Loop Quantum Gravity

An Introduction

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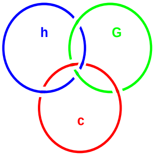
Praha 2007



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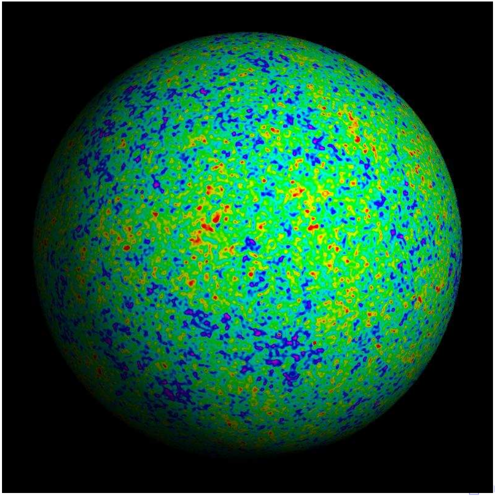
Cosmological Puzzles A: Isotropy of the CMB



Horizon Problem:

- On large scales: Universe = flat ($k = 0$) FRW

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 =: a(\eta)^2[-d\eta^2 + d\vec{x}^2]$$

- For eqn. of state $p = w\rho$, cosm. const. Λ, curvature k, Einstein's eqns. yield f. small t:

$$a(t) \propto t^{[1+(1+3w)/2]^{-1}}$$

- If strong energy cond. $1 + 3w > 0$ holds

$$\lim_{t \to 0} a(t) = 0 \quad \text{und} \quad \lim_{t \to 0} |\eta(t)| < \infty$$

- Big bang singularity: Time had a beginning.

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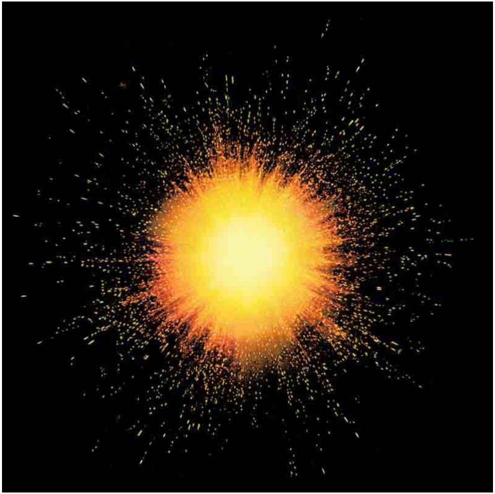

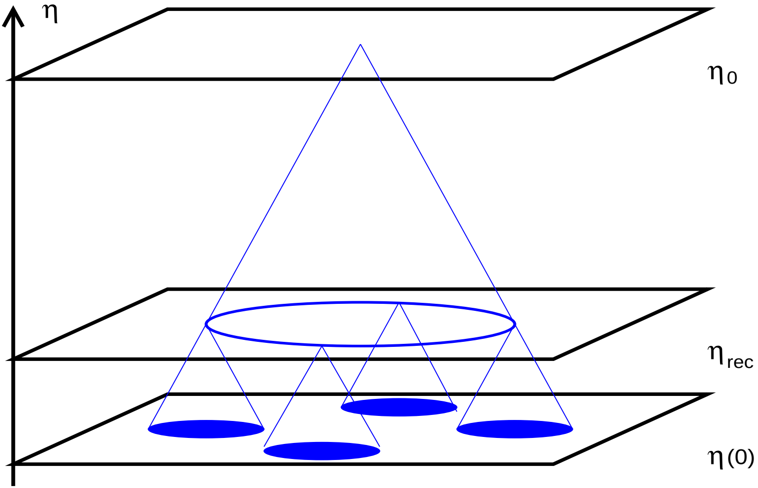
η
η0
ηrec
η(0)

Problem:

- For normal matter ($w > -\frac{1}{3}$), e.g. photons ($w = \frac{1}{3}$), time difference $\eta_{\rm rec} - \eta(0)$ too short.
- Idea of Inflation:
  Use exotic matter ($w < -\frac{1}{3}$) to extend $\eta_{\rm rec} - \eta(0)$ during $t \in [t_i \approx t_P, t_f]$.
- Popular model: Inflaton with extremely flat potential

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \;\Rightarrow\; \dot{\phi} \approx 0 \;\Rightarrow\; w = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \approx -1$$

- Implies expontial growth of scale factor

$$H^2 := (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho = \text{const.}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

