

Distribuce (zobecněná funkce)

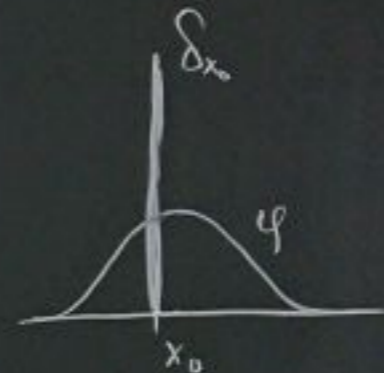
Def. spojitý lineární funkcionál
na prostoru testovacích funkcí

$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x) \varphi(x) dx$$

" $T[\varphi]$

$$T[\varphi + \omega \psi] = T[\varphi] + \omega T[\psi]$$

$$\varphi_n \rightarrow \varphi \quad T[\varphi_n] \rightarrow T[\varphi]$$



$$\delta_{x_0}(x) \quad \delta_{x_0}$$

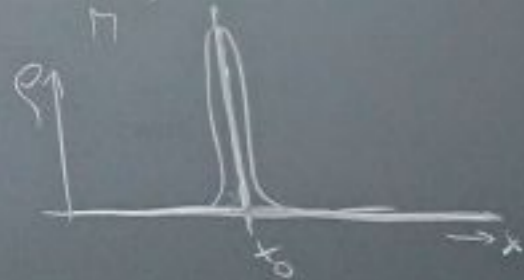
$$\langle \delta_{x_0}, \varphi \rangle = \varphi(x_0)$$

$$\int \delta_{x_0}(x) \varphi(x) dx = \varphi(x_0) \int \delta_{x_0}(x) \frac{\varphi(x)}{\varphi(x_0)} dx$$

