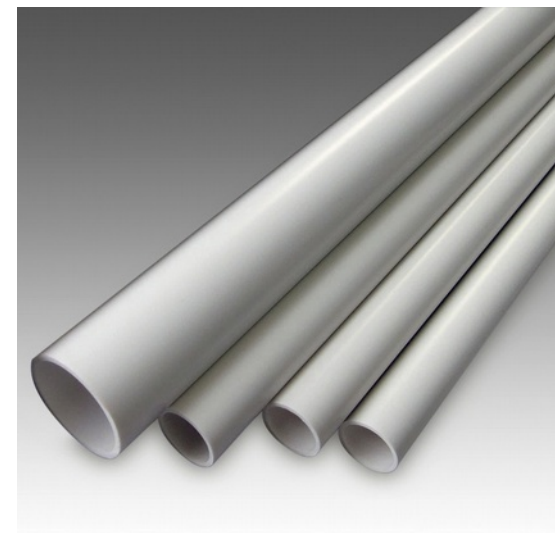
Problém?



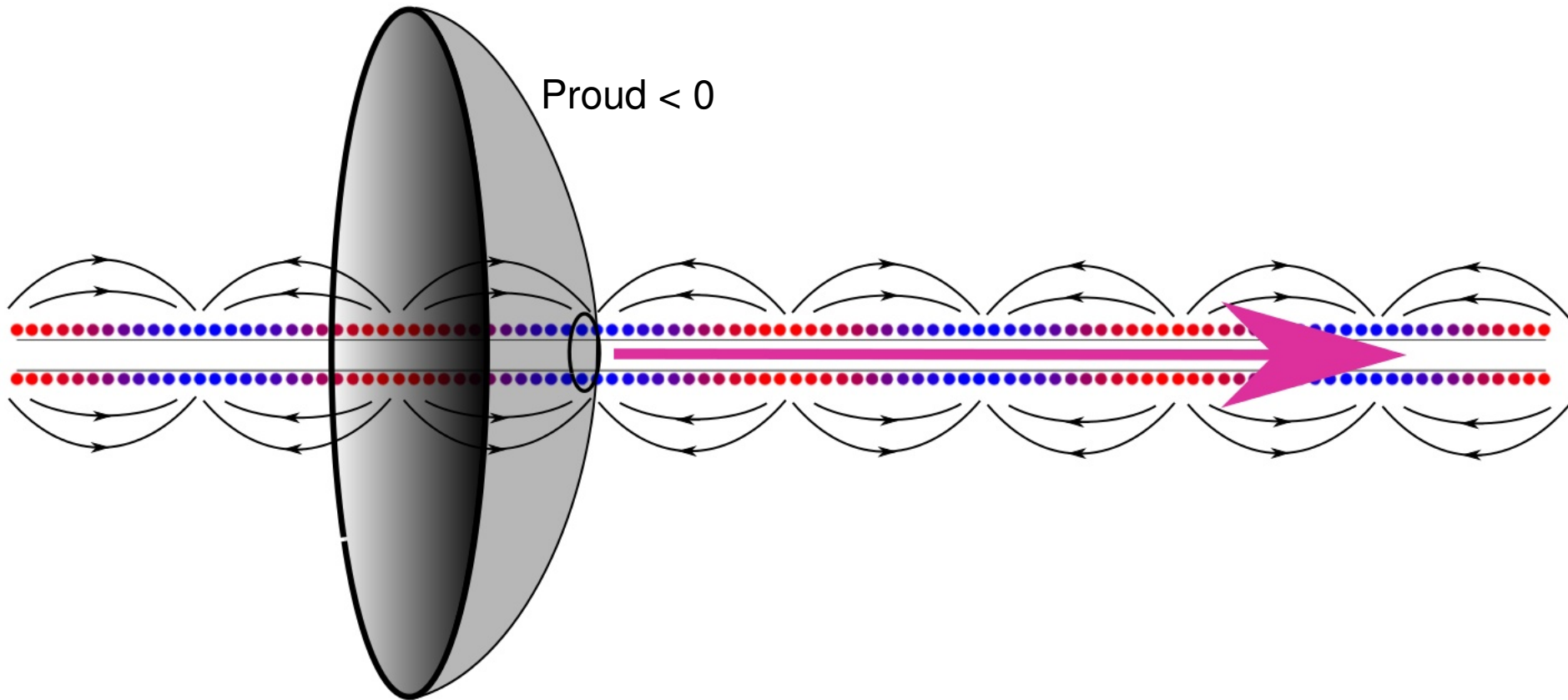
Problém !



Cirkulace magnetického pole by měla být dána proudy
(ale kterými ?)

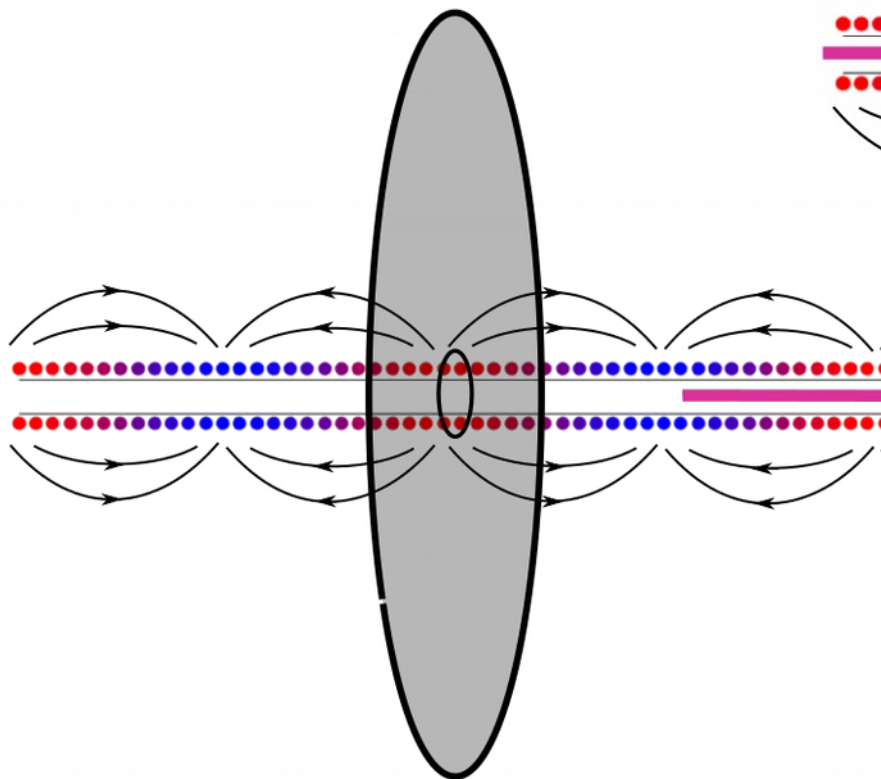


Problém !

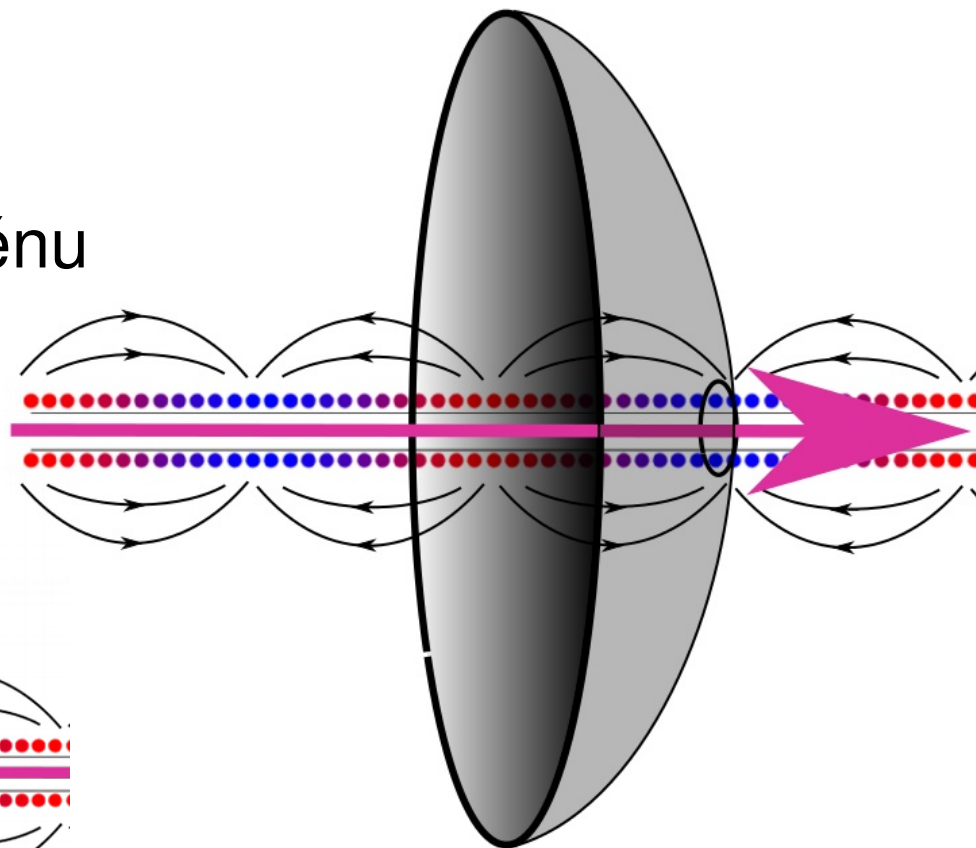


Maxwell:

přidejte k proudu ještě změnu
toku elektrického pole



proud > 0
změna toku $E < 0$



proud > 0
změna toku $E < 0$

Mimořádně:
Zachování elektrického náboje

Maxwellovy rovnice (schematicky)

$$\begin{aligned} \text{Circular } \mathbf{B} &= \mu \mathbf{j} + \varepsilon \mu \text{ rychlost změny } \text{Radial } \mathbf{E} \\ \text{Radial } \mathbf{E} &= Q \\ \text{Circular } \mathbf{E} &= - \text{ rychlost změny } \text{Radial } \mathbf{B} \\ \text{Radial } \mathbf{B} &= 0 \end{aligned}$$

Toto je onen slavný příspěvek k proudu,
na který přišel Maxwell)

Maxwellovy vlny !

$$\begin{aligned}
 & \text{Círková B} = \cancel{\text{Círková}} + \frac{1}{c^2} \text{ rychlost změny } \text{Círková E} \\
 & \text{Círková E} = \cancel{0} \\
 & \text{Círková E} = - \text{ rychlost změny } \text{Círková B} \\
 & \text{Círková B} = 0
 \end{aligned}$$

Najednou nejsou potřeba proudy,
aby mohlo existovat pole!

Maxwellovy vlny !

$$\text{rot } \mathbf{B} = -\frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

Diagram illustrating the curl of the magnetic field $\mathbf{B}$ (circular arrows) is equal to the negative of the rate of change of the electric field $\mathbf{E}$ (radial arrows) divided by c^2 . The diagram shows a circular arrow labeled B on the left, followed by an equals sign, a crossed-out diagram of a magnetic field (circular arrows) with a red 'X' over it, then a plus sign, the fraction $\frac{1}{c^2}$, the text "rychlost změny" (rate of change), and finally a diagram of an electric field (radial arrows) labeled E .

$$\text{div } \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Diagram illustrating the divergence of the electric field $\mathbf{E}$ (radial arrows) is equal to the charge density ρ divided by the permittivity of free space ϵ_0 . The diagram shows a radial arrow labeled E on the left, followed by an equals sign, a crossed-out diagram of a magnetic field (circular arrows) with a red 'X' over it, and then a diagram of a charge density ρ .

$$\text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Diagram illustrating the curl of the electric field $\mathbf{E}$ (circular arrows) is equal to the negative of the rate of change of the magnetic field $\mathbf{B}$ (radial arrows). The diagram shows a circular arrow labeled E on the left, followed by an equals sign, the text "- rychlost změny" (negative rate of change), and finally a diagram of a magnetic field (radial arrows) labeled B .

$$\text{div } \mathbf{B} = 0$$

Diagram illustrating the divergence of the magnetic field $\mathbf{B}$ (radial arrows) is equal to zero. The diagram shows a radial arrow labeled B on the left, followed by an equals sign, the number 0, and the text "(již od -13 772 124 201 př.n.l. !)" (since -13 772 124 201 B.C. !).

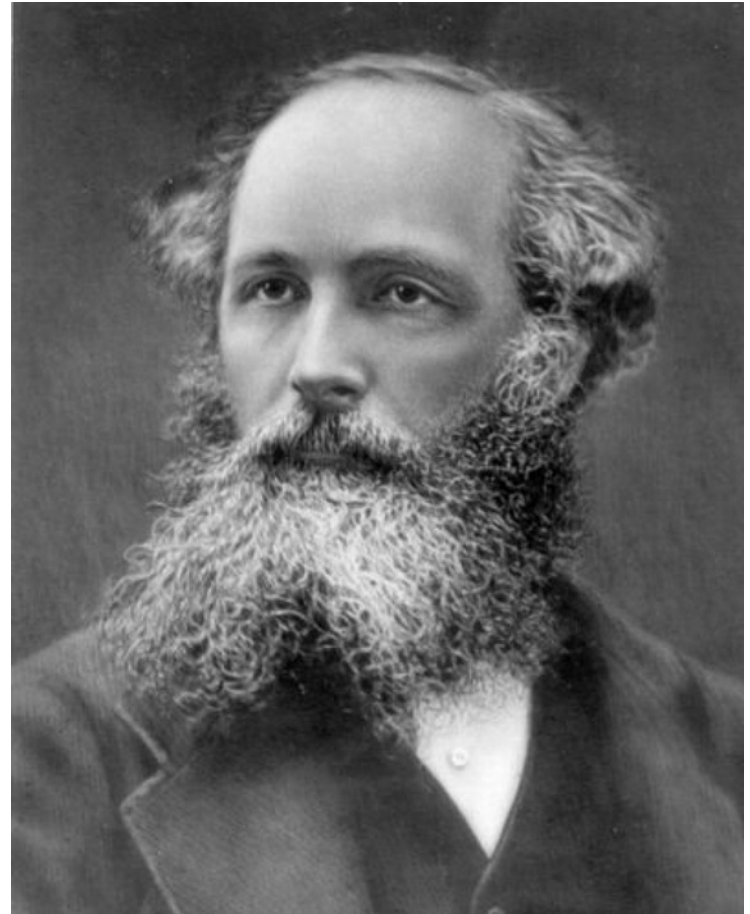
J. C. Maxwell

Another theory of electricity

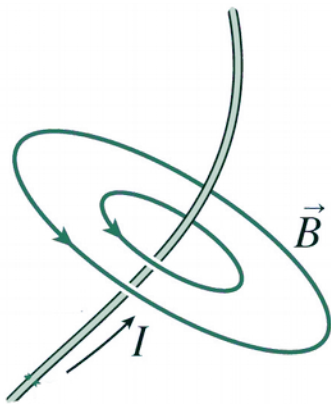
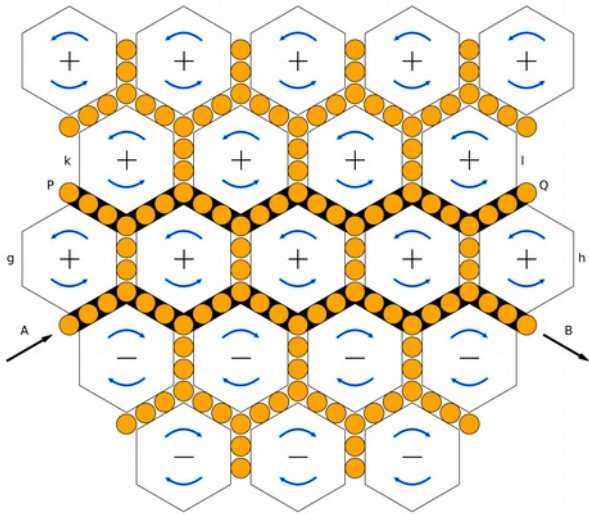
- which I prefer
- denies action at a distance
- and attributes electric action to
- tensions and pressures
- in an all-pervading medium

these stresses being the same in kind with those familiar to engineers, and the medium being identical with that in which light is supposed to be propagated

Shrnutí: elektromagnetické pole si lze představit jako „deformace“ pružného média. Změna v rozložení nábojů a proudů vede ke změně pole, která se postupně, rychlostí světla, šíří dále do prostoru.