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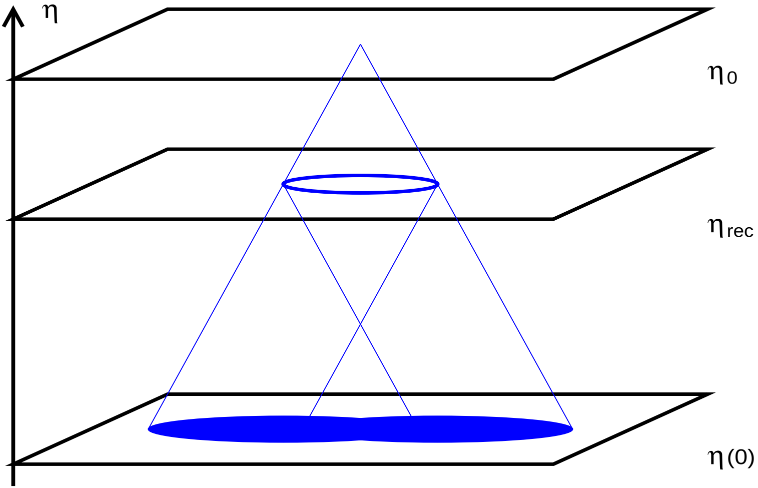
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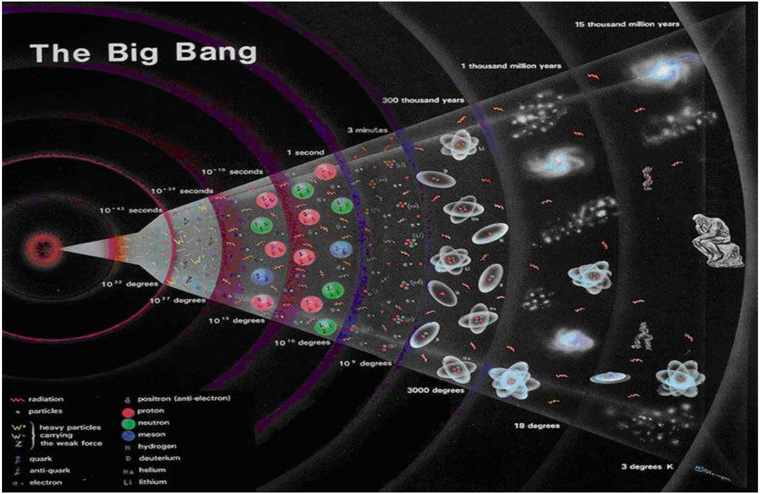
The Big Bang
15 thousand million years
1 thousand million years
300 thousand years
3 minutes
1 second
10⁻¹⁰ seconds
10⁻³⁴ seconds
10⁻⁴³ seconds
10³² degrees
10²⁷ degrees
10¹⁵ degrees
10¹⁰ degrees
10⁹ degrees
3000 degrees
18 degrees
3 degrees K
radiation
particles
heavy particles carrying the weak force
quark
anti-quark
electron
positron (anti-electron)
proton
neutron
meson
hydrogen
deuterium
helium
lithium

Inflation also explains temperature fluctuations, but:

- What is the inflaton? How did it decay? Necessary?
- Fine tuning of FRW initial data $\leftrightarrow$ fine tuning of V(ϕ).
- Is GR really valid until t_P? What happens before?
- Quantum avoidance of horizon problem due to breakdown of notion of class. spacetime at t_P?

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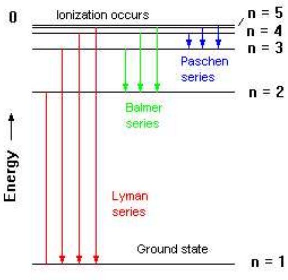

- Pre – big bang?

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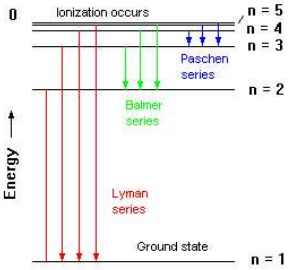

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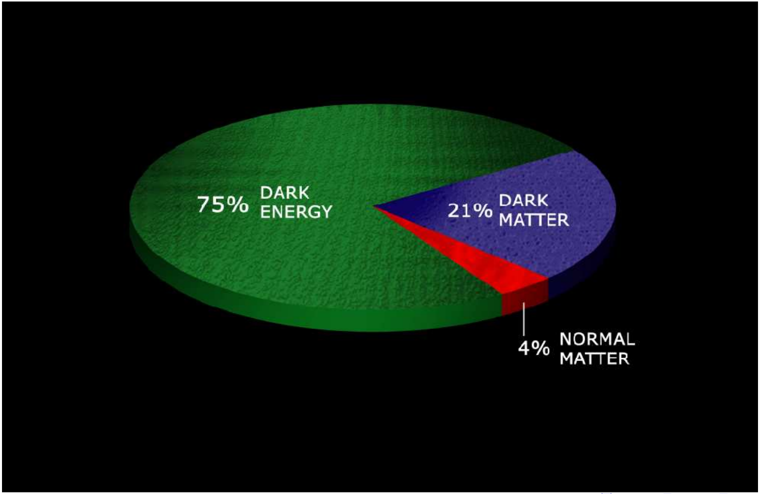
Cosmological Puzzle B: Dark Energy



Problem of cosmological constant:

- Dark energy = vacuum fluct.? E.g. zero point energy

$$< \widehat{H} >_{\text{scalar}} - <: \widehat{H} :>_{\text{scalar}} = \frac{\hbar}{2} \int_{R^3} d^3x \int_{R^3} d^3k \, |k|$$

- Quantum gravity: Cut – off at $k\ell_P \approx 1$ where $\ell_P^2 = \hbar G$?

$$< \widehat{H} > - <: \widehat{H} :> = \frac{\hbar}{2} \int_{R^3} d^3x \, \ell_P^{-4}$$

- Comparison with cosmological term

$$H_{\text{cosmo}} = \frac{\Lambda}{G} \int_{R^3} d^3x \sqrt{|\det(g)|} \quad \Rightarrow \quad \Lambda \ell_P^2 \approx 1$$

- Worst prediction in history of physics:
  Experimentally: $\Lambda \ell_P^2 \approx 10^{-120}$.