$$x_0 \cdot Q$$

$$x \neq x_0 \quad \varrho(x) = 0$$

$$\int_V \varrho dV = Q$$



\mathcal{D} prostor test fce
- hladké fce s kompaktní podporou

$$\varphi_n \xrightarrow{\mathcal{D}} \varphi$$

\exists komp. $K \quad \forall n \text{ supp } \varphi_n \subset K$

$$\varphi_n^{[k]} \xrightarrow{K} \varphi^{[k]}$$

\mathcal{D}' prostor distribucí

$$\mathcal{D} \subset \mathcal{D}'$$

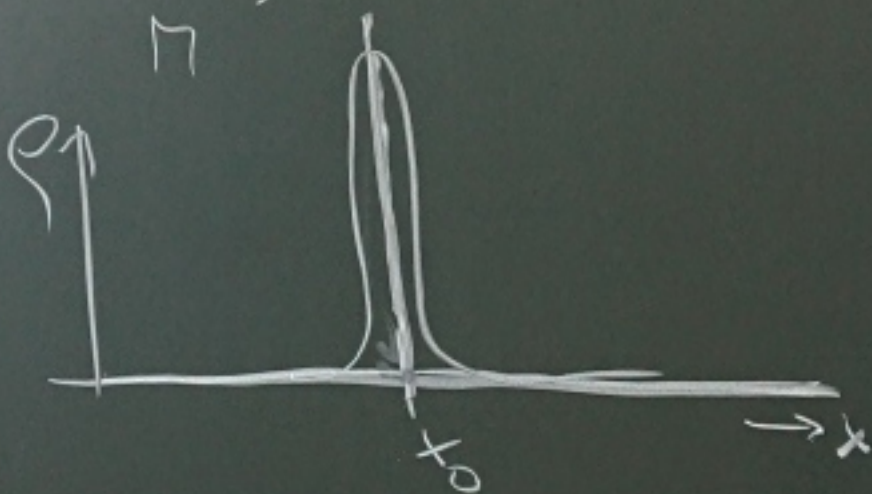
$$\langle T, \varphi \rangle$$

~~$$T[\varphi]$$~~

$$x_0 \cdot Q$$

$$x \neq x_0 \quad \rho(x) = 0$$

$$\int \rho \, dV = Q$$

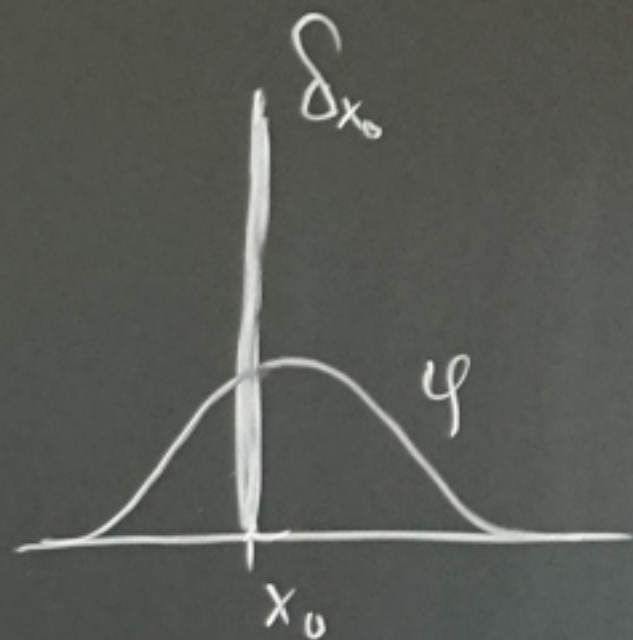
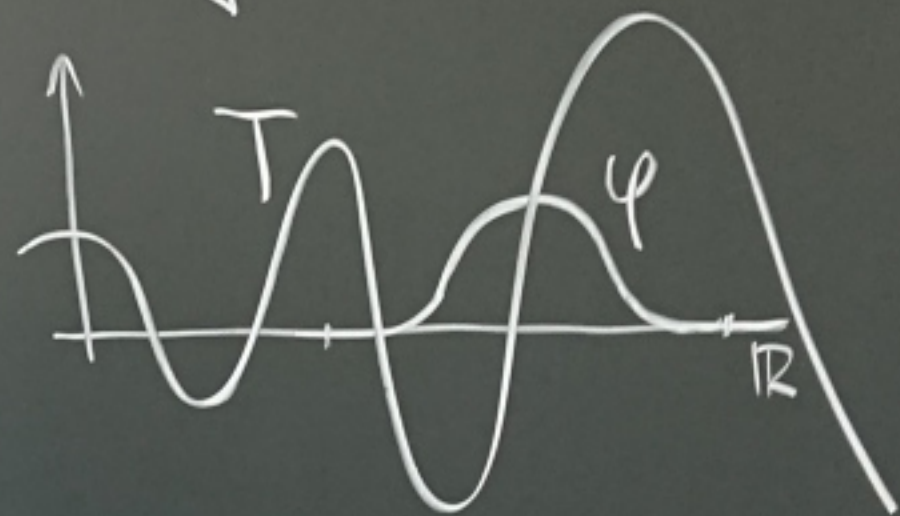


Distribuce (zobecněná funkce)

Df. spojitý lineární funkcionál
na prostoru testovacích funkcí

$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x) \varphi(x) dx$$

= $T[\varphi]$



$$T[\varphi + \omega \varphi] = T[\varphi] + \omega T[\varphi]$$

$$\varphi_n \rightarrow \varphi$$
$$T[\varphi_n] \rightarrow T[\varphi]$$

$$\delta_{x_0}(x) \quad \delta_{x_0}$$

$$\langle \delta_{x_0}, \varphi \rangle = \varphi(x_0)$$

$$\int \delta_{x_0}(x) \varphi(x) dx = \varphi(x_0) \int \delta_{x_0}(x) \frac{\varphi(x)}{\varphi(x_0)} dx$$

\mathcal{D} prostor test. fun.

- hladké fun. s komp. nosičem

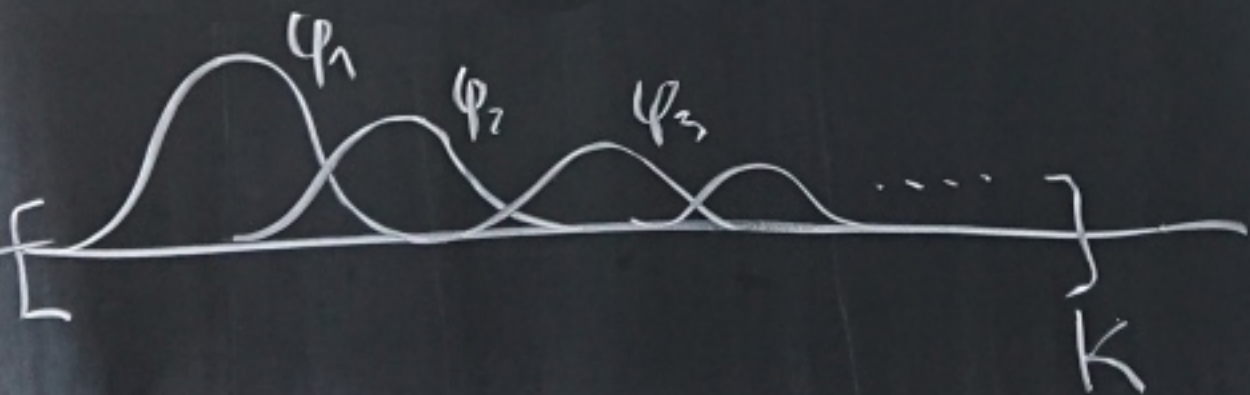
$$\varphi_n \xrightarrow{\mathcal{D}} \varphi$$

\exists komp. $K \quad \forall n \text{ supp } \varphi_n \subset K$

$$\varphi_n^{[k]} \xrightarrow{K} \varphi^{[k]}$$

\mathcal{D}' prostor distribucí

$$\mathcal{D} \subset \mathcal{D}'$$



\mathcal{D} prostor test. fun.