Maxwellův mechanický model



- Rovnice EM pole představovaly
~1850 ze 3/4 vyplněnou křížovku
- známá políčka představovala "pomalé" jevy
- kromě tajejny ale nebylo jasné ani zadání / jazyk
- mechanický model
(vždyť napsal "On the Equilibrium of Elastic Solids")
- už delší dobu se hledalo řešení s "vlnami"
- pečlivým studiem by na ně přišel někdo další
- "dimenzionální analýza" by brzy provalila c^2
(opět Maxwell)

v kontinuu byly známý jen gravitační vlny podléhající

Equations of Currents.

(62) It is known from experiment that the motion of a magnetic pole in the electromagnetic field in a closed circuit **cannot generate work** unless the circuit which the pole describes passes round an electric current. Hence, except in the space occupied by the electric currents,

$$\alpha dx + \beta dy + \gamma dz = d\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

a complete differential of ϕ , the magnetic potential.

The quantity ϕ may be susceptible of an indefinite number of distinct values, according to the number of times that the exploring point passes round electric currents in its course, the difference between successive values of ϕ corresponding to a passage completely round a current of strength c being $4\pi c$.

Hence if there is no electric current,

$$dx - \frac{d\beta}{dz} = 0 ;$$

but if there is a current p' ,

$$\frac{d\gamma}{dy} - \frac{d\beta}{dz} = 4\pi p'.$$

Similarly,

$$\frac{d\alpha}{dz} - \frac{d\gamma}{dx} = 4\pi q',$$

$$\frac{d\beta}{dx} - \frac{d\alpha}{dy} = 4\pi r'.$$

. (C)

We may call these the **Equations of Currents.**

$$\left. \begin{aligned} p' &= p + \frac{df}{dt} \\ q' &= q + \frac{dg}{dt} \\ r' &= r + \frac{dh}{dt} \end{aligned} \right\} \rightarrow \mathbf{J} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\left. \begin{aligned} P &= \mu \left(\gamma \frac{dy}{dt} - \beta \frac{dz}{dt} \right) - \frac{dF}{dt} - \frac{d\Psi}{dx} \\ Q &= \mu \left(\alpha \frac{dz}{dt} - \gamma \frac{dx}{dt} \right) - \frac{dG}{dt} - \frac{d\Psi}{dy} \\ R &= \mu \left(\beta \frac{dx}{dt} - \alpha \frac{dy}{dt} \right) - \frac{dH}{dt} - \frac{d\Psi}{dz} \end{aligned} \right\} \rightarrow \mathbf{E} = \mu (\mathbf{v} \times \mathbf{H}) - \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi$$

$$\left. \begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz} \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx} \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy} \end{aligned} \right\} \rightarrow \mu \mathbf{H} = \nabla \times \mathbf{A}$$

$$\left. \begin{aligned} P &= k f \\ Q &= k g \\ R &= k h \end{aligned} \right\} \rightarrow \epsilon \mathbf{E} = \mathbf{D}$$

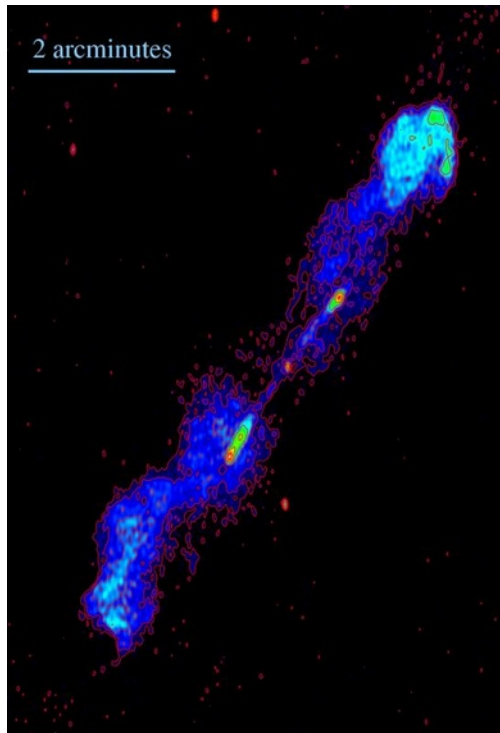
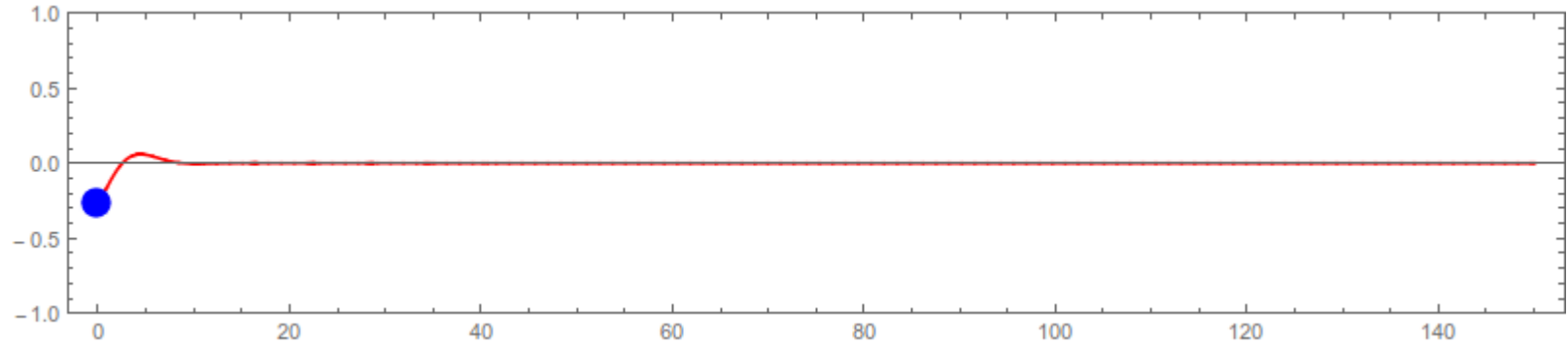
$$\left. \begin{aligned} P &= -\zeta p \\ Q &= -\zeta q \\ R &= -\zeta r \end{aligned} \right\} \rightarrow \sigma \mathbf{E} = \mathbf{j}$$

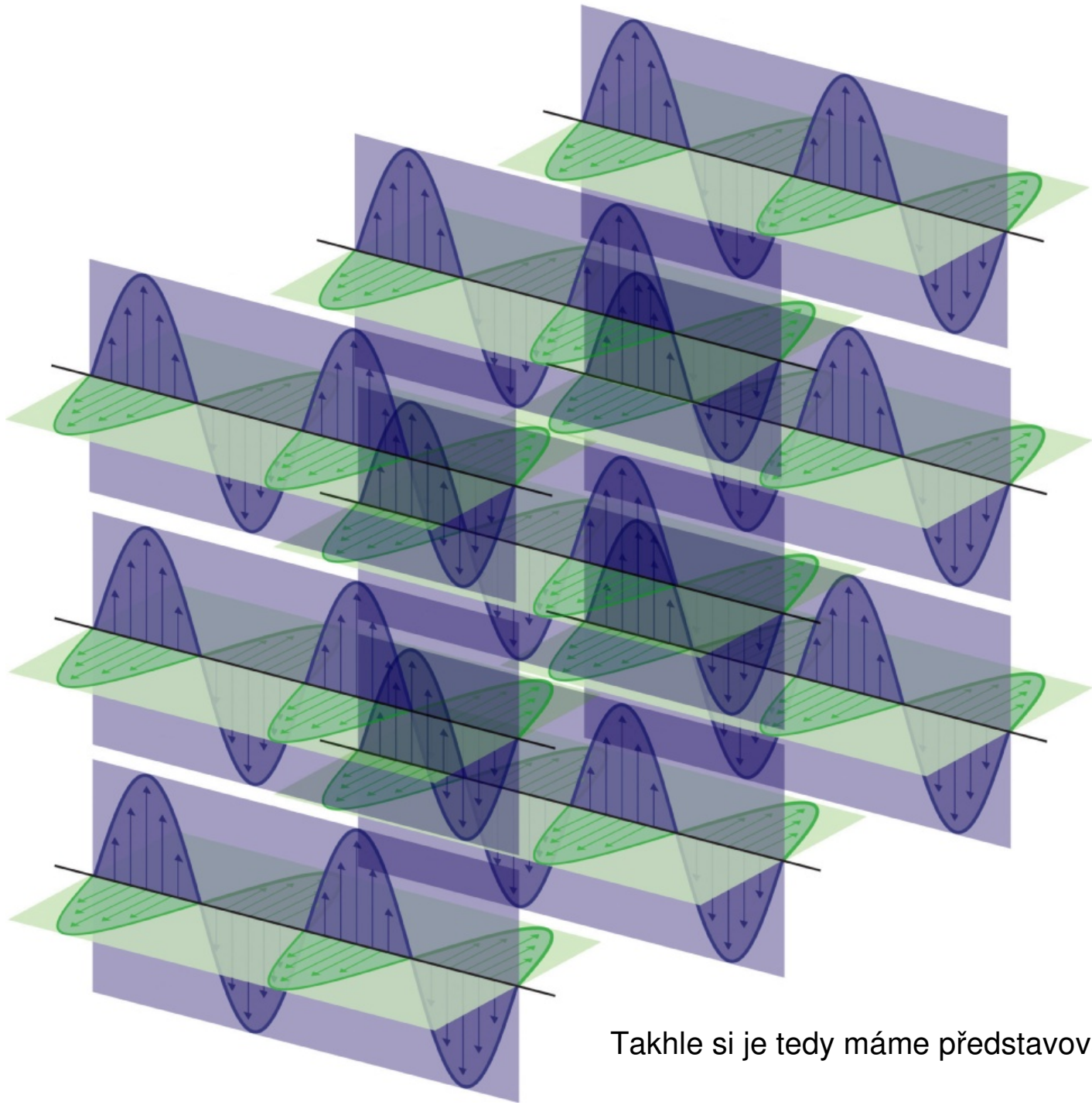
$$\left. \begin{aligned} \frac{d\gamma}{dy} - \frac{d\beta}{dz} &= 4\pi p' \\ \frac{d\alpha}{dz} - \frac{d\gamma}{dx} &= 4\pi q' \\ \frac{d\beta}{dx} - \frac{d\alpha}{dy} &= 4\pi r' \end{aligned} \right\} \rightarrow \nabla \times \mathbf{H} = \mathbf{J}$$

$$e + \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = 0 \rightarrow -\rho = \nabla \cdot \mathbf{D}$$

$$\frac{de}{dt} + \frac{dp}{dx} + \frac{dq}{dy} + \frac{dr}{dz} = 0 \rightarrow -\frac{\partial \rho}{\partial t} = \nabla \cdot \mathbf{j}$$

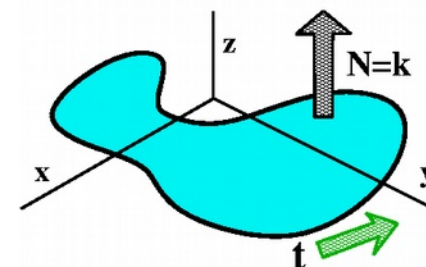
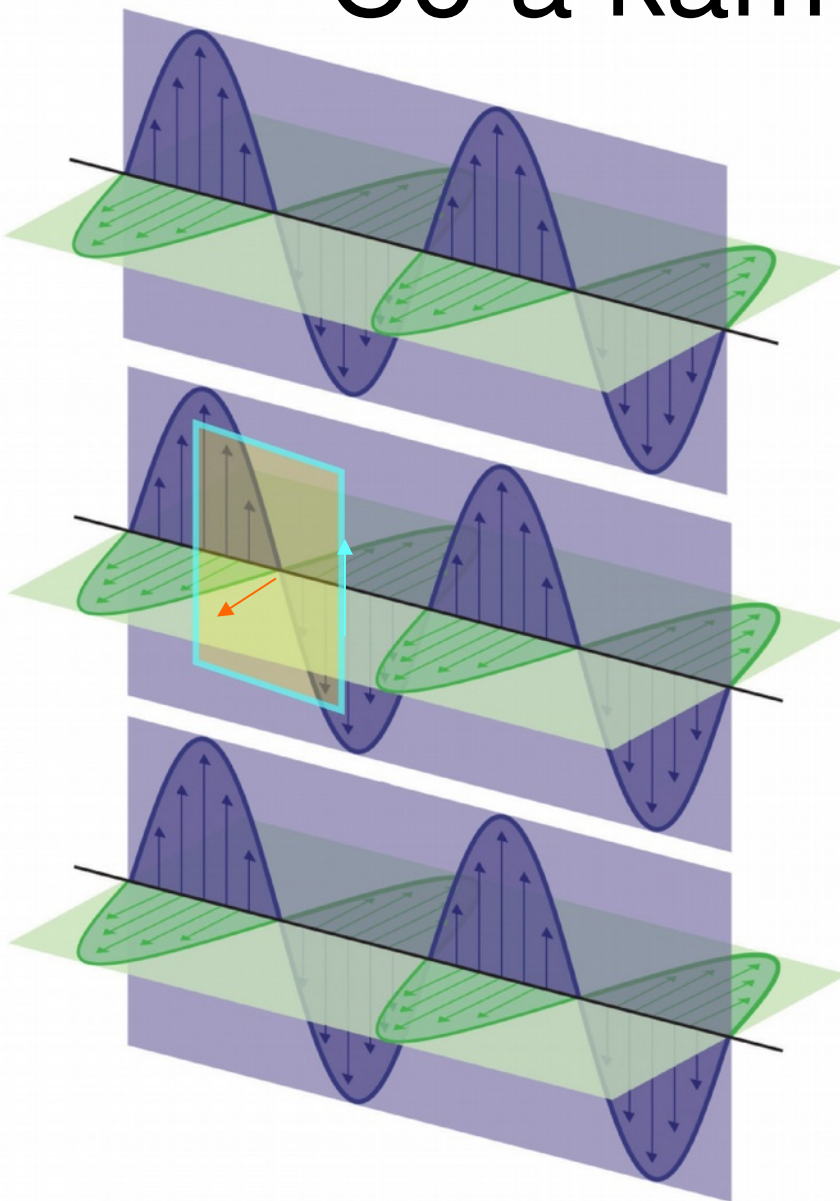
VlNy – no a co ?