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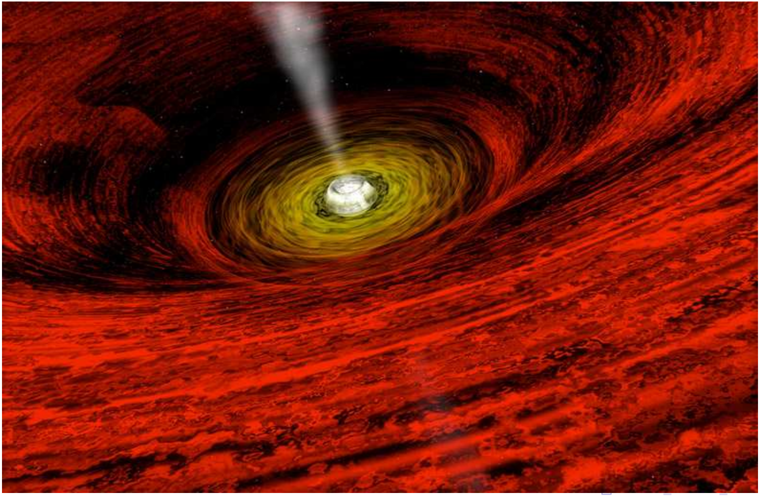
Astrophysical Puzzles: Black Holes



Entropy of Black Holes

- Class. GR (Penrose & Hawking): $\delta\text{Ar}(H) \geq 0 \Rightarrow S \propto \text{Ar}(H)$ (cf. 2nd law).
- QFT on CST (Hawking – effect): Black holes = black radiators $kT = \hbar\omega \approx \hbar c/r$.
- Entropy (Bekenstein): For Schwarzschild solution $r = 2Gm/c^2$ with $S = E/(kT) = mc^2/(kT)$

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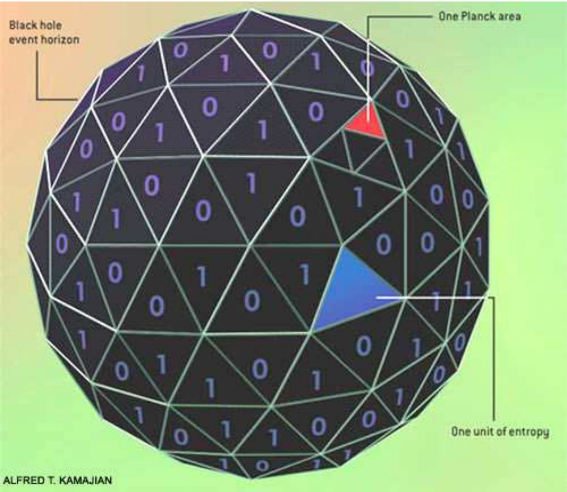
Microscopical Explanation of BH – Entropy? ['t Hooft, Susskind 92]



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- Microscopical explanation of BH – entropy?
- Hawking effect correct? Due to blue shift

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- Information paradox: Pure states $\to$ mixed states, unitarity?
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Principle of Background Independence

- It is widely believed that only a full fledged quantum theory of gravity can answer these fundamental questions.
- For more than 70 years physicists are looking for a unified theory of general relativity and quantum mechanics – so far w/o success.
- Why?

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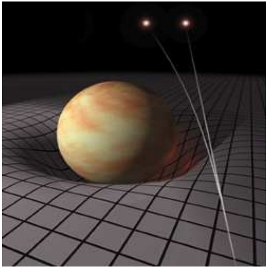

- Einstein's equations

$$R_{\mu\nu}[g] - \frac{1}{2} R[g] \, g_{\mu\nu} = 8\pi G \, T_{\mu\nu}[g]$$

- Background independence: Geometry g not prescribed but dynamically determined by matter energy density T.

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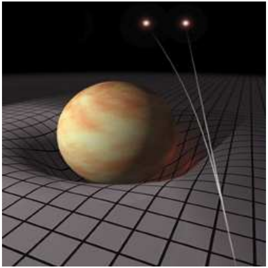

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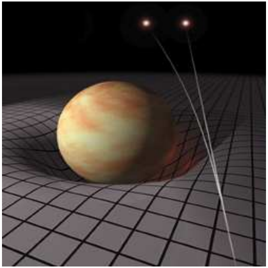

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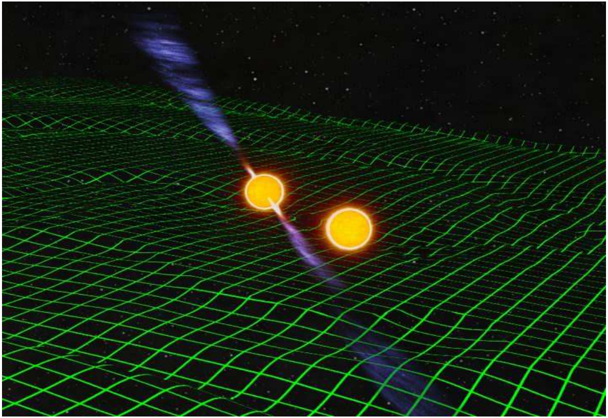
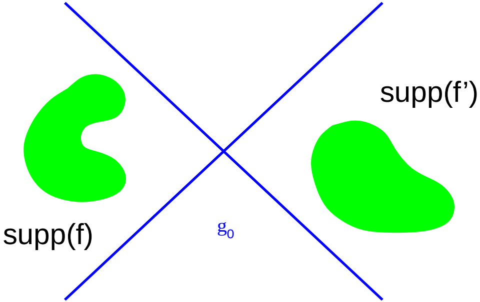
Background independence and backreaction (gravitational waves)