- lokalni fun. s kompak. nosicem

$$\varphi_n \xrightarrow{\mathcal{D}} \varphi$$

\exists komp. $K \quad \forall n \quad \text{supp } \varphi_n \subset K$

$$\varphi_n^{[2]} \xrightarrow{K} \varphi^{[2]}$$

\mathcal{D}' prostor distribucij

$$\mathcal{D} \subset \mathcal{D}'$$

$\langle T, \varphi \rangle$

~~$T(\varphi)$~~

Distribuce a operace na nich

\mathcal{D}' topol. dual \mathcal{D}

$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x) \varphi(x) dx$$

$$T_1 + T_2 \quad \langle T_1 + T_2, \varphi \rangle = \langle T_1, \varphi \rangle + \langle T_2, \varphi \rangle$$

$$rT \quad \langle rT, \varphi \rangle = r \langle T, \varphi \rangle$$

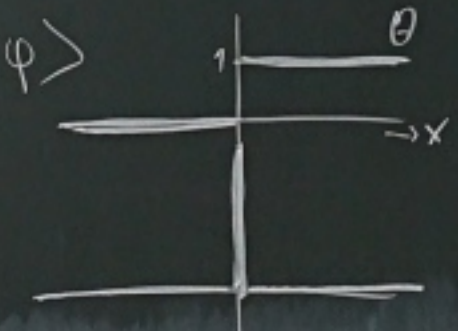
$$fT \quad \langle fT, \varphi \rangle = \langle T, f\varphi \rangle$$

kladné fee $\int fT \varphi dx = \int T(f\varphi) dx$

$$\int_{\mathbb{R}} T' \varphi dx = \int_{\mathbb{R}} (T\varphi)' dx - \int_{\mathbb{R}} T \varphi' dx \quad \Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

$$\langle T', \varphi \rangle \stackrel{\text{def}}{=} - \langle T, \varphi' \rangle$$

$$\begin{aligned} \langle (fT)', \varphi \rangle &= - \langle fT, \varphi' \rangle \\ &= - \langle T, f\varphi' \rangle + \langle T, f'\varphi \rangle \\ &= \langle T', f\varphi \rangle + \langle f'T, \varphi \rangle \\ &= \langle \underline{fT' + f'T}, \varphi \rangle \end{aligned}$$



$$\begin{aligned} \langle \Theta, \varphi \rangle &= \int_{\mathbb{R}} \Theta(x) \varphi(x) dx = \int_{\mathbb{R}^+} \varphi(x) dx \\ \langle \Theta', \varphi \rangle &= - \langle \Theta, \varphi' \rangle \\ &= - \int_0^{\infty} \varphi'(x) dx = - [\varphi]_0^{\infty} = \varphi(0) \\ &= \langle \delta, \varphi \rangle \\ \Theta' &= \delta \end{aligned}$$

\mathcal{D} prostor test fee
- kladné fee a zamy nozice

$$\varphi_n \xrightarrow{\mathcal{D}} \varphi$$

\exists zam. $K \quad \forall m \text{ supp } \varphi_n \subset K$

$$\varphi_n^{[k]} \xrightarrow{K} \varphi^{[k]}$$

\mathcal{D}' prostor distribucí
 $\mathcal{D} \subset \mathcal{D}'$

Distribuce a operace na nich

\mathcal{D}' topol. dual k \mathcal{D}

$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x) \varphi(x) dx$$

$$T_1 + T_2 \quad \langle T_1 + T_2, \varphi \rangle = \langle T_1, \varphi \rangle + \langle T_2, \varphi \rangle$$

$$rT \quad \langle rT, \varphi \rangle = r \langle T, \varphi \rangle$$

$$fT \quad \langle fT, \varphi \rangle = \langle T, f\varphi \rangle$$

↑
kladně fce

$$\int fT \varphi dx = \int T(f\varphi) dx$$

$$\int_{\mathbb{R}} T' \varphi \, dx = \int_{\mathbb{R}} (T\varphi)' \, dx - \int_{\mathbb{R}} T \varphi' \, dx$$