Takhle si je tedy máme představovat?

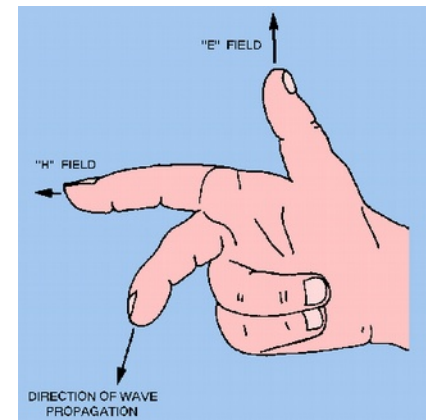
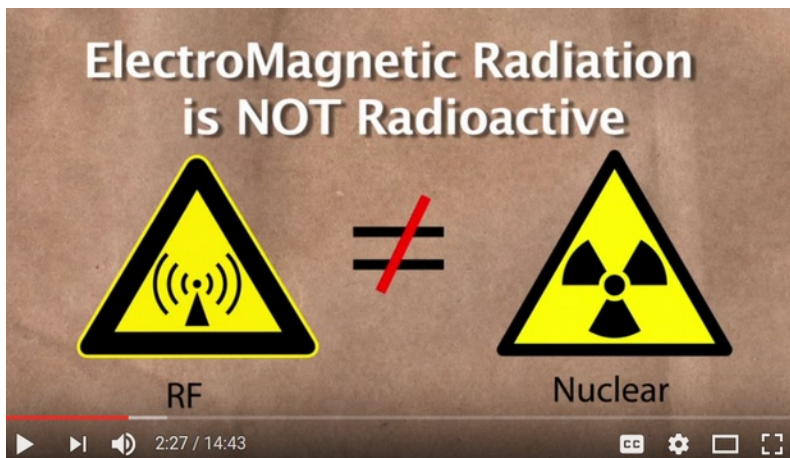
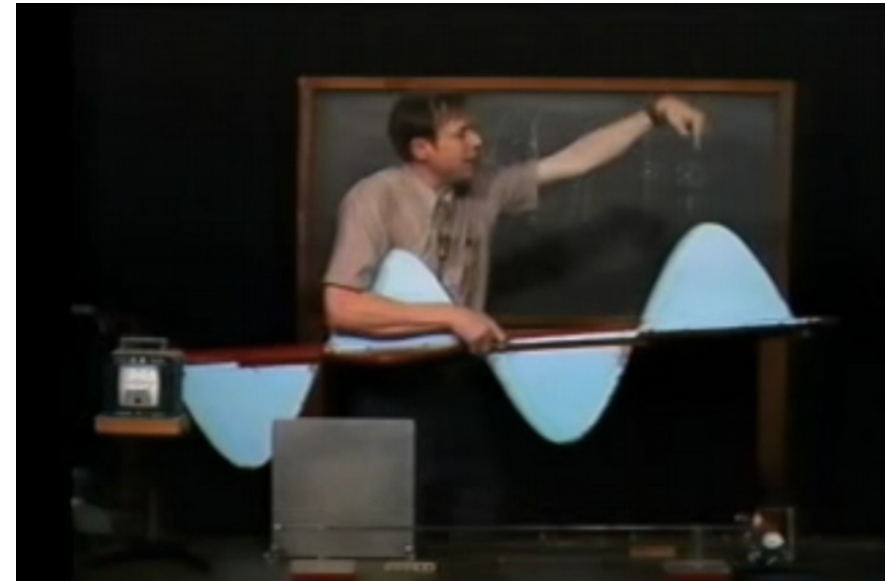
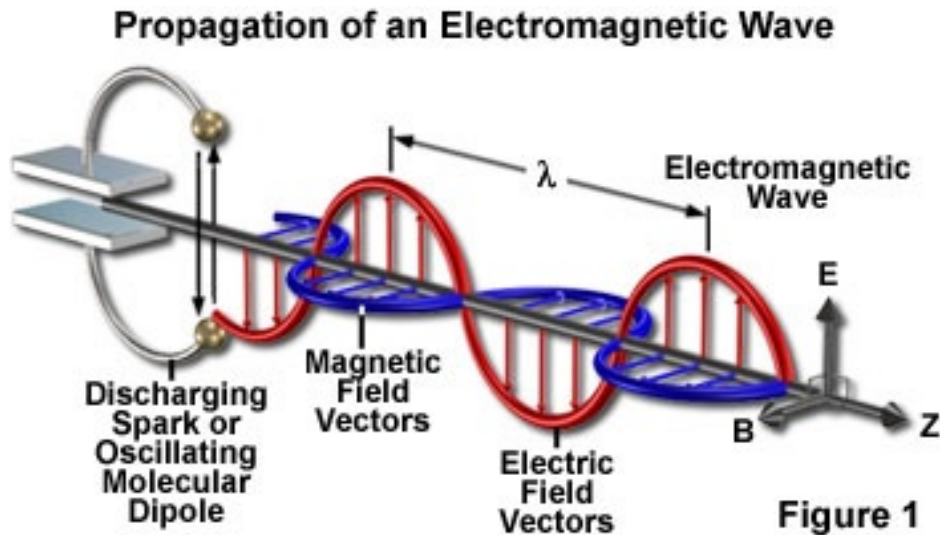
Co a kam letí ?



Stokes !

$\otimes$ x < 0
 $\nwarrow$ y $\nearrow$ se mění:
 roste $\rightarrow$ E , B
 klesá $\leftarrow$ B , E

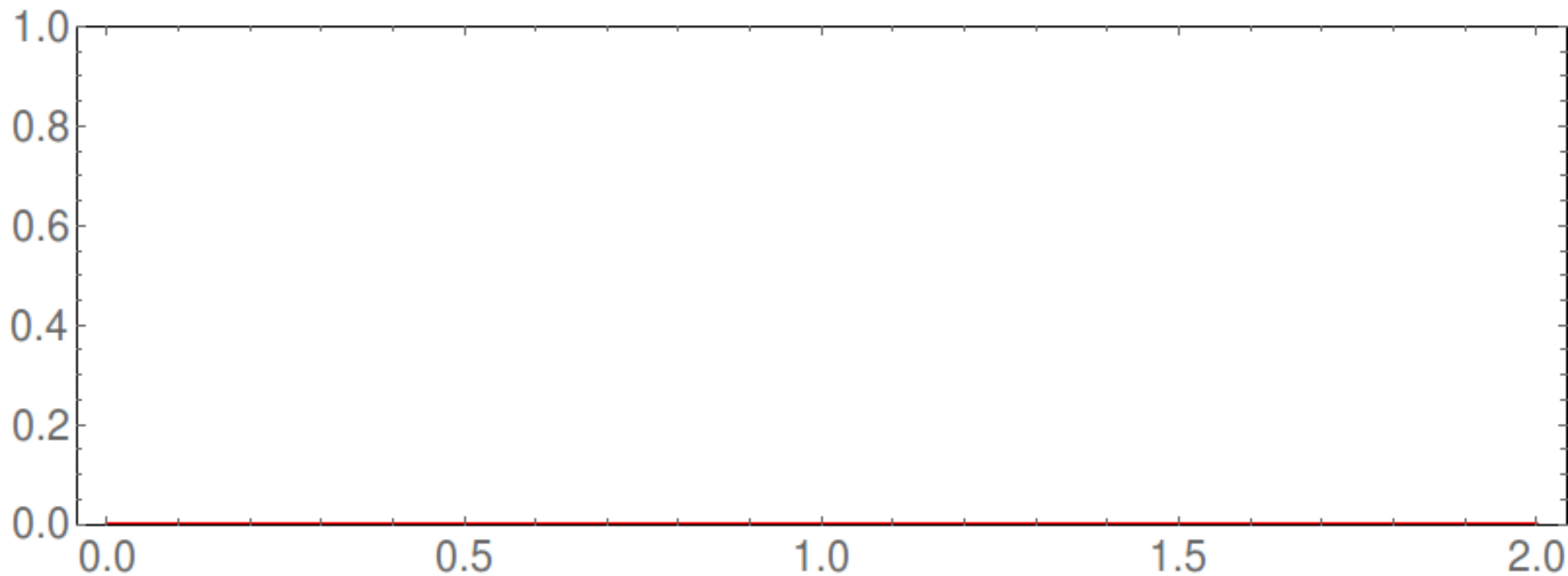
Takže, co to jsou ty EM vlny?



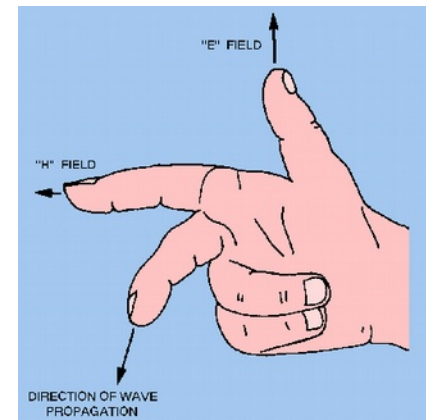
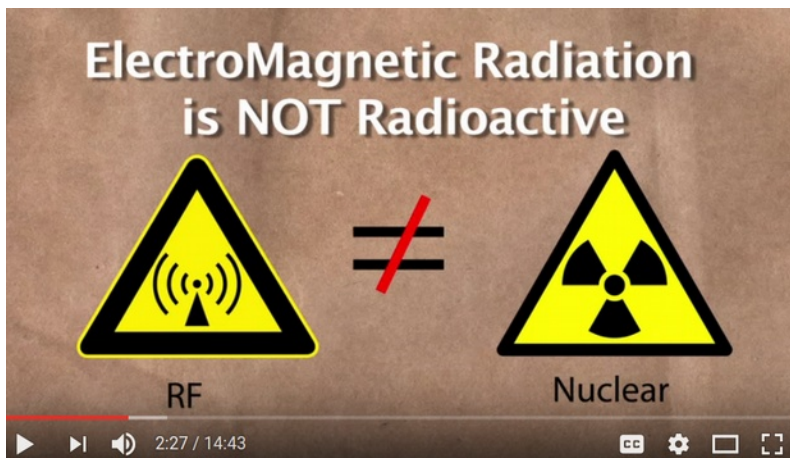
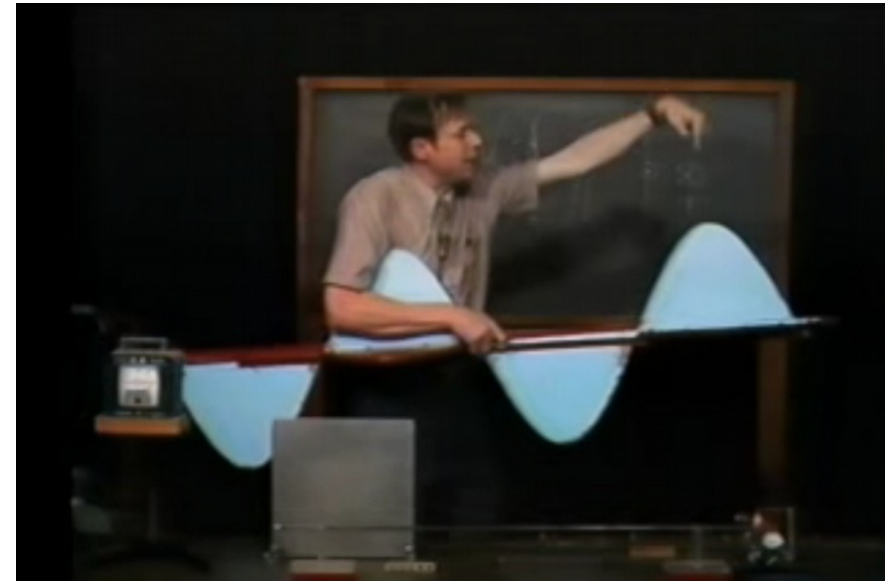
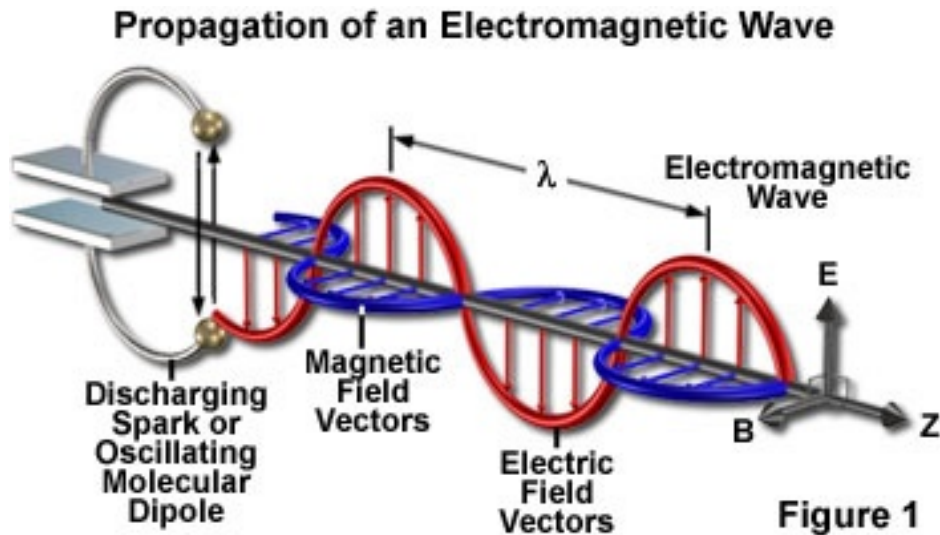
Elektromagnetické vlny

- ano, jsou to **vlny**
 - mají rychlost a směr šíření
 - frekvenci odpovídá vlnová délka
 - přenášejí energii a hybnost
- jsou to stupně volnosti elektromagnetického pole
(něco, co zbyde, když náboje a proudy zaniknou)
- Maxwellova teorie říká jak
 - **zdroje** vytvářejí EM pole
 - toto pole se **šíří** (na delší vzdálenosti v podobě vln)
 - ovlivňuje „**detektor**“
(někdy se toto působení zdá okamžité, jindy to trvá)

Obvod s žárovkou (se nezdá)



Takže, co to jsou ty EM vlny?



