$x_0 \cdot Q$

$x \neq x_0 \quad \rho(x) = 0$

$\int \rho \, dV = Q$

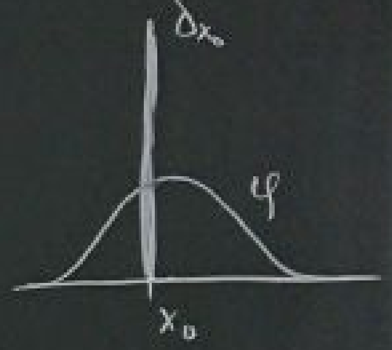
$\nexists$

Distribuce (zobecněná funkce)

Def. spojitý lineární funkcionál na prostoru testovacích funkcí

$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x)\, \varphi(x)\, dx$$

$$= T[\varphi]$$



$$T[\varphi + \lambda \psi] = T[\varphi] + \lambda T[\psi], \quad \lambda \in \mathbb{R}$$

$$\varphi_n \to \varphi$$

$$T[\varphi_n] \to T[\varphi]$$

$\delta_{x_0}(x) \qquad \delta_{x_0}$

$$\langle \delta_{x_0}, \varphi \rangle = \varphi(x_0)$$

$$\int \delta_{x_0}(x)\, \varphi(x)\, dx = \varphi(x_0) \underbrace{\int \delta_{x_0}(x) \frac{\varphi(x)}{\varphi(x_0)}\, dx}_{1}$$

$\mathcal{D}$ prostor test fcí

- hladké fce s komp. nosičem

$$\varphi_n \xrightarrow{\mathcal{D}} \varphi$$

$$\exists \text{ komp. } K \ \forall n \ \operatorname{supp} \varphi_n \subset K$$

$$\varphi_n^{[k]} \overset{K}{\rightrightarrows} \varphi^{[k]}$$

$\mathcal{D}'$ prostor distribucí

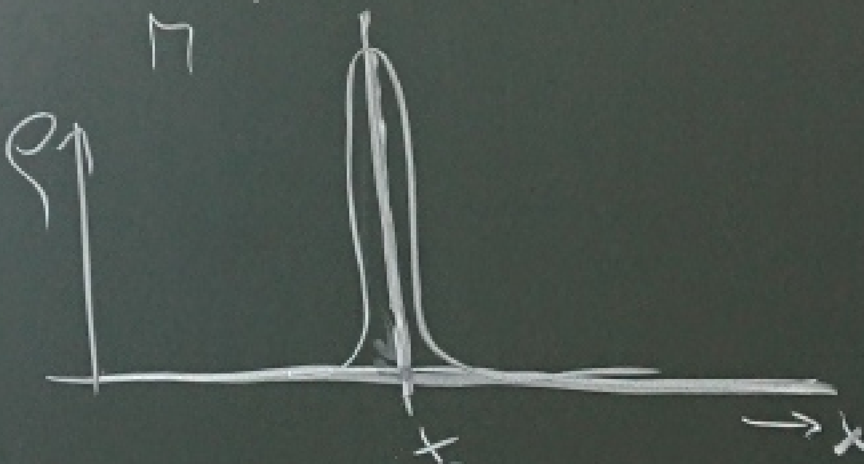
$$\mathcal{D} \subset \mathcal{D}'$$

$$\langle T, \varphi \rangle \qquad \cancel{T(x)}$$

x_0 • Q

$$x \neq x_0 \quad \rho(x) = 0$$

$$\int_{\mathbb{R}} \rho \, dV = Q$$



Distribuce (zobecněná funkce)