- Usual QFT (on CST) background dependent



- E.g. via causality axiom:
  If supp(f), supp(f') spacelike separated wrt g_0 then

$$[\hat{\phi}(f), \hat{\phi}(f')] = 0, \quad \hat{\phi}(f) := \int_M d^4x \;\; f(x) \;\; \hat{\phi}(x)$$

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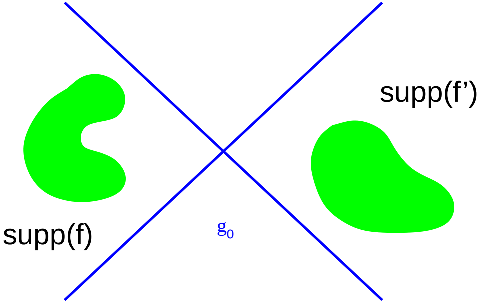

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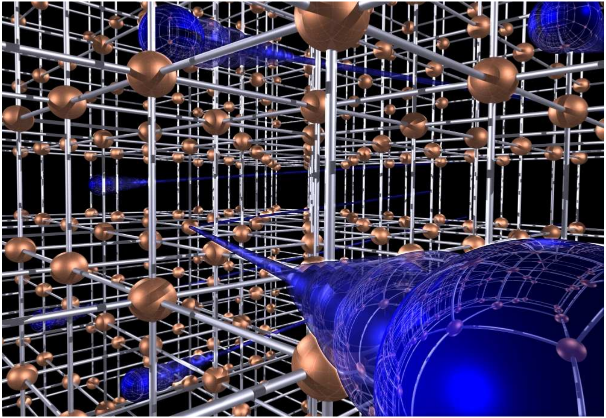
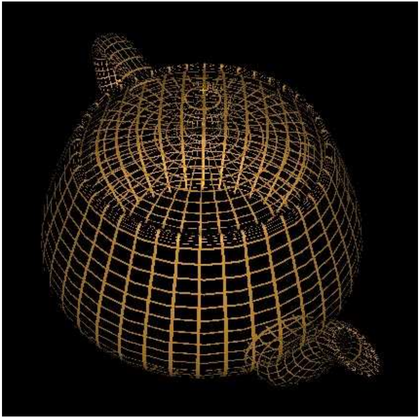
Ordinary QFT: matter propagates on rigid spacetime



BI QFT: Matter can only exist where geometry is excited



- The structure crucial for ordinary QFT

$$\underset{\text{Background}}{g_0} \Rightarrow \underset{\text{Light Cone}}{(x-y)^2 < 0} \Rightarrow \underset{\text{Algebra}}{\mathfrak{A}}$$

- collapses when g_0 not available.
- ignores gravitational backreaction.
- invalid approx. in extreme cosm. & astrophys. situat.
- Perturbative approach

$$\underset{\text{Total Metric}}{\overset{\uparrow}{g}} = \underset{\text{Background}}{\overset{\uparrow}{g_0}} + \underset{\text{Perturbation (Graviton)}}{\overset{\uparrow}{h}}$$

violates BI, inacceptable due to non – renormalisability, merely effect. graviton QFT over g_0.

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- collapses when g_0 not available.
- ignores gravitational backreaction.
- invalid approx. in extreme cosm. & astrophys. situat.
- Perturbative approach

$$\underset{\text{Total Metric}}{\underset{\uparrow}{g}} \quad = \quad \underset{\text{Background}}{\underset{\uparrow}{g_0}} \quad + \quad \underset{\text{Perturbation (Graviton)}}{\underset{\uparrow}{h}}$$

violates BI, inacceptable due to non – renormalisability, merely effect. graviton QFT over g_0.

- We need the

$$\widehat{R}_{\mu\nu} \; - \; \frac{1}{2}\,\widehat{R}\,\widehat{g}_{\mu\nu} \; = \; 8\pi G\,\widehat{T}_{\mu\nu}(\widehat{g})$$

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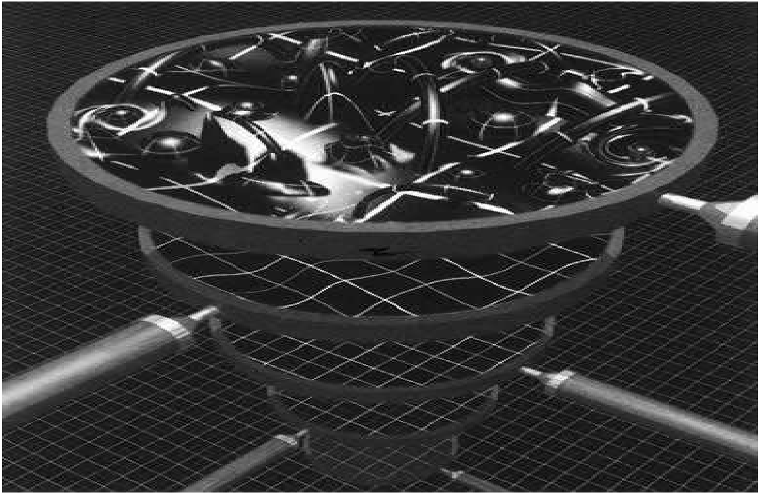
Principle of Background Independence