Mocniny vzdálenosti



Pole

magnetické

elektrické

elektromagnetické

1T

100 000V/m

40 V/m

1km

1/100 000 000

100 pV/m

1mV/m

10km

1/100 000 000 000

0.1 pV/m

0.1mV/m

magn. pole země

(ve Vakuu)

Mocniny vzdálenosti



magnetické

1T



elektrické

100 000V/m



elektromagnetické

40 V/m

1km

1/100 000 000

1mV/m

1mV/m

10km

1/100 000 000 000

0.01mV/m

0.1mV/m

magn. pole země

(ve Vakuu)

Heinrich Hertz 1887

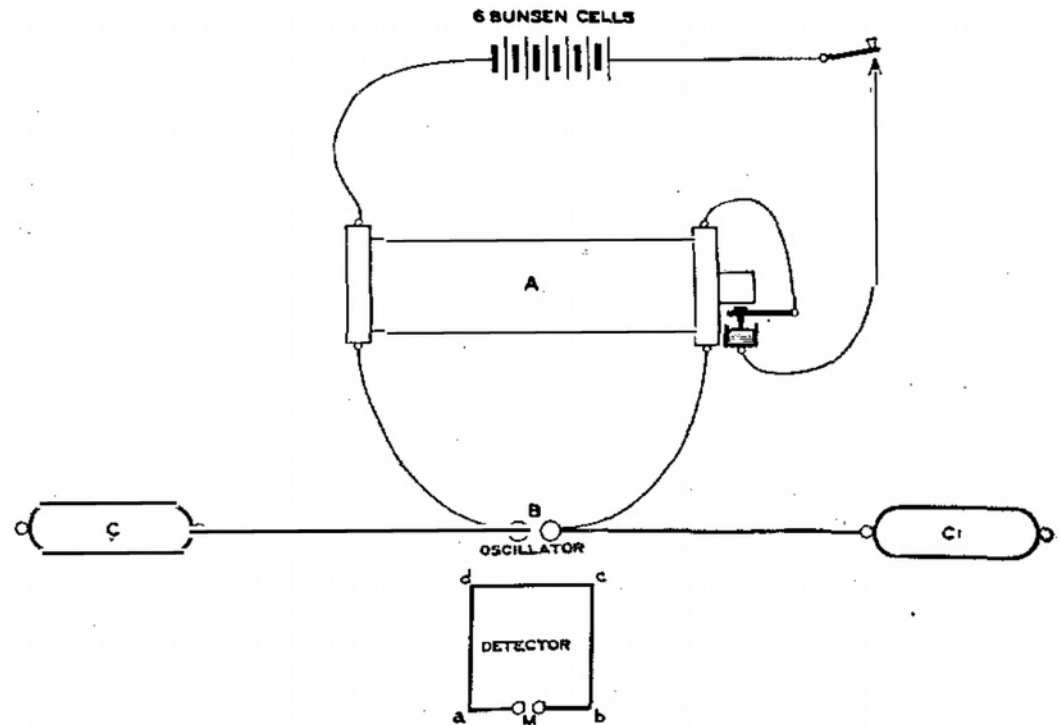
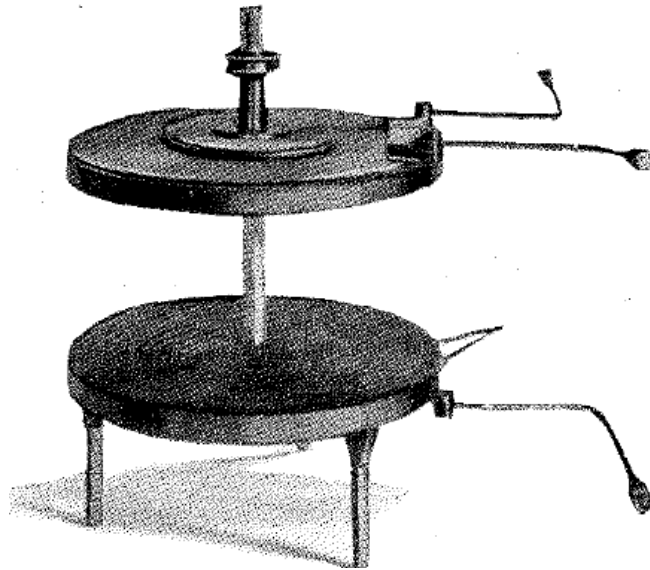
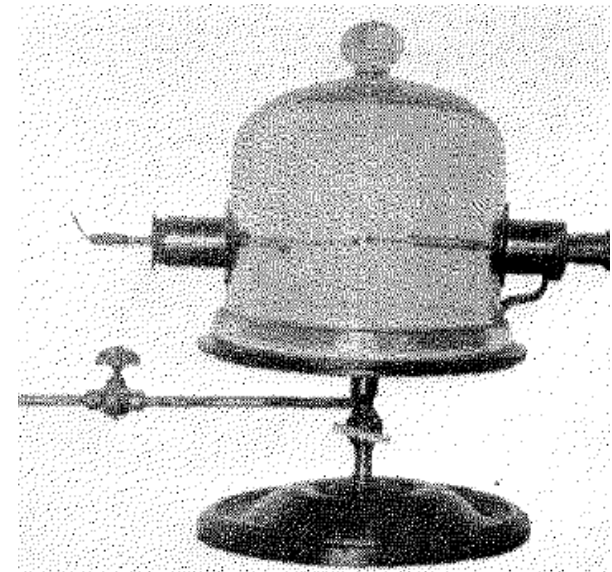


Figure 3—The First Oscillator of Hertz. Two copper wires, each 1 metre in length, supported on rods of sealing wax. The large spheres are of sheet zinc, and are 30 centimetres in diameter. Base 260 x 7.5 centimetres

Heinrich Hertz 1886-7

- as a detector the spark directly observed at the gap in a resonator,
- measured by the spark-micrometer (hot-wire galvanometer)
- determined the positions of nodes, anti-nodes, wavelength
- placed respectively at the principal foci of two parabolic mirrors
- up to a distance of about 16 metres
- the waves could penetrate a wooden door
- laws of reflection were proved
- refraction - prism of pitch (800kilogrammes)



Hertzovy vlny

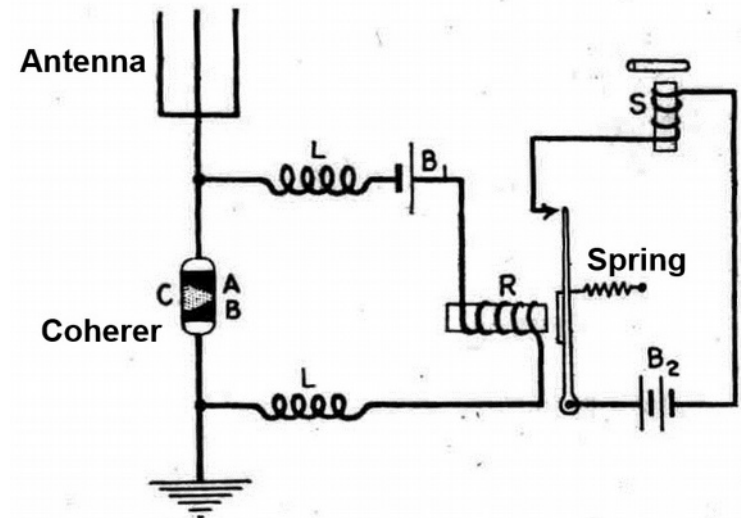
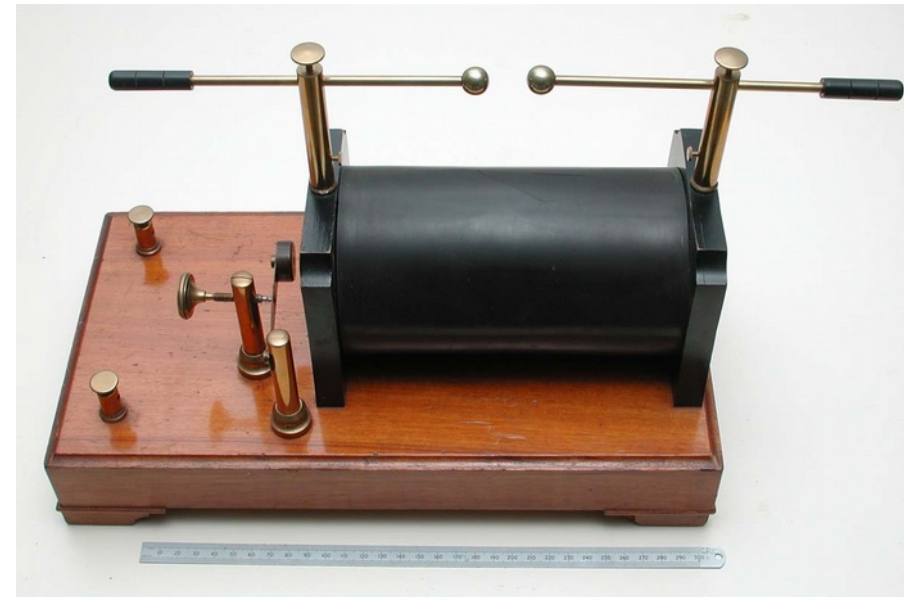
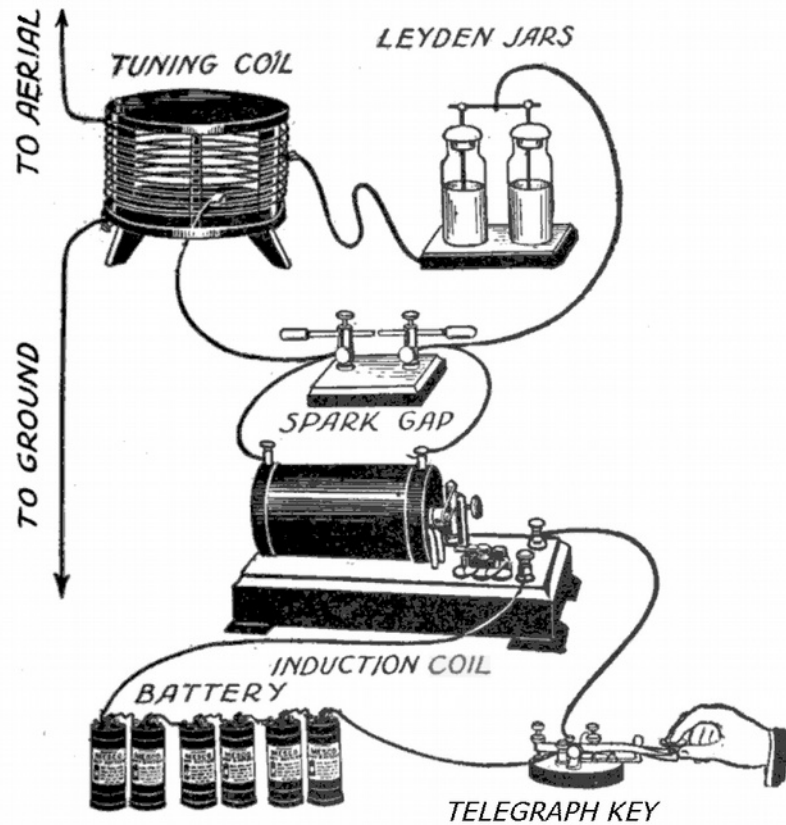
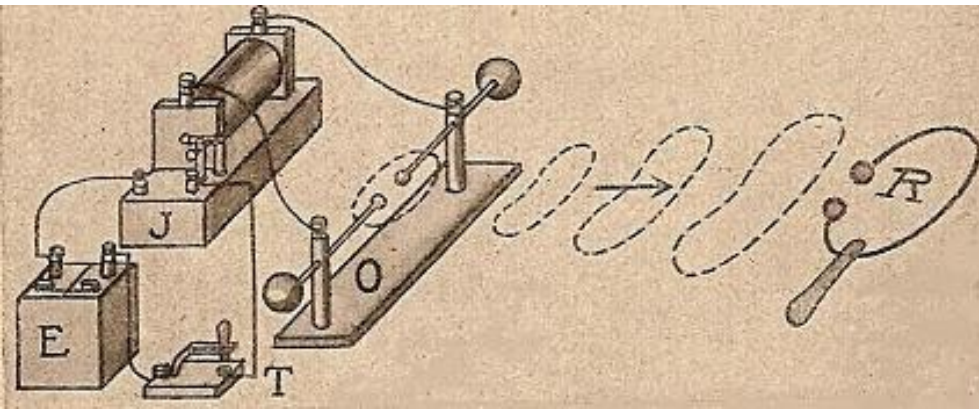


Fig. 101. Marconi 1896 Receiver.

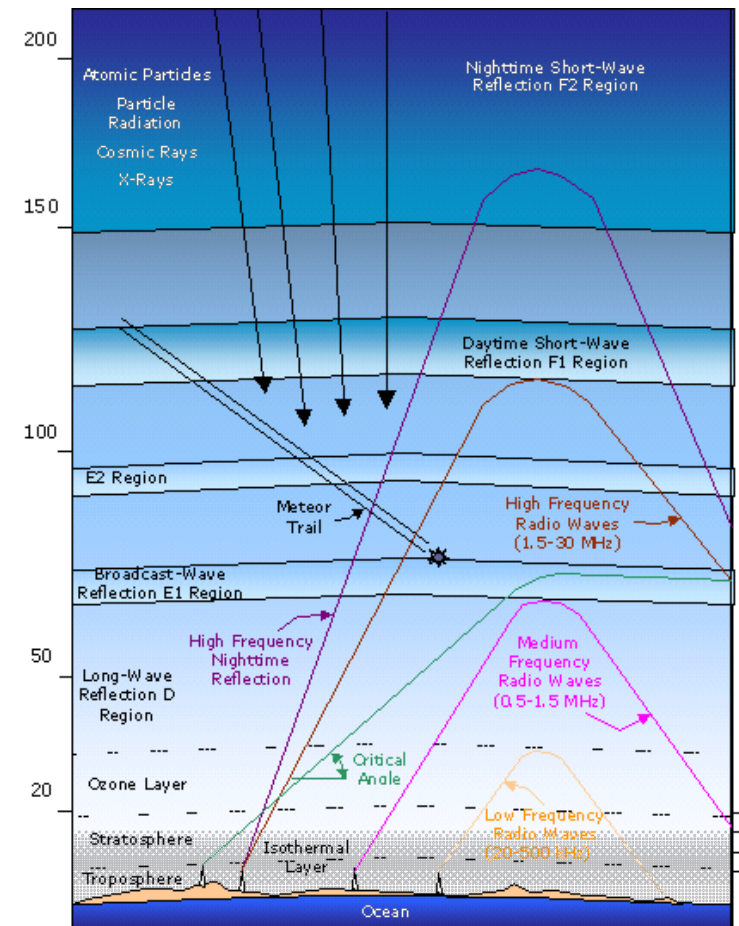
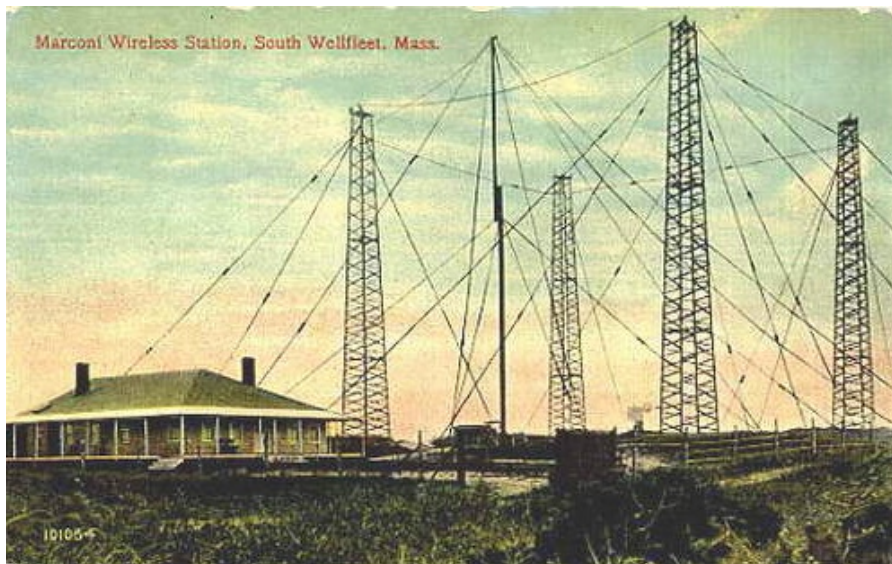
Guglielmo Marconi

Wireless telegraphic communication

Nobel Lecture, December 11, 1909

I have also recently been able to confirm the statement made by Prof. Fleming in his book *The Principles of Electric Wave Telegraphy*, 1906, page 555, that with a power of 8 watts in the aerial it is possible to communicate to distances of over 100 miles.