Def. spojitý lineární funkcionál na prostoru testovacích funkcí

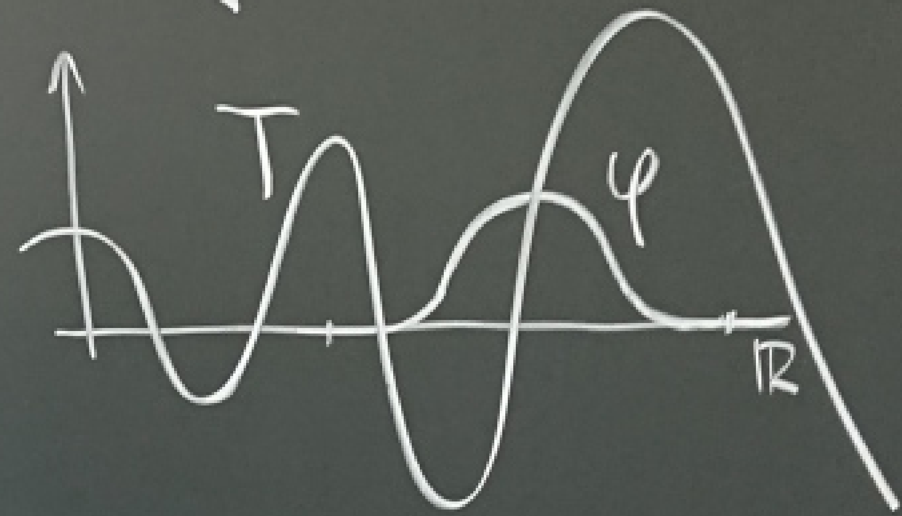
$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x)\,\varphi(x)\,dx$$

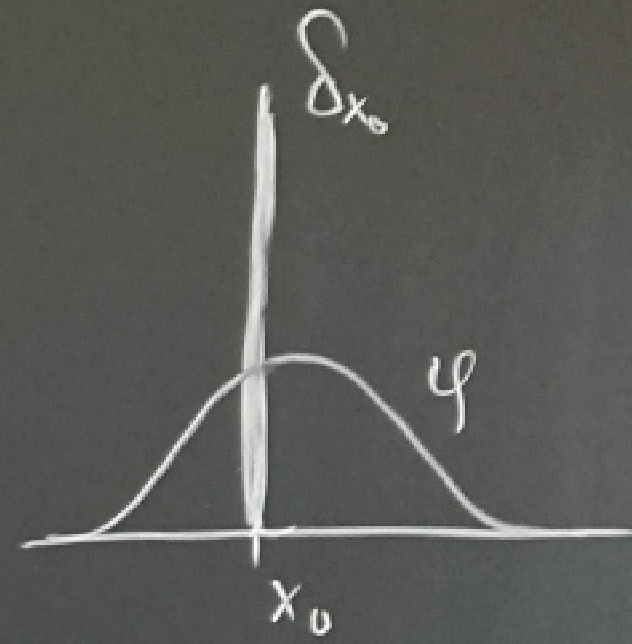
$$\langle T, \varphi \rangle = T[\varphi]$$

$$T[\varphi + \Omega \psi] = T[\varphi] + \Omega\, T[\psi], \quad \Omega \in \mathbb{R}$$

$$\varphi_n \to \varphi$$

$$T[\varphi_n] \to T[\varphi]$$





$\delta_{x_0}(x)$ $\qquad$ δ_{x_0}

$$\langle \delta_{x_0}, \varphi \rangle = \varphi(x_0)$$

$$\int \delta_{x_0}(x)\,\varphi(x)\,dx = \varphi(x_0) \underbrace{\int \delta_{x_0}(x)\,\frac{\varphi(x)}{\varphi(x_0)}\,dx}_{1}$$