Classical Formulation

Analogy with SU(2) – Yang – Mills Theory

- Canonical variables: $(A_a^j, \quad E_j^a)$
- Poisson brackets: $\{A_a^j(x), E_k^b(y)\} = g^2 \ \delta_a^b \delta_k^j \delta(x, y)$
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- Hamilton – functional (on Minkowski space $M \cong R \times R^3$)

$$H = \frac{1}{2g^2} \int_{R^3} d^3x \ \delta_{ab} \mathrm{Tr}[E^a E^b + B^a B^b]$$

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- Gauge invariant (Dirac) observables

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Observables & Physical Hamiltonian

Relational Ansatz [Rovelli 92 – 03], [Dittrich 04 – 06], [T.T. 04 – 06]

- Suitable scalar matter Φ: Brown – Kuchař deparam.

$$\mathbf{M} = 0 \quad \Leftrightarrow \quad \pi(x) + H(x) = 0$$

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Lattice – inspired canon. quantisation

- Magnet. dof.: Holonomy (Wilson – Loop)

$$A(e) = \mathcal{P} \ \exp(\int_e A)$$

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$$E_j(S) = \int_S \epsilon_{abc} \, E_j^a \, dx^b \wedge dx^c$$

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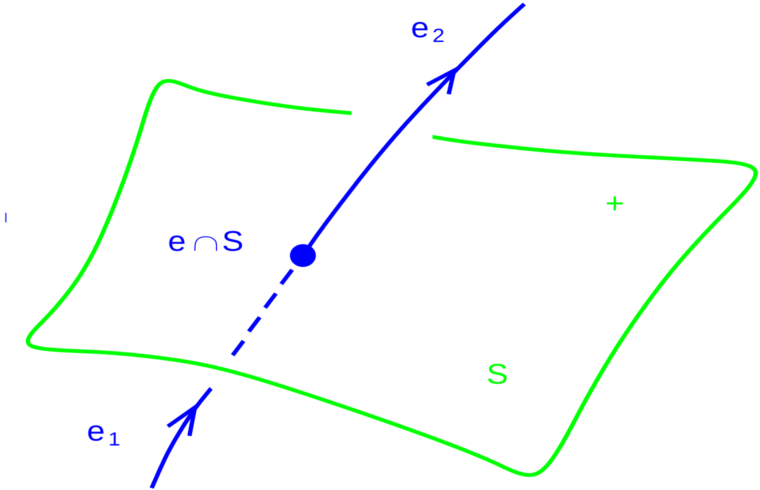
e 2
e ∩S
+
S
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Hilbert space representation

[Ashtekar, Isham, Lewandowski 92-93], [Lewandowski, Okolow, Sahlmann, T.T. 03-05], [Fleischhack 04]

- Diff(σ) inv. rep. of hol. – flux algebra $\mathfrak{A}$ unique.
- wave functions

$$\psi(A) = \psi_\gamma(A(e_1), .., A(e_N)), \quad \psi_\gamma : \; SU(2)^N \to C$$

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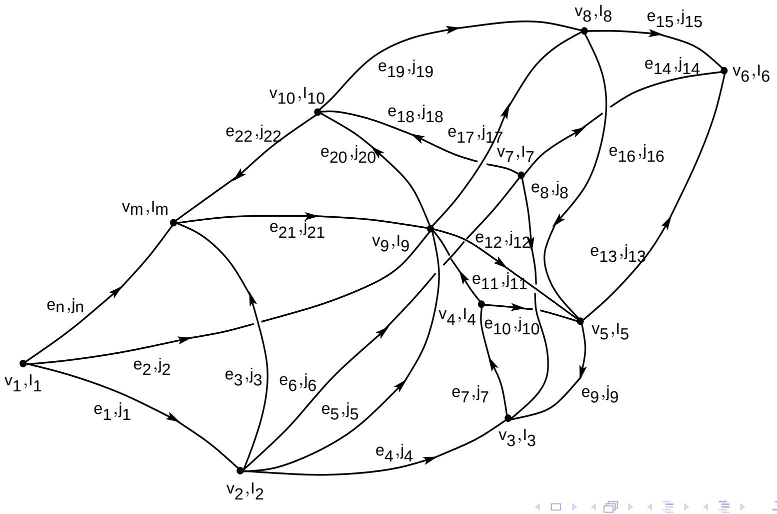
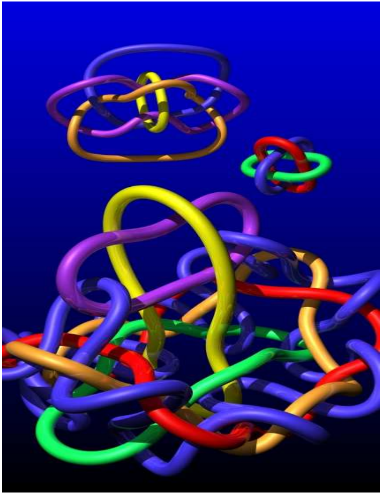
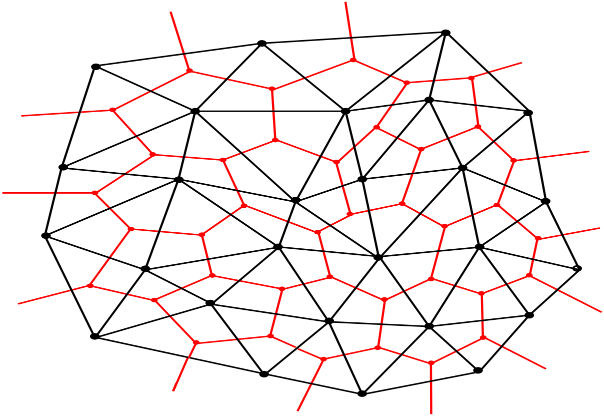
Spin Network Basis $T_{\gamma,j,l} \sim H_j$ Hermite Polynomials