On many occasions last winter, the S.S. "Caronia" of the Cunard Line, carrying a station utilizing about 1/2 kilowatt, when in the Mediterranean, off the coast of Sicily, failed to obtain communication with the Italian stations, but had no difficulty whatsoever in transmitting and receiving messages to and from the coasts of England and Holland, although these latter stations were considerably more than 1,000 miles away, and a large part of the continent of Europe and the Alps lay between them and the ship.



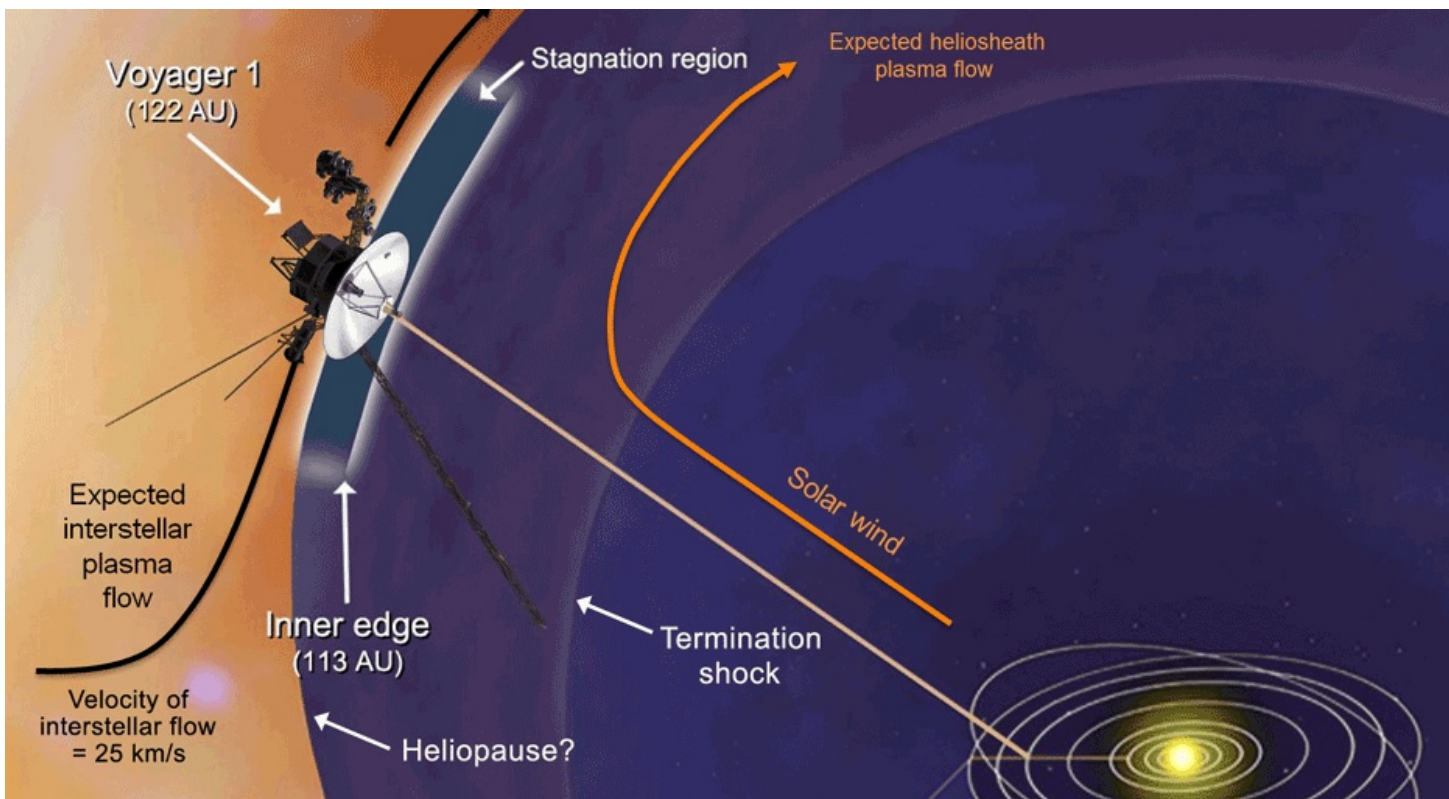
Voyager 1

T = 2012

P = 23 W

L = 18 000 000 000 km

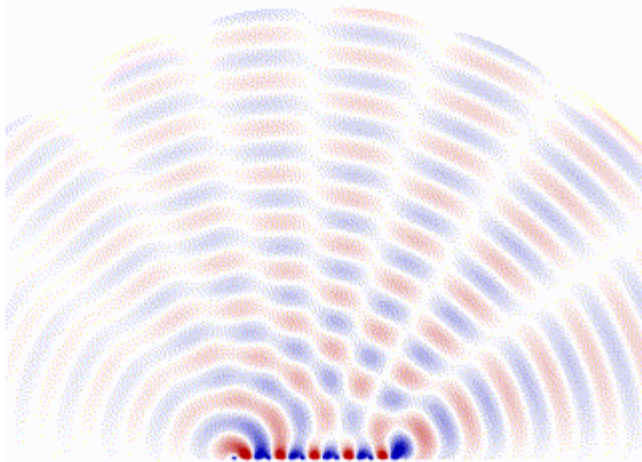
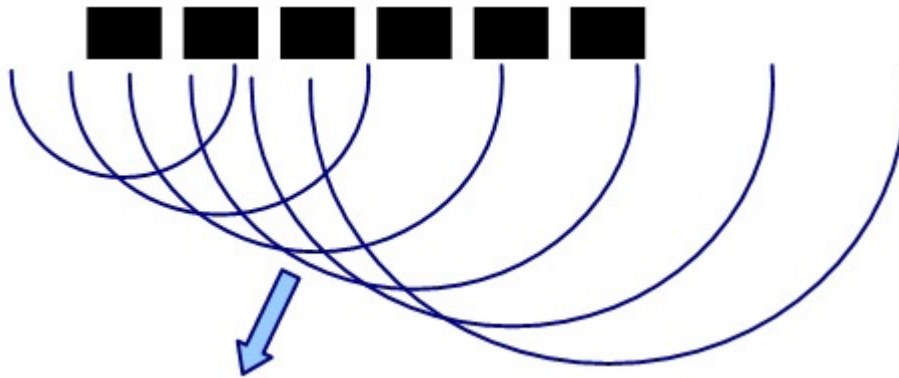
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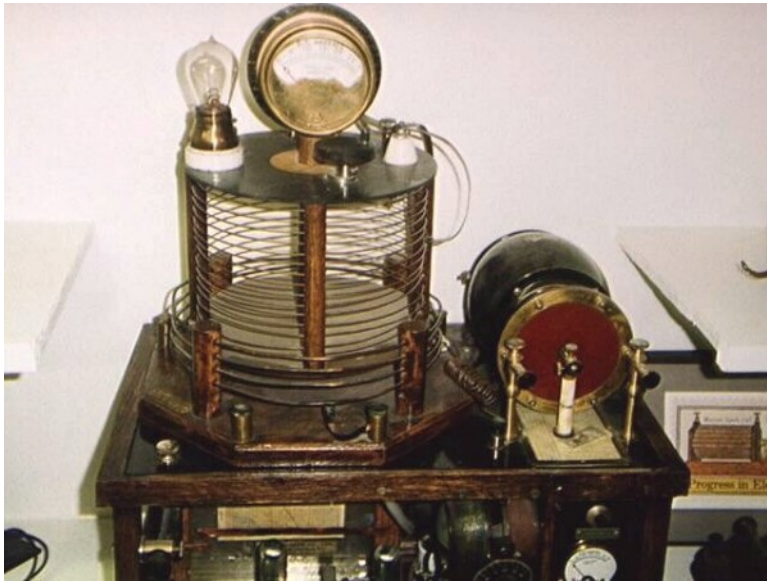
Karl Ferdinand Braun

Wireless telegraphic communication

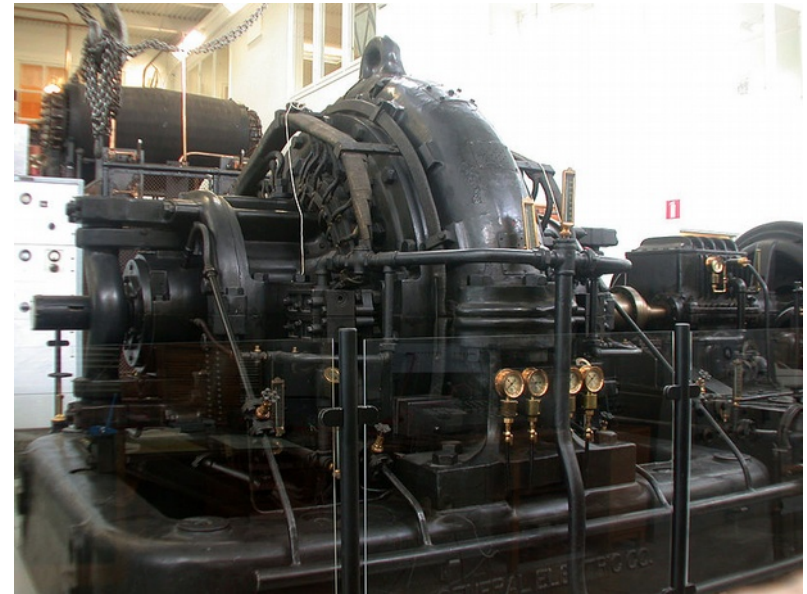
Nobel Lecture, December 11, 1909



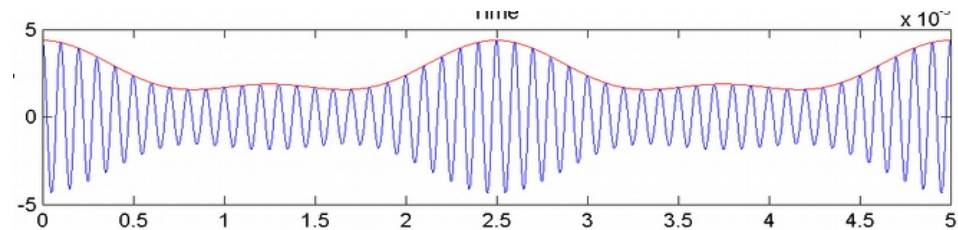
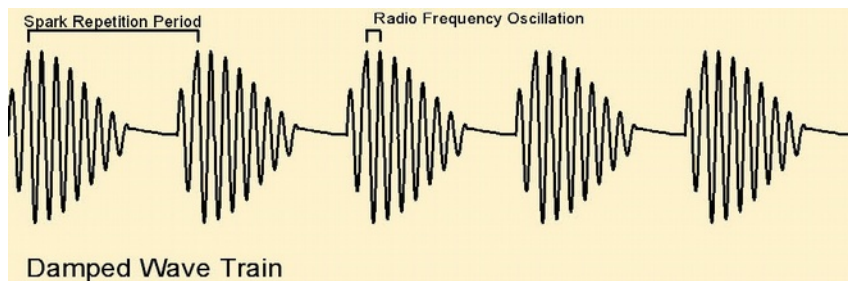
Počátky radiotelegrafie



Jiskrový vysílač

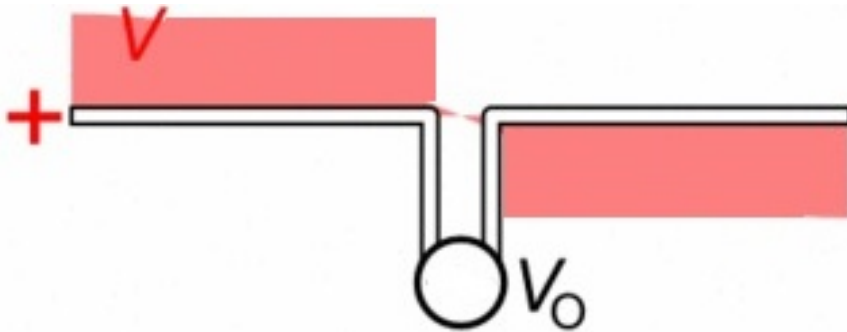


Alexandersonův alternátor

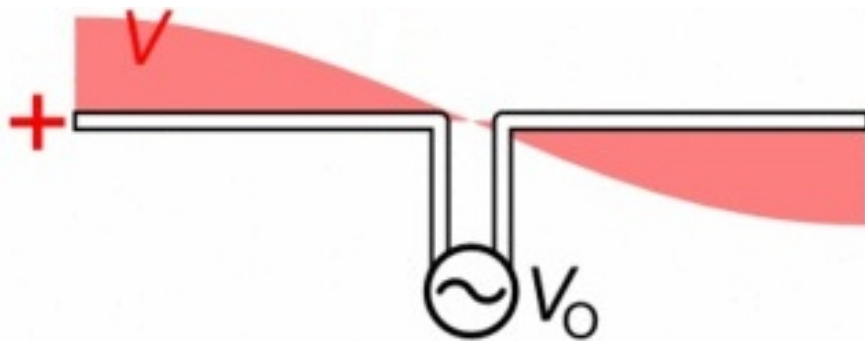


Anténa ?