$0 \quad [T\varphi]_{-\infty}^{+\infty}$

$$\langle T', \varphi \rangle \stackrel{\text{def}}{=} - \langle T, \varphi' \rangle$$

$$\langle \underline{(fT)'} , \varphi \rangle = - \langle fT, \varphi' \rangle$$

$$= - \langle T, f\varphi' \rangle = - \langle T, (f\varphi)' \rangle + \langle T, f'\varphi \rangle$$

$$= \langle T', f\varphi \rangle + \langle f'T, \varphi \rangle$$

$$= \underline{\underline{\langle fT' + f'T, \varphi \rangle}}$$

$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

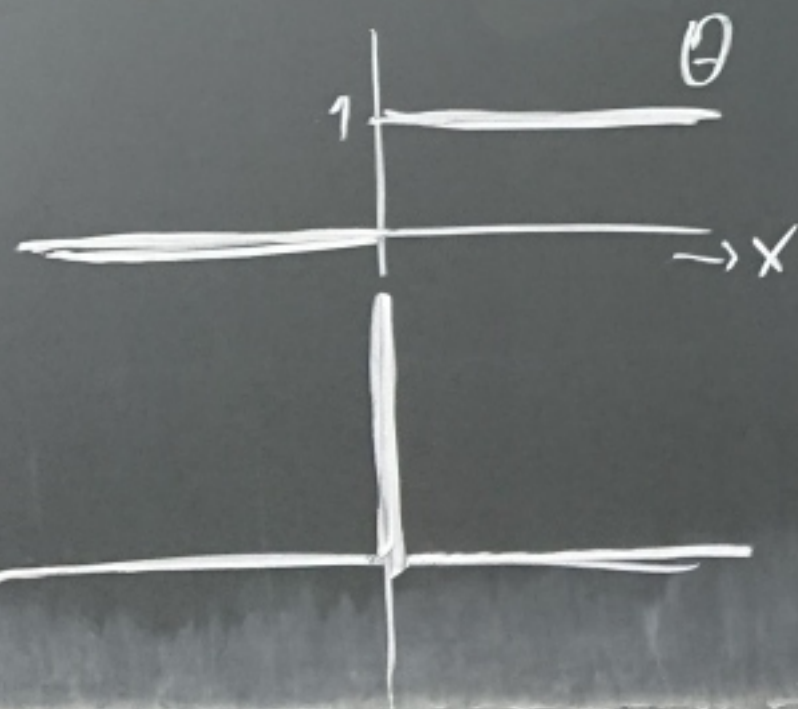
$$\begin{aligned} \langle \Theta, \varphi \rangle &= \int_{\mathbb{R}} \Theta(x) \varphi(x) dx = \\ &= \int_{\mathbb{R}^+} \varphi(x) dx \end{aligned}$$

$$\begin{aligned} \langle \Theta', \varphi \rangle &= - \langle \Theta, \varphi' \rangle \\ &= - \int_0^{\infty} \varphi'(x) dx = \end{aligned}$$

$$= - [\varphi]_0^{\infty} = \varphi(0)$$

$$= \langle \delta, \varphi \rangle$$

$$\Theta' = \delta$$



$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x) \varphi(x) dx$$

\mathcal{D} hladké fce s kompaktní nosič → \mathcal{D}' mězené distribuce

\mathcal{E} hladké fce → \mathcal{E}' distrib s kompaktní nosičem

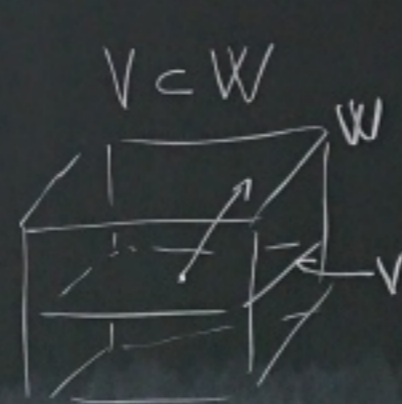
\mathcal{S} rychle kles. fce → \mathcal{S}' temperované distrib

\mathcal{L}^2 kvadr. integ. fce → $\mathcal{L}^{2'} = \mathcal{L}^2$

hustota

$\mathcal{D} \subset \mathcal{S}$

$\mathcal{S}' \subset \mathcal{D}'$



$V^* \subset W^*$

\mathcal{D} prostor test fce

- hladké fce s kompaktní nosičem

$\varphi_n \xrightarrow{\mathcal{D}} \varphi$

\exists kompaktní K $\forall n$ $\text{supp } \varphi_n \subset K$

$\varphi_n^{[k]} \xrightarrow{K} \varphi^{[k]}$

\mathcal{D}' prostor distribucí

$\mathcal{D} \subset \mathcal{D}'$

\mathcal{D} hladké fce s řepnou nosičem $\rightarrow \mathcal{D}'$ mezní distribuce