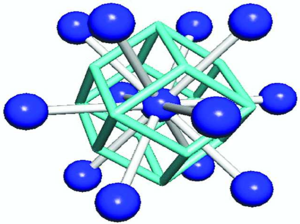
Colour Coding of Spin Quantum Numbers

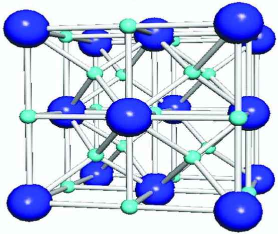


Dual Description in 2D (Dirichlet Voronoi)



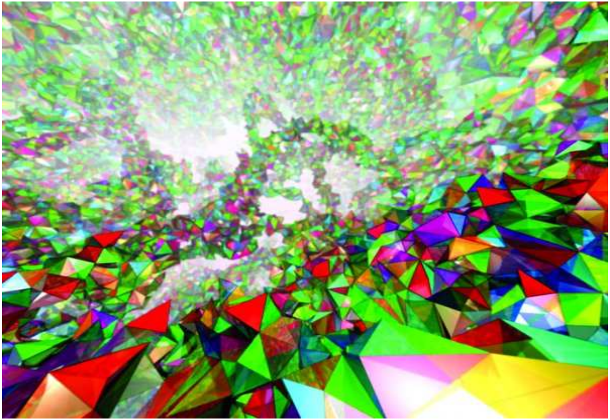
Dual Description in 3D


(a)

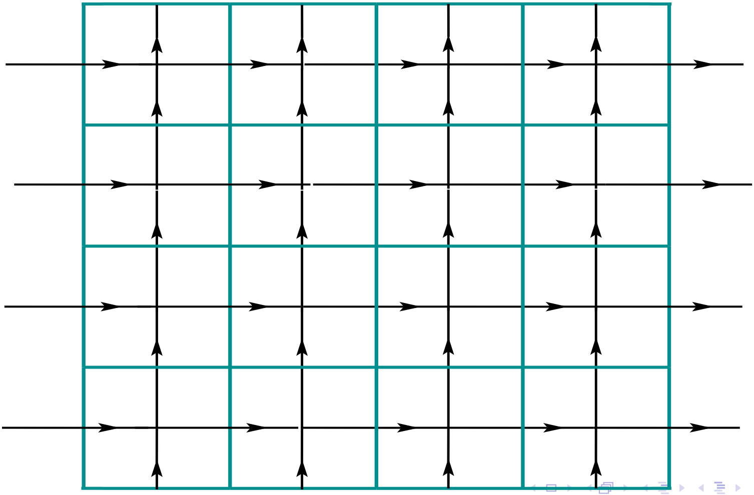

(b)

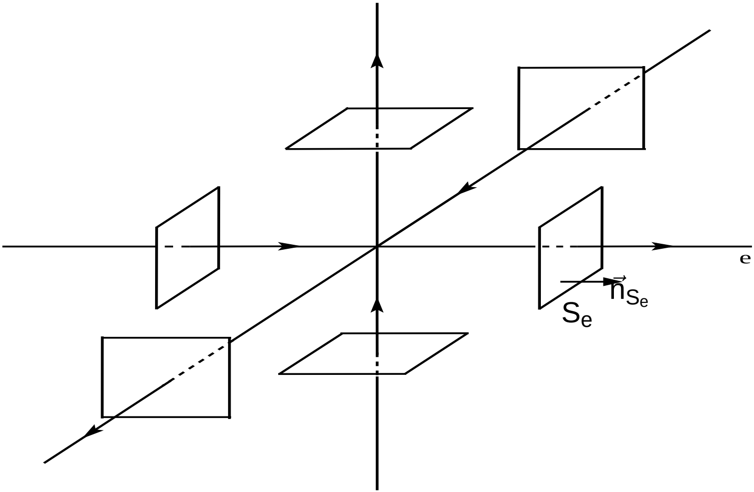
Special Case: 4 – valent Graph (diamond lattice)

Dual Diamond: Coloured, simplicial cell complex (Triangulation)



Consequences of BI




e
$\vec{n}_{S_e}$
S_e

Difference: BD & BI theories on cubic lattice

[T.T. 96 – 05, Giesel & T.T. 05]

- Yang – Mills on (R^4, η)

$$\mathrm{H} = \frac{\hbar}{2\,\mathrm{g}^2\,\epsilon} \sum_{\mathrm{v}\in\mathrm{V}(\gamma)} \sum_{\mathrm{a}=1}^{3} \mathrm{Tr}\left(\mathrm{E}(\mathrm{S}_\mathrm{v}^\mathrm{a})^2 + [2 - \mathrm{A}(\alpha_\mathrm{v}^\mathrm{a}) - \mathrm{A}(\alpha_\mathrm{v}^\mathrm{a})^{-1}]\right)$$