$$E_C = \frac{1}{2}CU^2 \quad E_L = \frac{1}{2}LI^2$$



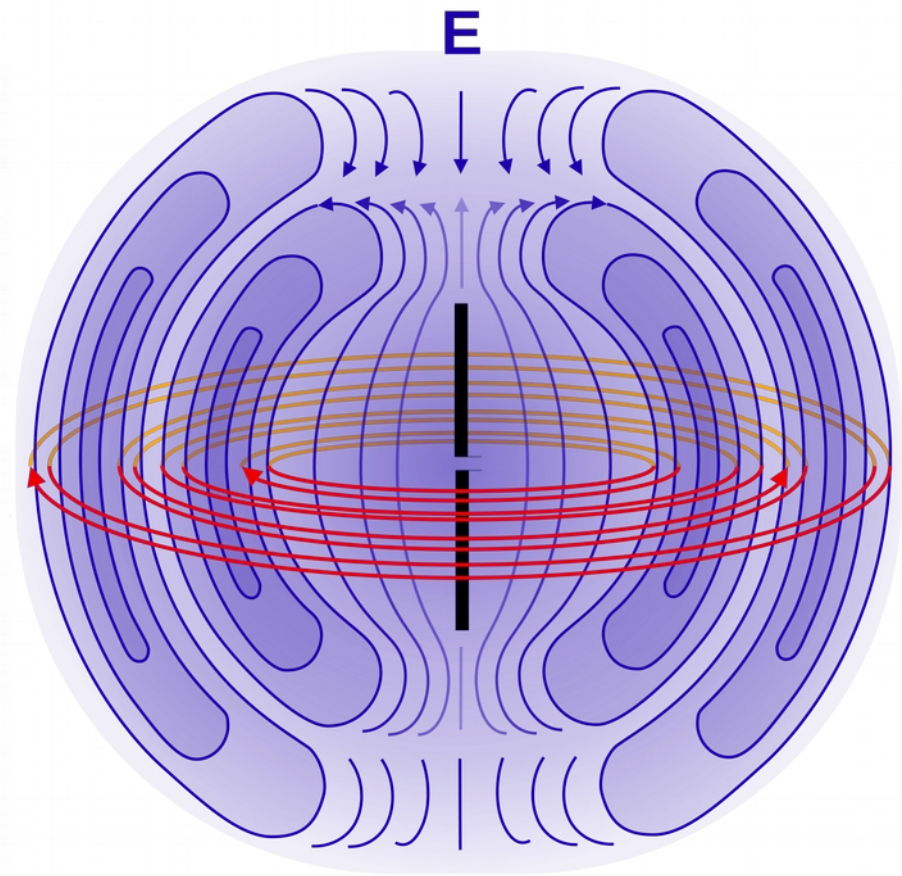
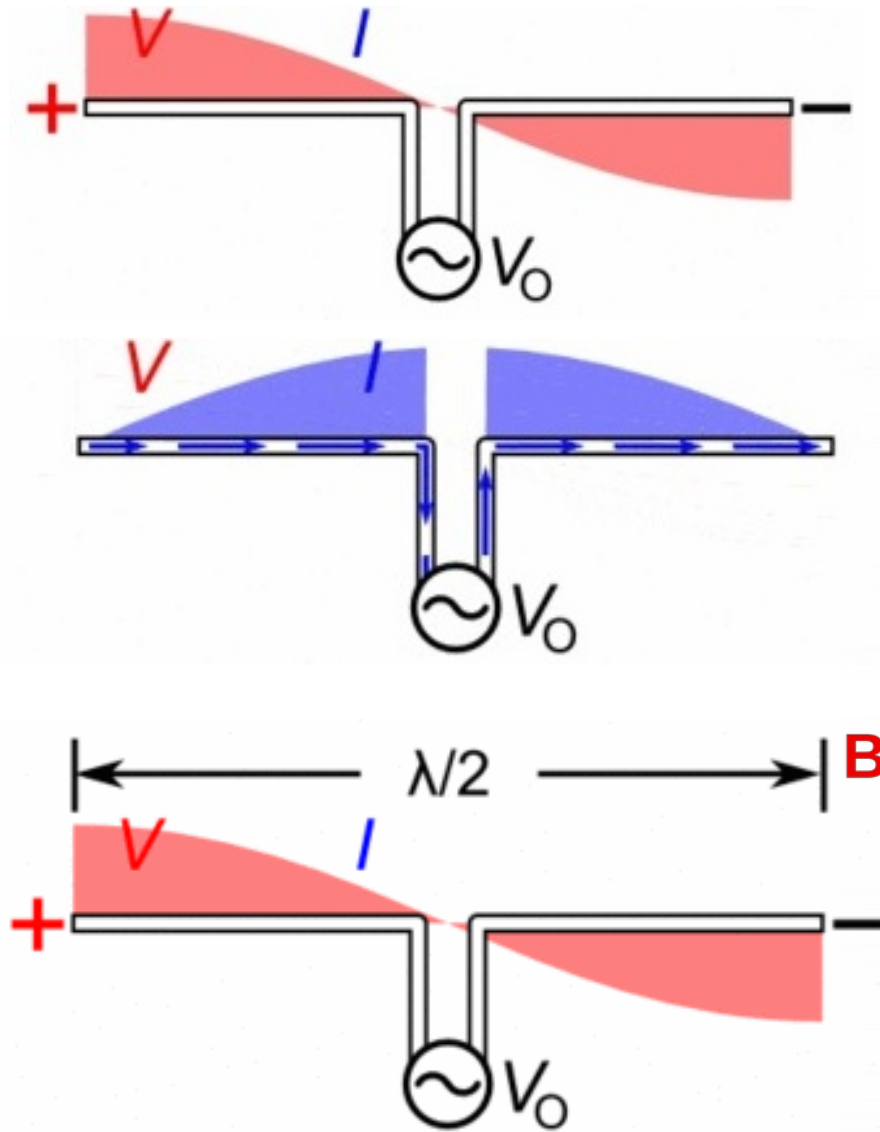
V Hetzově anténě již nejsou náboje rozloženy podle pravidel elektrostatiky a proudy nejsou stacionární!

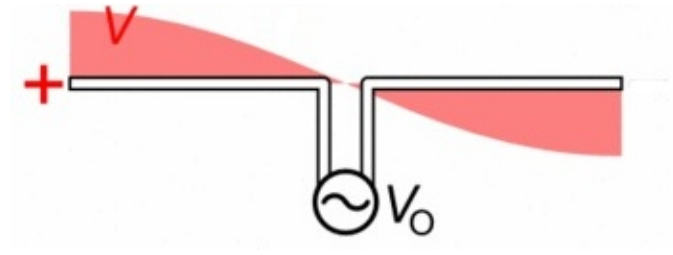
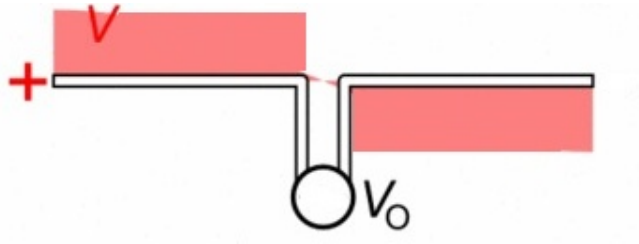


Na rozdíl od kondenzátoru a cívky tak nad nimi (a jejich energií) ztrácíme kontrolu – ta může odletět v podobě vln pryč.

Pozor – to červené není napětí, ale náboj

Pozor – to červené není napětí, ale náboj





$$\mathbf{B}(\mathbf{r}, t) = -\frac{\mu_0}{4\pi} \hat{\mathbf{r}} \times \left\{ \frac{\dot{\mathbf{p}}_{\text{ret}}}{r^2} + \frac{\ddot{\mathbf{p}}_{\text{ret}}}{cr} \right\}$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \left\{ \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{p}_{\text{ret}}) - \mathbf{p}_{\text{ret}}}{r^3} + \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \dot{\mathbf{p}}_{\text{ret}}) - \dot{\mathbf{p}}_{\text{ret}}}{cr^2} + \frac{\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \ddot{\mathbf{p}}_{\text{ret}}) - \ddot{\mathbf{p}}_{\text{ret}}}{c^2r} \right\}$$

$$P_{E1}(t) = \frac{1}{4\pi\epsilon_0} \frac{2|\ddot{\mathbf{p}}_{\text{ret}}|^2}{3c^3}$$

$$P_{M1}(t) = \frac{\mu_0}{4\pi} \frac{2|\ddot{\mathbf{m}}_{\text{ret}}|^2}{3c^3},$$

Vyzářený výkon lze dobře odhadnout

- * p součin náboje a délky
- * m součin proudu a plochy

.. $(2\pi f)^2$

- * ret vlnám to chvíli trvá

Friisova rovnice

$$P_R = A_R \mathcal{P}_T = \frac{P_T G_T A_R}{4\pi r^2}$$

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi R} \right)^2$$

$$G = \frac{A_{eff}}{\lambda^2/4\pi} = \frac{4\pi A_{phys} e_a}{\lambda^2}$$

Example 15.6.1: A geosynchronous satellite is transmitting a TV signal to an earth-based station at a distance of 40,000 km. Assume that the dish antennas of the satellite and the earth station have diameters of 0.5 m and 5 m, and aperture efficiencies of 60%. If the satellite's transmitter power is 6 W and the downlink frequency 4 GHz, calculate the antenna gains in dB and the amount of received power.

Solution: The wavelength at 4 GHz is $\lambda = 7.5$ cm. The antenna gains are calculated by:

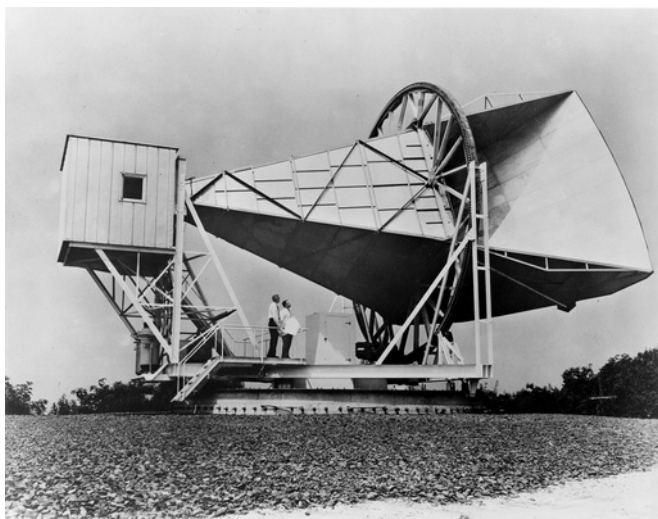
$$G = e_a \left(\frac{\pi d}{\lambda} \right)^2 \Rightarrow G_{sat} = 263.2 = 24 \text{ dB}, \quad G_{earth} = 26320 = 44 \text{ dB}$$

Because the ratio of the earth and satellite antenna diameters is 10, the corresponding gains will differ by a ratio of 100, or 20 dB. The satellite's transmitter power is in dB, $P_T = 10 \log_{10}(6) = 8$ dBW, and the free-space loss and gain:

$$L_f = \left(\frac{4\pi r}{\lambda} \right)^2 = 4 \times 10^{19} \Rightarrow L_f = 196 \text{ dB}, \quad G_f = -196 \text{ dB}$$

It follows that the received power will be in dB:

$$P_R = P_T + G_T - L_f + G_R = 8 + 24 - 196 + 44 = -120 \text{ dBW} \Rightarrow P_R = 10^{-12} \text{ W}$$

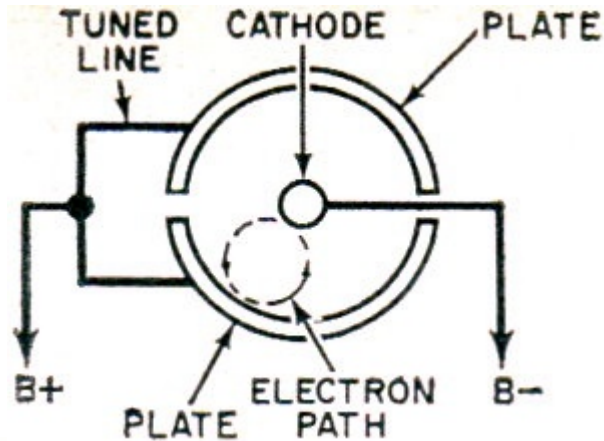


Ukázkový problém z učebnice

S.J. Orfanidis: *Electromagnetic Waves and Antennas*

Výkon !

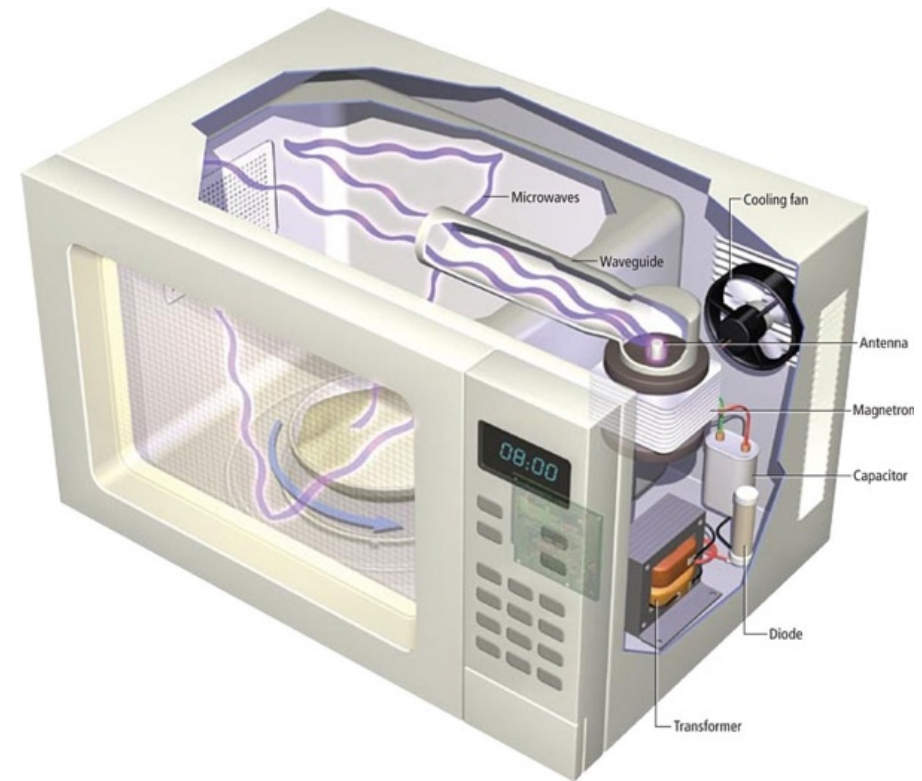
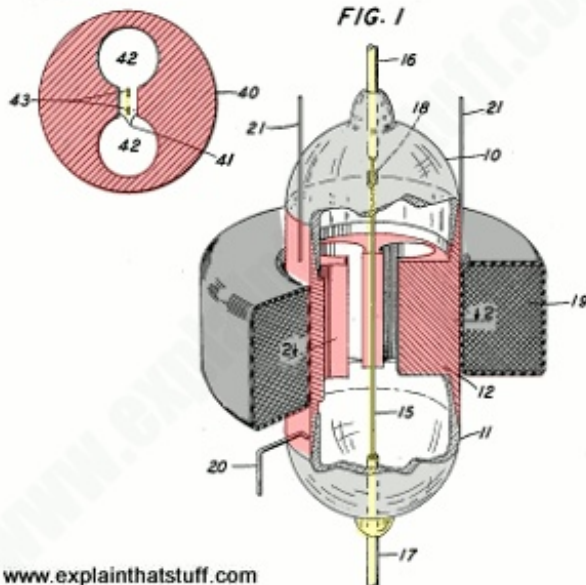
Elektromagnetické vlny nejsou jen ony slabé vlnky v éteru



Dec. 8, 1936. A. L. SAMUEL 2,063,342

ELECTRON DISCHARGE DEVICE

FIG. 5



Výkon !