\mathcal{E} hladké fce $\rightarrow \mathcal{E}'$ distrib s řepnou nosičem

\mathcal{S} rychle kles. fce $\rightarrow \mathcal{S}'$ temperované distrib

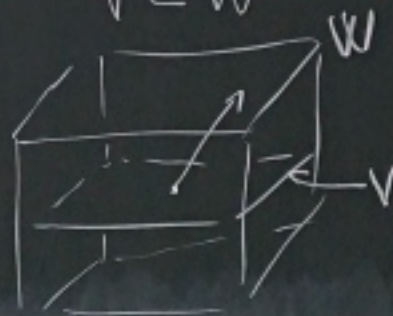
\mathcal{L}^2 kvadr. integro. fce $\rightarrow \mathcal{L}^{2'} = \mathcal{L}^2$

hustota
 $\mathcal{D} \subset \mathcal{S}$

$\mathcal{S}' \subset \mathcal{D}'$

$V \subset W$

$V^* \subset W^*$



Derivate distribuci

$$\langle fT, \varphi \rangle = \langle T, f\varphi \rangle$$

$$\langle T', \varphi \rangle = -\langle T, \varphi' \rangle$$

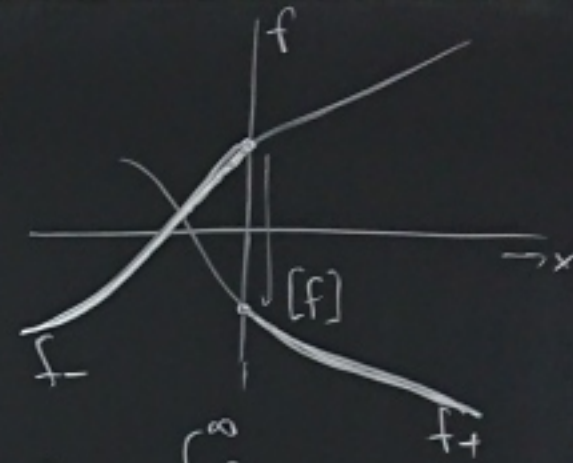
$$\langle T^{[k]}, \varphi \rangle = (-1)^k \langle T, \varphi^{[k]} \rangle$$

$$\theta' = \delta$$

$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x) \varphi(x) dx$$

$$f = \begin{cases} f_- & x < 0 \\ f_+ & x > 0 \end{cases} \quad [f] = f_+(0) - f_-(0)$$

$$f^{(k)} = \begin{cases} f_-^{(k)} & x < 0 \\ f_+^{(k)} & x > 0 \end{cases} \quad \boxed{f' = f^{(1)} + [f] \delta}$$



$$\langle f', \varphi \rangle = -\langle f, \varphi' \rangle = -\int_{\mathbb{R}} f(x) \varphi'(x) dx = -\int_{-\infty}^0 f_-(x) \varphi'(x) dx - \int_0^{\infty} f_+(x) \varphi'(x) dx$$

$$= -\int_{-\infty}^0 (f_- \varphi)' dx + \int_{-\infty}^0 f_- \varphi dx - \int_0^{\infty} (f_+ \varphi)' dx + \int_0^{\infty} f_+ \varphi dx$$

$$= \langle f^{(1)}, \varphi \rangle - [f_- \varphi]_{-\infty}^0 - [f_+ \varphi]_0^{\infty} = \langle f^{(1)}, \varphi \rangle + (-f_-(0) + f_+(0)) \varphi(0)$$

$$= \langle f^{(1)}, \varphi \rangle + [f] \langle \delta, \varphi \rangle = \langle f^{(1)} + [f] \delta, \varphi \rangle$$

$$f' = f^{(1)} + [f] \delta$$

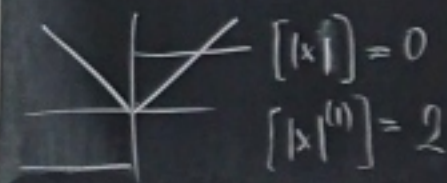
$$f'' = f^{(2)} + [f^{(1)}] \delta + [f] \delta'$$

$$\dots$$
$$f^{[k]} = f^{(k)} + [f^{(k-1)}] \delta + [f^{(k-2)}] \delta' + \dots + [f] \delta^{[k-1]}$$

$$\langle \delta', \varphi \rangle = -\langle \delta, \varphi' \rangle = -\varphi'(0)$$

$$\langle \delta'', \varphi \rangle = -\langle \delta', \varphi' \rangle = \langle \delta, \varphi'' \rangle = \varphi''(0)$$