- Gravity on $\mathrm{R} \times \sigma$

$$\mathbf{M} = \frac{\hbar}{\ell_\mathrm{P}^4} \sum_{\mathrm{v}\in\mathrm{V}(\gamma)} \sum_{\mathrm{j}=0}^{3} \left|\sum_{\mathrm{a}=1}^{3} \mathrm{Tr}\left(\tau_\mathrm{j}\,\mathrm{A}(\alpha_\mathrm{v}^\mathrm{a})\,\mathrm{A}(\mathrm{e}_\mathrm{v}^\mathrm{a})\,[\mathrm{A}(\mathrm{e}_\mathrm{v}^\mathrm{a})^{-1}, \mathrm{V}_\mathrm{v}^{1/2}]\right)\right|^2$$

- Volume operator

$$\mathrm{V}_\mathrm{v} = \sqrt{\left|\epsilon_{\mathrm{abc}} \mathrm{Tr}\left(\mathrm{E}(\mathrm{S}_\mathrm{v}^\mathrm{a})\,\mathrm{E}(\mathrm{S}_\mathrm{v}^\mathrm{b})\,\mathrm{E}(\mathrm{S}_\mathrm{v}^\mathrm{c})\right)\right|}$$

- Lattice spacing ϵ disappears, automat. UV finite.

Difference: BD & BI theories on cubic lattice

[T.T. 96 – 05, Giesel & T.T. 05]

- Yang – Mills on (R^4, η)

$$H = \frac{\hbar}{2\, g^2\, \epsilon} \sum_{v \in V(\gamma)} \sum_{a=1}^{3} \mathrm{Tr}\left(E(S_v^a)^2 + [2 - A(\alpha_v^a) - A(\alpha_v^a)^{-1}]\right)$$

- Gravity on $R \times \sigma$

$$\mathbf{M} = \frac{\hbar}{\ell_P^4} \sum_{v \in V(\gamma)} \sum_{j=0}^{3} \left| \sum_{a=1}^{3} \mathrm{Tr}\left(\tau_j\, A(\alpha_v^a)\, A(e_v^a)\, [A(e_v^a)^{-1}, V_v^{1/2}]\right) \right|^2$$

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Physical Hilbert Space [Dittrich, T.T. 04]

- Kinemat. HS decomposes into separable **M** inv. sectors

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- Display $\mathcal{H}_\theta$ as direct integral (Fourier decomp.)

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Semiclassical Limit

Non – pert. approach, no pert. theory, rather:

[T.T. 00], [T.T., Winkler 00 – 02]

- Construct minimal uncertainty states f. $\mathfrak{A}$
- F. each point (A_0, E_0) obtain $\psi_{(A_0,E_0)}$ s.t.

$$< \psi_{A_0,E_0}, \widehat{A(e)}\psi_{A_0,E_0} >= A_0(e), \quad < \psi_{A_0,E_0}, \widehat{E_j(S)}\psi_{A_0,E_0} >= E_{j0}(S)$$

- and

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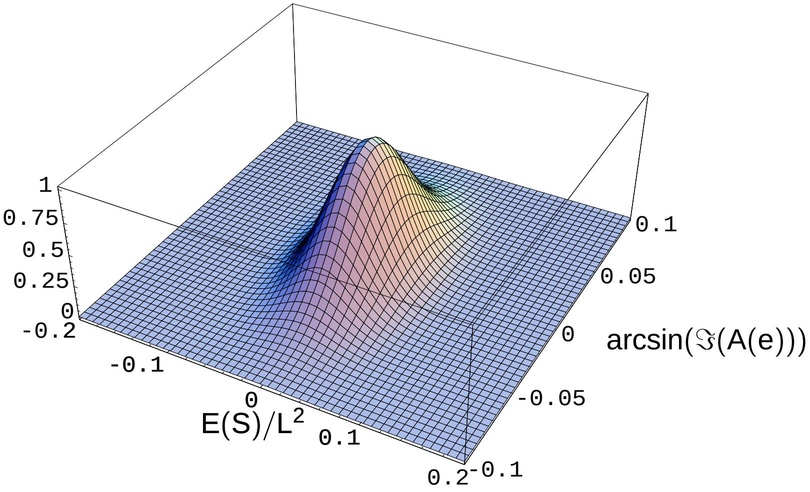
Coherent States



Semiclassical Limit of Quantum Dynamics

- **Theorem** [Giesel & T.T. 06]
  For any (A_0, E_0)
  1. Expectation Value
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- **Corollary**
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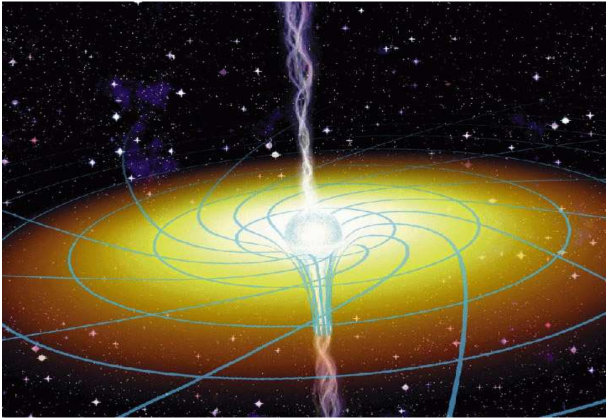
Singularity avoidance

Singularities: Divergence of Riemann Tensor $R^{(4)}_{\mu\nu\rho\sigma}$



Idea: [Brunnemann, T.T. 05]