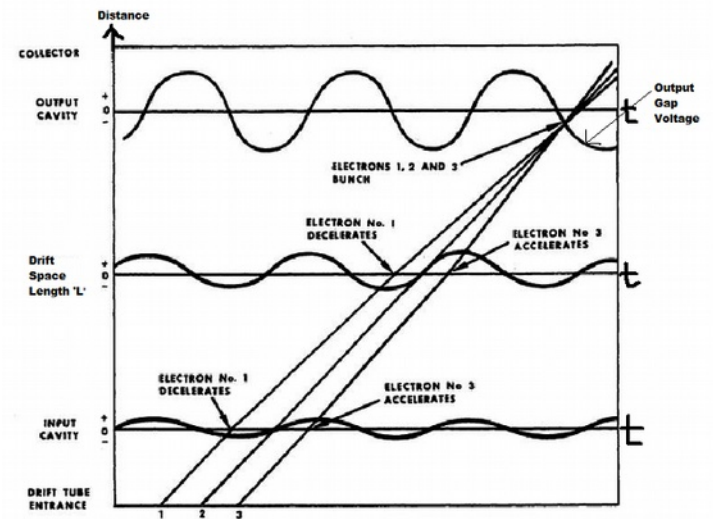
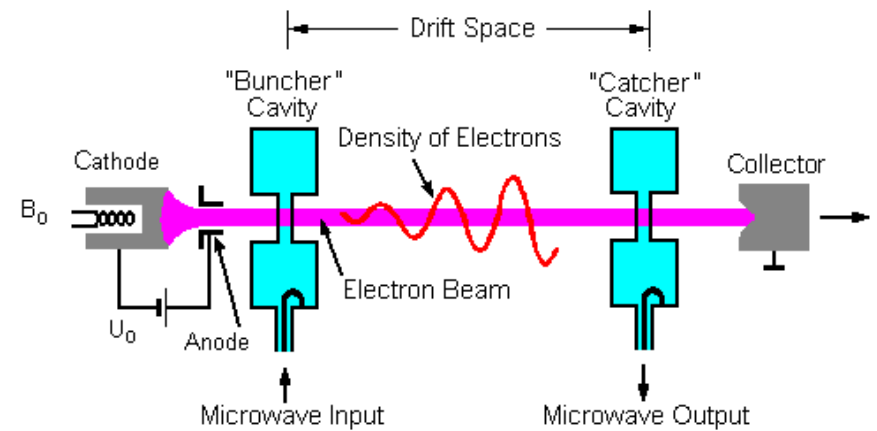


Fig. 19 The SLAC 11.4 GHz, 75-MW PPM klystron

Výkon !

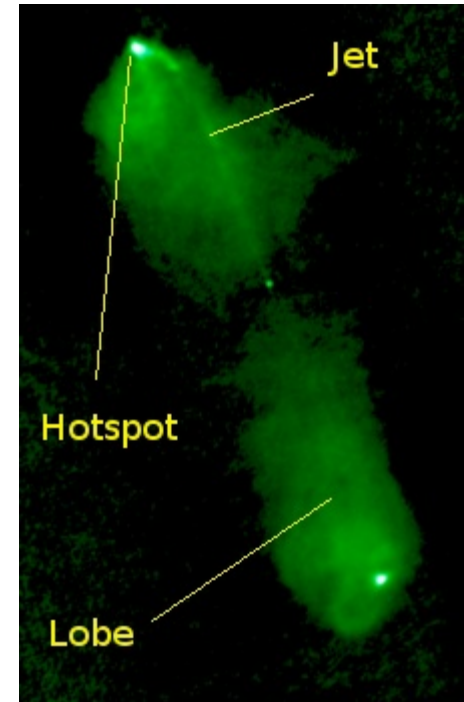
Rádiové rekordy

Rádiové galaxie:

10^{39} W v pásmu 10 MHz – 100 GHz

Pulsary

10^{24} W v pásmu 200 – 600 MHz



Transatlantické telefonování

1928: Bell Labs

výzkum atmosférických poruch a vlastností ionosféry
krátké vlny (15 MHz, anténa rozměr 30 m)

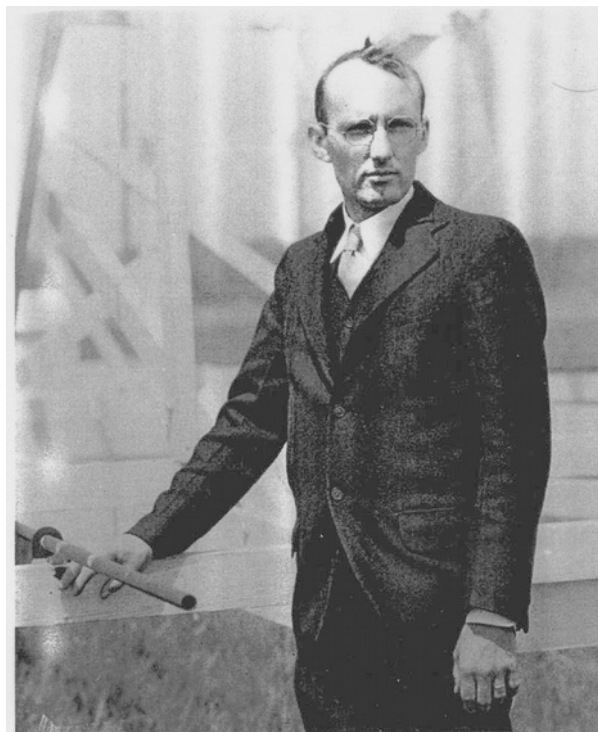
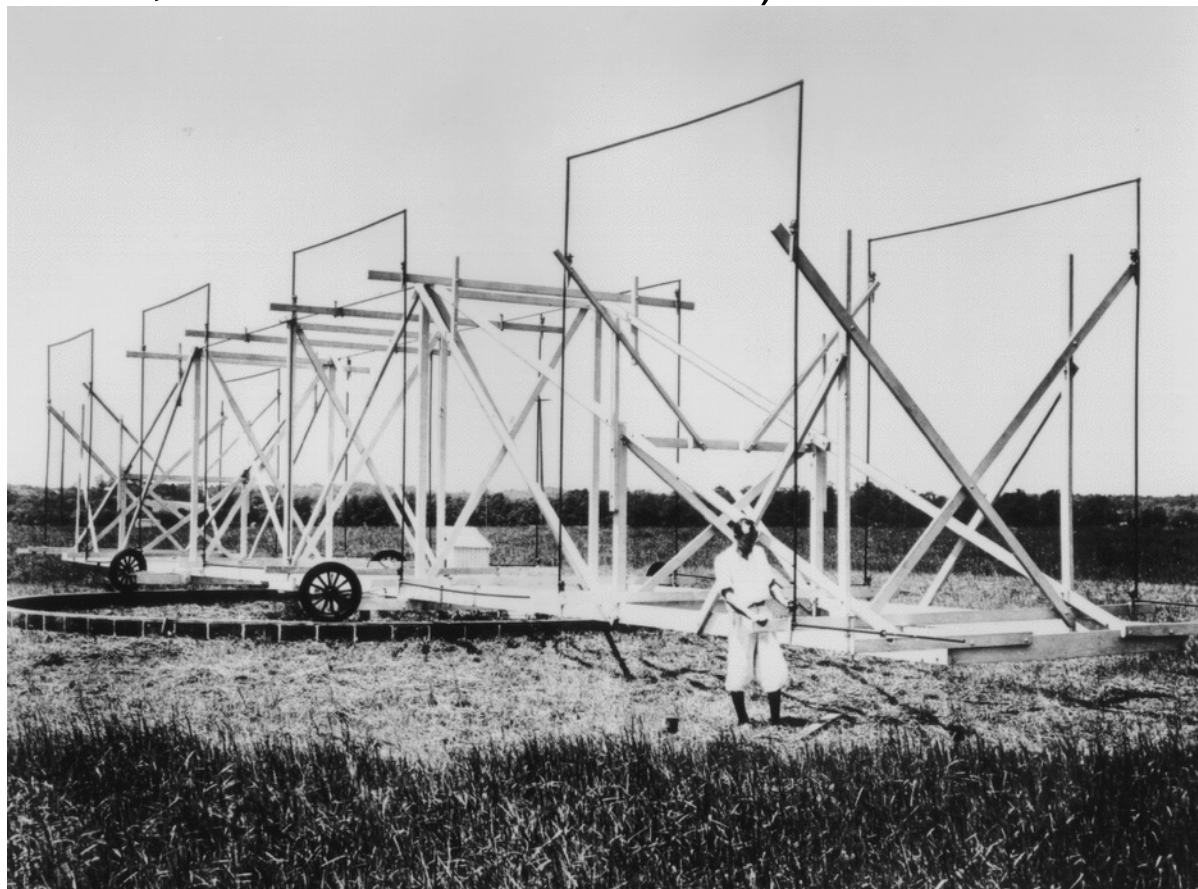
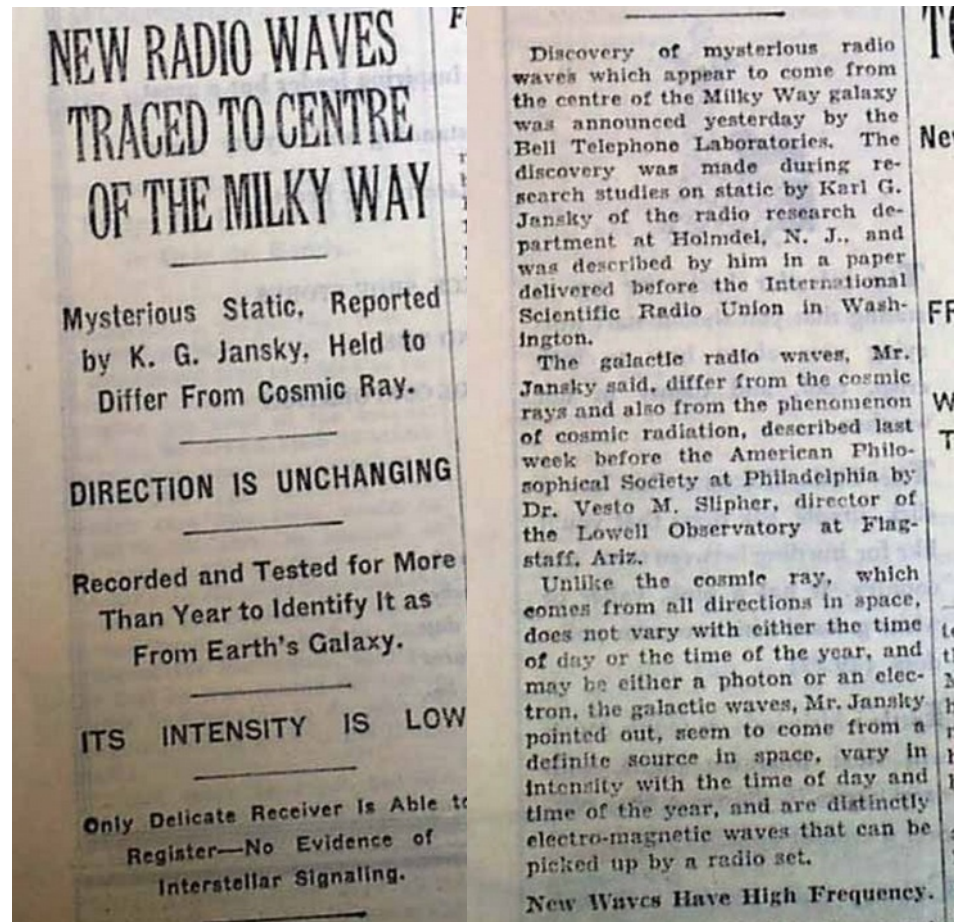


FIG. 1—Karl Guthe Jansky, about 1933.