$$|x|' = \text{sign } x \quad |x|'' = 0 + 2\delta$$



Derivace distribuci

$$\langle fT, \varphi \rangle = \langle T, f\varphi \rangle$$

$$\langle T', \varphi \rangle = -\langle T, \varphi' \rangle$$

$$\langle T^{[k]}, \varphi \rangle = (-1)^k \langle T, \varphi^{[k]} \rangle$$

$$\theta' = \delta$$

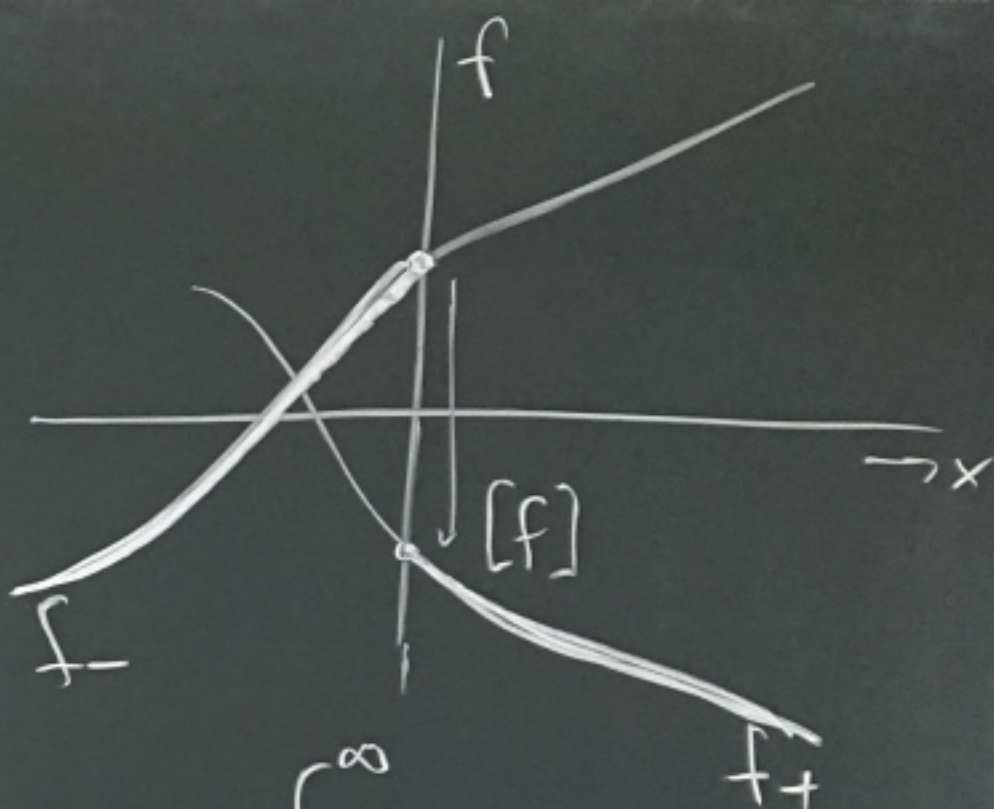
$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x) \varphi(x) dx$$

$$f = \begin{cases} f_- & x < 0 \\ f_+ & x > 0 \end{cases}$$

$$[f] = f_+(0) - f_-(0)$$

$$f^{(1)} = \begin{cases} f_-^{(1)} & x < 0 \\ f_+^{(1)} & x > 0 \end{cases}$$

$$f' = f^{(1)} + [f] \delta$$



$$\begin{aligned} \langle f', \varphi \rangle &= -\langle f, \varphi' \rangle = -\int_{\mathbb{R}} f(x) \varphi'(x) dx = -\int_{-\infty}^0 f_-(x) \varphi'(x) dx - \int_0^{\infty} f_+(x) \varphi'(x) dx \\ &= \int_{-\infty}^0 (f_- \varphi)' dx + \int_{-\infty}^0 f_- \varphi dx - \int_0^{\infty} (f_+ \varphi)' dx + \int_0^{\infty} f_+ \varphi dx \end{aligned}$$

$$= \langle f^{(1)}, \varphi \rangle - [f \varphi]_{-\infty}^0 - [f_+ \varphi]_0^{\infty} = \langle f^{(1)}, \varphi \rangle + (-f_-(0) + f_+(0)) \varphi(0)$$

$$= \langle f^{(1)}, \varphi \rangle + [f] \langle \delta, \varphi \rangle = \langle f^{(1)} + [f] \delta, \varphi \rangle$$

$$f' = f^{(1)} + [f] \delta$$

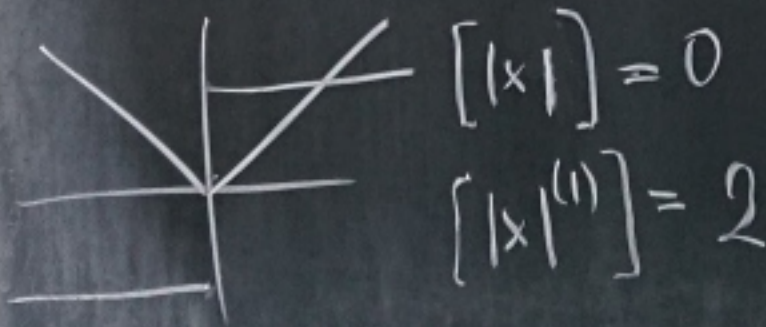
$$f'' = f^{(2)} + [f^{(1)}] \delta + [f] \delta'$$

$$\vdots$$
$$f^{[k]} = f^{(k)} + [f^{(k-1)}] \delta + [f^{(k-2)}] \delta' + \dots + [f] \delta^{(k-1)}$$

$$\langle \delta', \varphi \rangle = -\langle \delta, \varphi' \rangle = -\varphi'(0)$$