- Gauss – Codacci

$$\int_M d^4X \sqrt{|\det(g)|}\, R^{(4)} = \int_R dt \int_\sigma d^3x\, N \frac{\mathrm{Tr}\left(F_{ab}E^aE^b\right)}{\sqrt{|\det(E)|}} =: \int_R dt\, R(N)$$

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Result & Discussion:

- Expectation value huge but finite, curvature $\lesssim \ell_P^{-2} \approx 10^{66} \text{cm}^{-2}$.
- Curvature operator not bounded but EV bounded.
- Inhomogeneities & anisotropies taken into account (fluctuations).
- Similar calculation for Schwarzschild black hole since mathematically equivalent to homogenous Kantowski – Sachs model:
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Bekenstein – Hawking Entropy

Area – Operator

[Rovelli, Smolin 94]

- Class. Area functional for 2 – mfd. S

$$\mathrm{Ar}(S) = \int_S \sqrt{\mathrm{Tr}\left(\left[E^a \epsilon_{abc} dx^b \wedge dx^c\right]^2\right)}$$

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- Spectrum explicitly known, purely discrete. Eigenstates = spin – network – states $T_{\gamma,j,I}$
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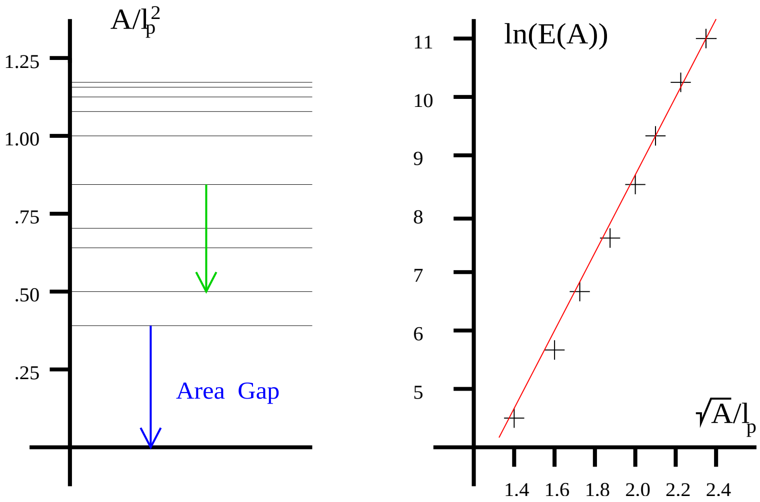
A/l_p^2
1.25
1.00
.75
.50
.25
Area Gap
ln(E(A))
11
10
9
8
7
6
5
√A/l_p
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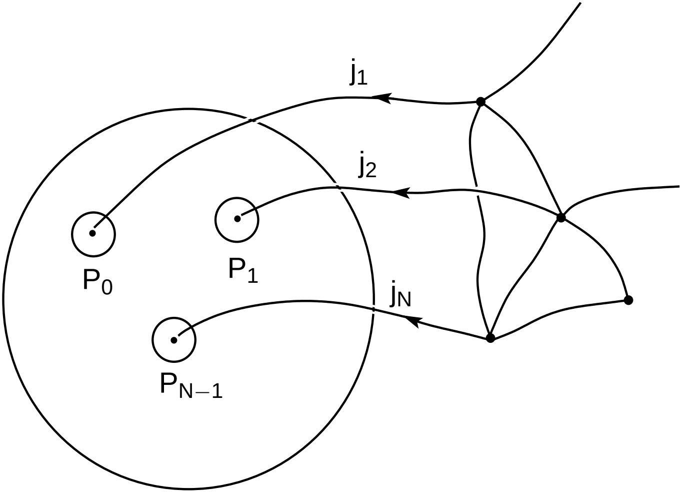
j1
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Entropy – Calculation

[Krasnov 95]

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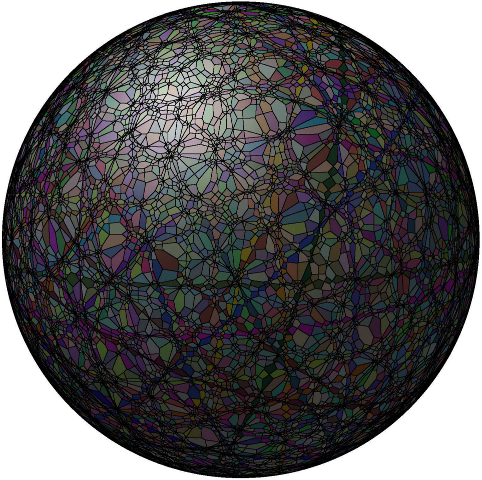
Warning: All spins contribute, no bit picture!



Dark Energy

[Sahlmann, T.T. 02]

- $$H_{EKG} = \frac{1}{2} \int_\sigma d^3x \; [\pi^2 + E^a_j E^b_j \phi_{,a} \phi_{,b}] \; [\sqrt{|\det(E)|}]^{-1}$$
- BD Q'ion: $E = E_0$ external field. Annihilator on $\mathcal{H}_{matter}$:
$$\hat{a} = 2^{-1/2} \, [\sqrt[4]{-\Delta_0} \; \hat{\phi} - i \sqrt[4]{-\Delta_0}^{-1} \; \hat{\pi}]$$
- BI Q'ion: $E = \hat{E}$ operator. Annihilator on $\mathcal{H}_{geo} \otimes \mathcal{H}_{matter}$:
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- Clear, simple conceptual setup, minimalistic.
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