Karl Jansky

- Bouřky
- Bouřky
- a Neznámý šum



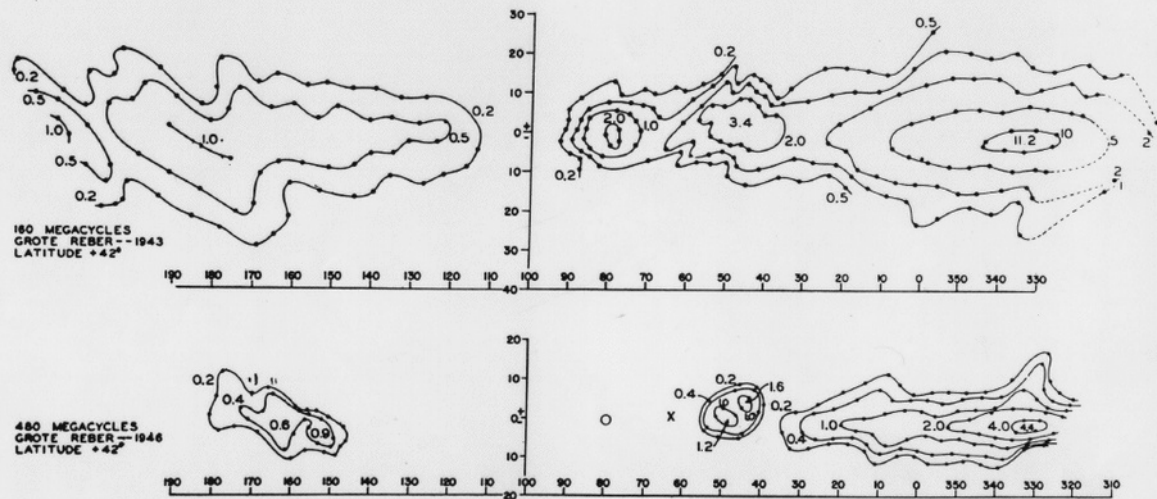
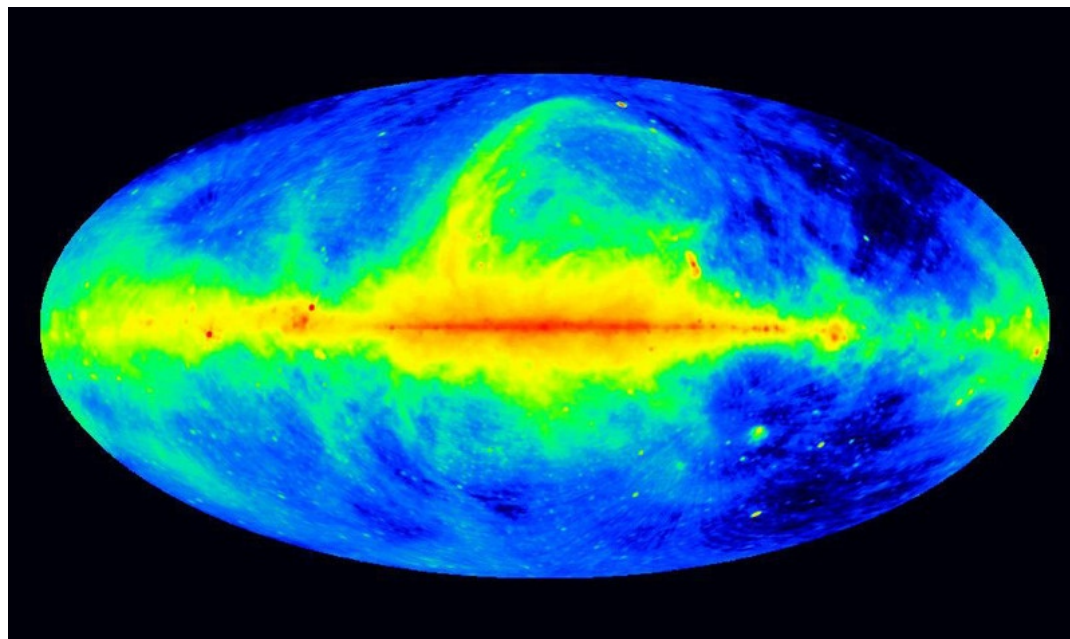


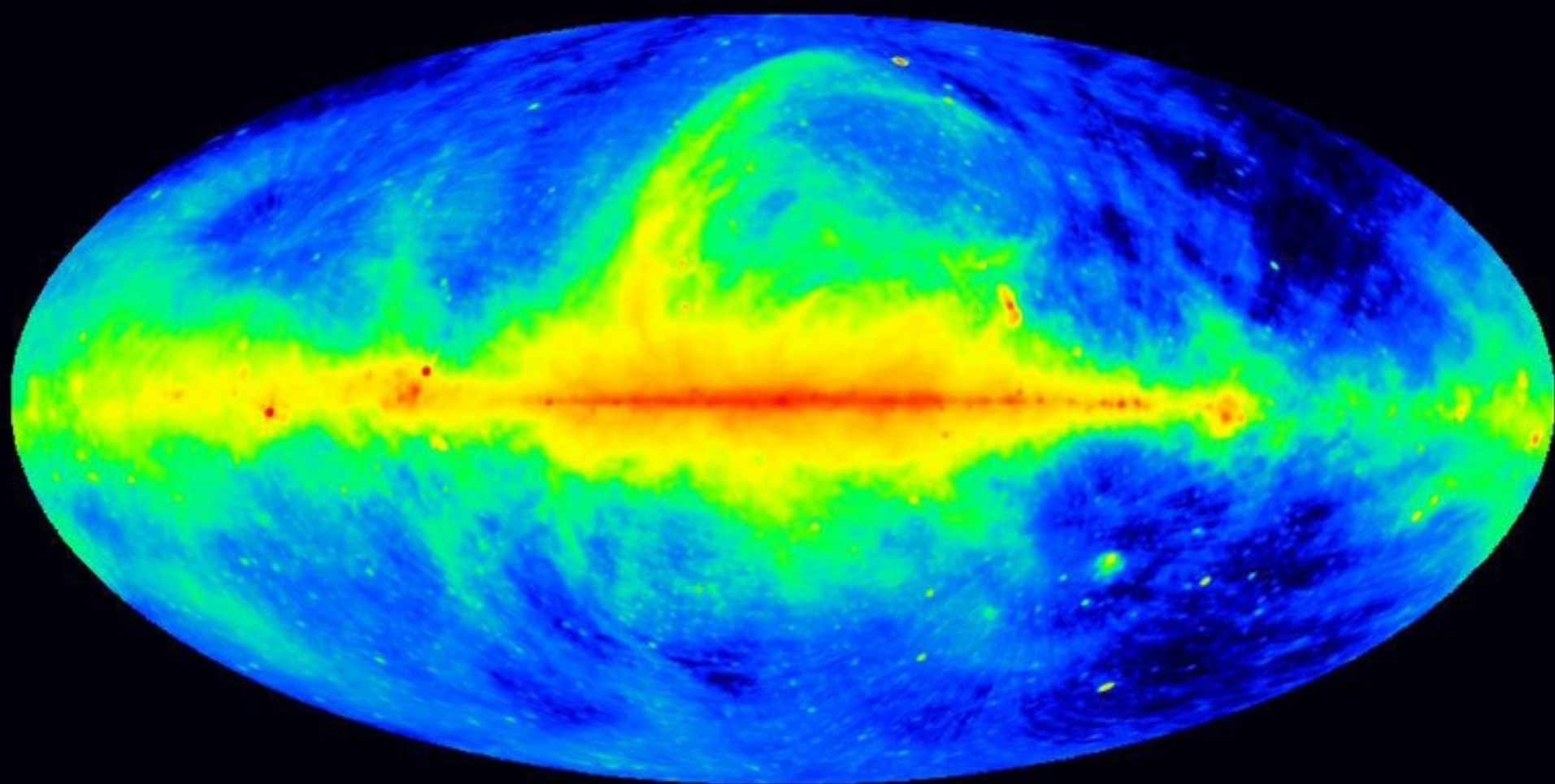
FIG. 7—Contours of constant intensity at 160 MHz and 480 MHz, taken at Wheaton, Illinois.



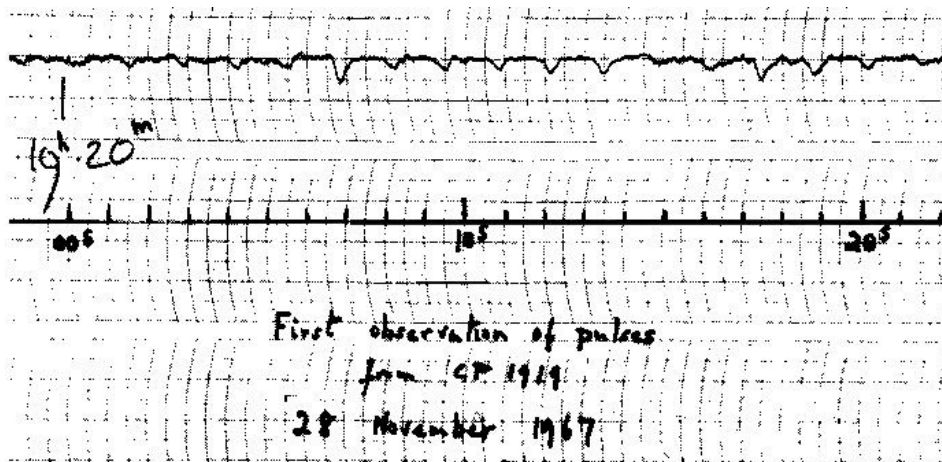
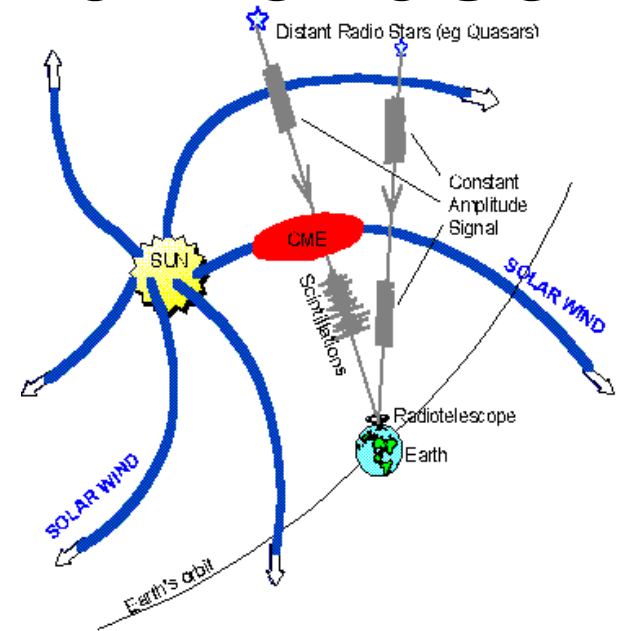
Grote Reber

408MHz
Haslam et al. 1982

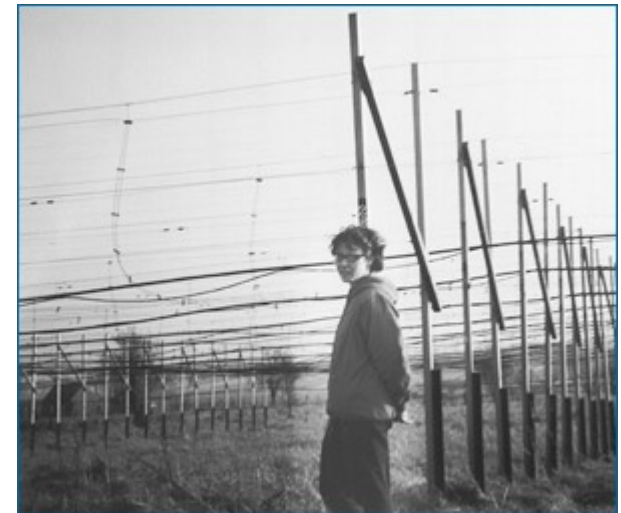




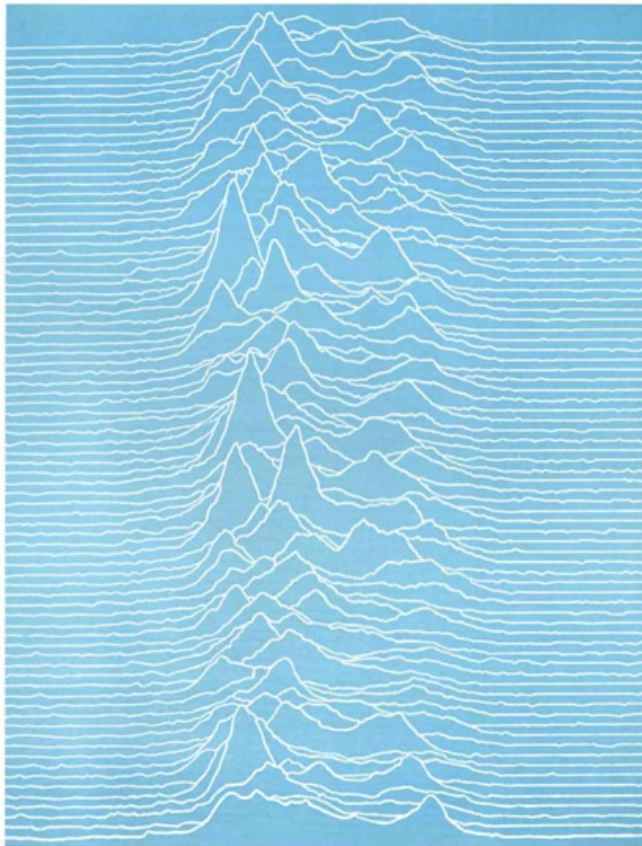
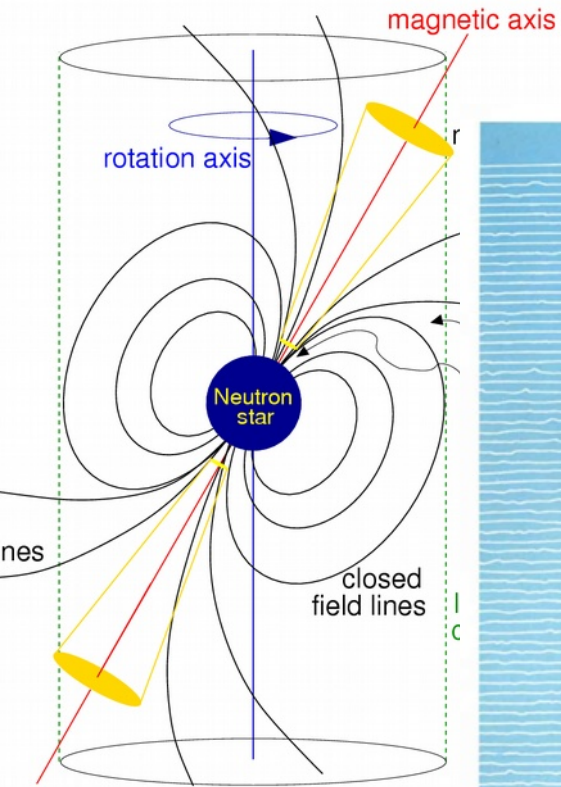
Jaké je dnes počasí u Venuše?



Jocelyn Bell

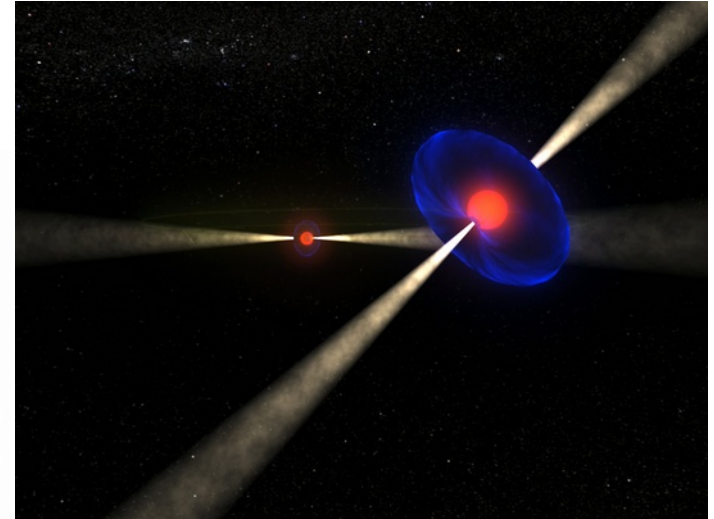


Pulsary



EIGHTY SUCCESSIVE PERIODS of the first pulsar observed, CP1919 (Cambridge pulsar at 19 hours 19 minutes right ascension), are stacked on top of one another using the average period of 1.33730 seconds in this computer-generated illustration produced at the Arecibo Radio Observatory in Puerto Rico. Although the lead-

ing edges of the radio pulses occur within a few thousandths of a second of the predicted times, the shape of the pulses is quite irregular. Some of this irregularity in radio reception is caused by the effects of transmission through the interstellar medium. The average pulse width is less than 50 thousandths of a second.



na víc už nám nezbyl čas ...