$$\langle \delta'', \varphi \rangle = -\langle \delta', \varphi' \rangle = \langle \delta, \varphi'' \rangle = \varphi''(0)$$

$$|x|' = \text{sign } x \quad |x|'' = 0 + 2\delta$$



Substituce v distribuci

$$y = y(x)$$

$$x = x(y)$$

$$x = x(y(x))$$

$$\begin{array}{ccc} f(x) & f(y(x)) & f \circ y \\ T & T \circ y & \end{array}$$

$$\delta_{x_0}(x) = \delta(x - x_0)$$

$$\langle T, \varphi \rangle = \int T(x) \varphi(x) dx$$

$$\langle T \circ y, \varphi \rangle = \int_{\mathbb{R}} T(y(x)) \varphi(x) dx$$

$y = y(x)$
 $x = x(y) \quad dx = \frac{dx}{dy} dy$

$$= \int_{\mathbb{R}} T(y) \varphi(x(y)) \left| \frac{dx}{dy}(y) \right| dy$$

$$= \langle T, \varphi \circ x \frac{dx}{dy} \rangle = \langle T, \frac{\varphi}{\left| \frac{dy}{dx} \right|} \circ x \rangle$$

$$\langle T \circ y, \varphi \rangle = \langle T, \frac{\varphi}{\left| \frac{dy}{dx} \right|} \circ x \rangle = \left\langle \frac{1}{\left| \frac{dy}{dx}(x_0) \right|} \delta_{x_0}, \varphi \right\rangle$$

$$\delta(x) \rightarrow (\delta \circ y)(x)$$

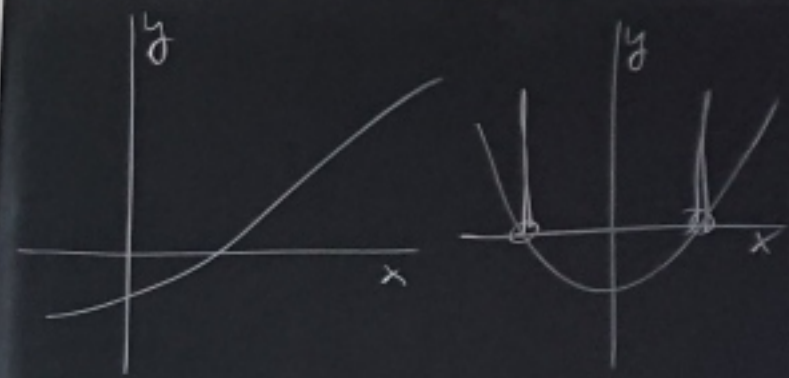
$$\langle \delta \circ y, \varphi \rangle = \left\langle \delta, \frac{\varphi}{\left| \frac{dy}{dx} \right|} \circ x \right\rangle$$

$$= \frac{\varphi}{\left| \frac{dy}{dx} \right|}(x_0) \delta(y(x))$$

$$= \frac{\varphi(x_0)}{\left| \frac{dy}{dx}(x_0) \right|} \delta(y(x))$$

$x_0 = x(0)$
 $y(x_0) = 0$

$$\delta \circ y = \frac{1}{\left| \frac{dy}{dx}(x_0) \right|} \delta_{x_0} \quad \delta(y(x)) = \frac{1}{\left| \frac{dy}{dx}(x_0) \right|} \delta_{x_0}(x)$$



$\delta(y)$

$$\delta(y(x)) = \sum_{x_0} \frac{1}{\left| \frac{dy}{dx}(x_0) \right|} \delta_{x_0}(x)$$

x_0 řešením $y(x_0) = 0$ $a > 0$

$$y(x) = x^2 - a^2 \quad x_0 = a \quad x_0 = -a$$

$$\delta(x^2 - a^2) = \frac{1}{2a} \delta(x - a) + \frac{1}{2a} \delta(x + a)$$

Substituce v distribuci

$$y = \gamma(x)$$

$$x = \alpha(y)$$

$$x = \alpha(\gamma(x))$$

$$f(x)$$

T

$$f(\gamma(x))$$

T \circ γ

$$f \circ \gamma$$

$$\langle T, \varphi \rangle = \int T(x) \varphi(x) dx$$

$$\langle T \circ \gamma, \varphi \rangle = \int_{\mathbb{R}} T(\gamma(x)) \varphi(x) dx$$

$$y = \gamma(x)$$

$$x = \alpha(y)$$

$$dx = \frac{dx}{dy} dy$$

$$= \int_{\mathbb{R}} T(y) \varphi(\alpha(y)) \left| \frac{dx}{dy}(y) \right| dy$$

$$= \langle T, \varphi \circ \alpha \frac{dx}{dy} \rangle = \langle T, \frac{\varphi}{\left| \frac{dx}{dy} \right|} \circ \alpha \rangle$$

$$\langle T \circ \gamma, \varphi \rangle = \langle T, \frac{\varphi}{\left| \frac{d\gamma}{dx} \right|} \circ \alpha \rangle$$

$$\delta(x) \rightarrow (\delta \circ \gamma)(x)$$

$$\langle \delta \circ \gamma, \varphi \rangle = \left\langle \delta, \frac{\varphi \circ \gamma}{\left| \frac{d\gamma}{dx} \right|} \right\rangle$$

$$= \frac{\varphi}{\left| \frac{d\gamma}{dx} \right|} (x(0))$$

$$= \frac{\varphi(x_0)}{\left| \frac{d\gamma}{dx}(x_0) \right|}$$

$$= \left\langle \frac{1}{\left| \frac{d\gamma}{dx}(x_0) \right|} \delta_{x_0}, \varphi \right\rangle$$

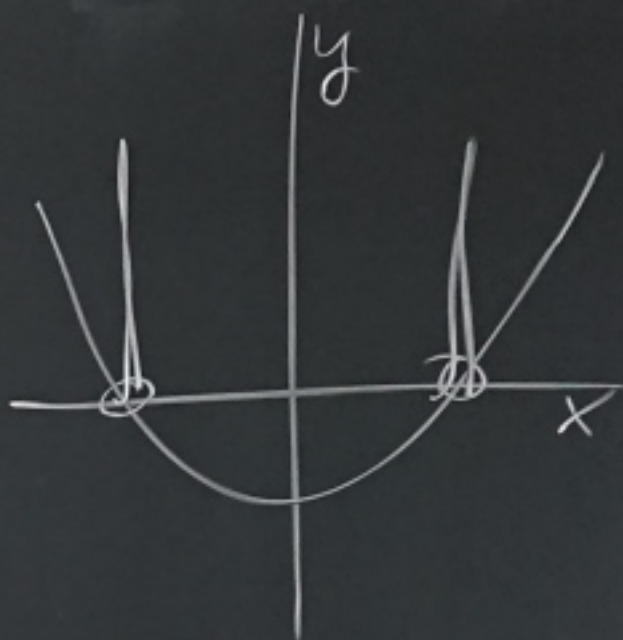
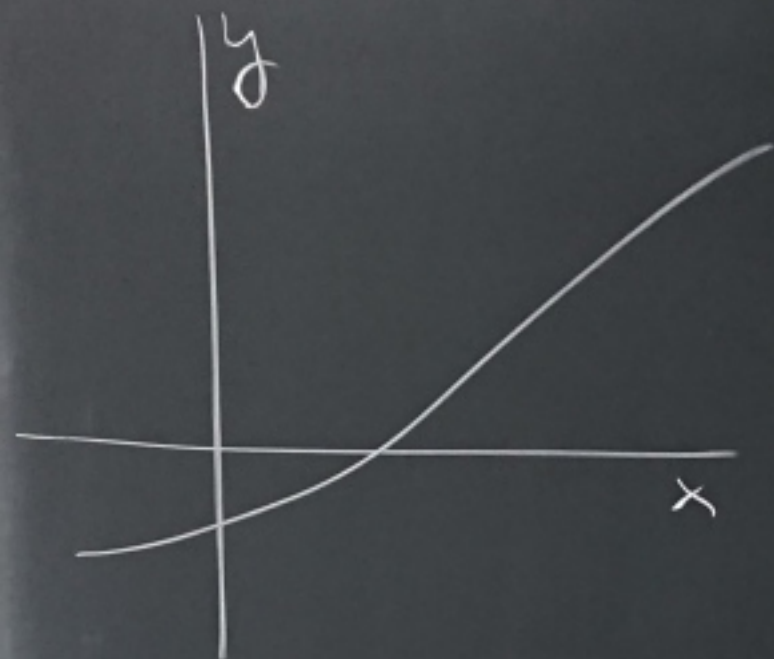
$$\delta \circ \gamma = \frac{1}{\left| \frac{d\gamma}{dx}(x_0) \right|} \delta_{x_0}$$

$$\delta(\gamma(x)) = \frac{1}{\left| \frac{d\gamma}{dx}(x_0) \right|} \delta_{x_0}(x)$$

$$\delta(\underbrace{\gamma(x)}_0)$$

$$x_0 = x(0)$$

$$\gamma(x_0) = 0$$



$\delta(y)$

$$\delta(y(x)) = \sum_{\substack{x_0 \text{ n\u00e9e o\u00e9n} \\ y(x_0) = 0}} \frac{1}{\left| \frac{dy}{dx}(x_0) \right|} \delta_{x_0}(x) \quad a > 0$$

$$y(x) = x^2 - a^2 \quad x_0 = a \quad x_0 = -a$$

$$\delta(x^2 - a^2) = \frac{1}{2a} \delta(x - a) + \frac{1}{2a} \delta(x + a)$$

$$\delta_{x_0}(x) = \delta(x - x_0)$$