$\mathcal{D}$ prostor test. fcí

– hladké fce s komp. nosičem

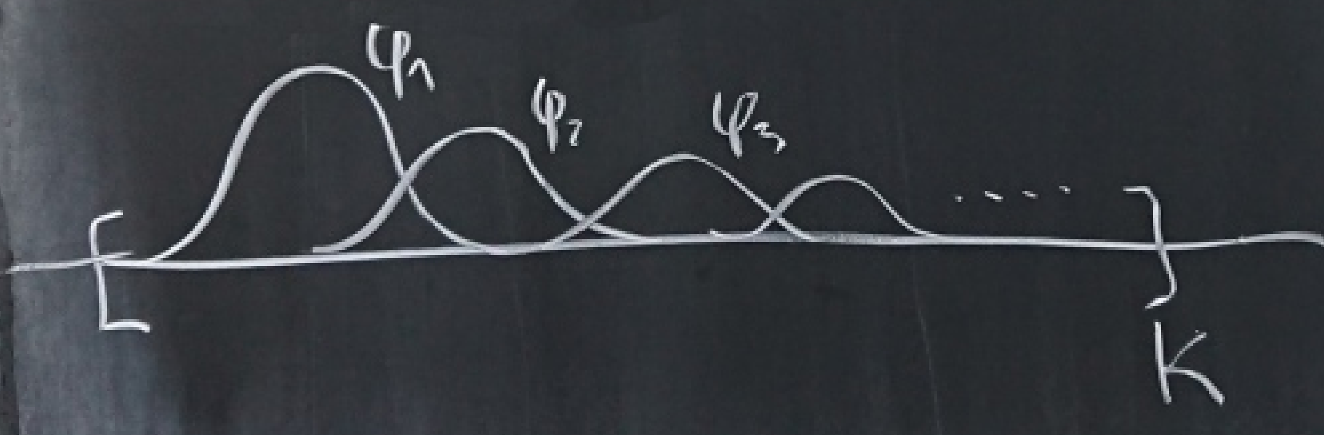
$\varphi_n \xrightarrow{\mathcal{D}} \varphi$

$\exists$ komp. K $\forall n$ $\operatorname{supp} \varphi_n \subset K$

$$\varphi_n^{[k]} \overset{K}{\rightrightarrows} \varphi^{[k]}$$

$\mathcal{D}'$ prostor distribucí

$\mathcal{D} \subset \mathcal{D}'$



$\mathcal{D}$ prostor test. fcí

– hladké fce s komp. nosičem

$$\varphi_n \xrightarrow{\mathcal{D}} \varphi$$

$$\exists \text{ komp. } K \quad \forall n \quad \operatorname{supp} \varphi_n \subset K$$

$$\varphi_n^{[k]} \overset{K}{\rightrightarrows} \varphi^{[k]}$$

$\mathcal{D}'$ prostor distribucí

$$\mathcal{D} \subset \mathcal{D}'$$

$$\langle T, \varphi \rangle \qquad \cancel{T(x)}$$

Distribuce a operace na nich

$\mathcal{D}'$ topol. duál k $\mathcal{D}$

$\langle T, \varphi \rangle =$ "$\int_{\mathbb{R}} T(x)\varphi(x)\,dx$"

$T_1 + T_2$ $\quad \langle T_1 + T_2, \varphi \rangle = \langle T_1, \varphi \rangle + \langle T_2, \varphi \rangle$

λT $\quad \langle \lambda T, \varphi \rangle = \lambda \langle T, \varphi \rangle$

fT $\quad \langle fT, \varphi \rangle = \langle T, f\varphi \rangle$

hladké fce $\quad \int fT\varphi\,dx = \int T(f\varphi)\,dx$

$$\int_{\mathbb{R}} T'\varphi\,dx = \cancel{\int_{\mathbb{R}} (T\varphi)'\,dx} - \int_{\mathbb{R}} T\varphi'\,dx$$

$$0 \quad [T\varphi]_{-\infty}^{+\infty}$$

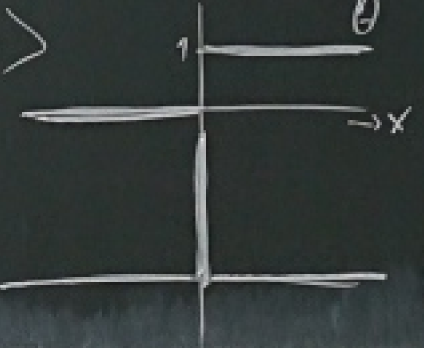
$$\langle T', \varphi \rangle \overset{\text{def}}{=} -\langle T, \varphi' \rangle$$

$$\langle \underline{(fT)'}, \varphi \rangle = -\langle fT, \varphi' \rangle$$

$$= -\langle T, f\varphi' \rangle = -\langle T, (f\varphi)' \rangle + \langle T, f'\varphi \rangle$$

$$= \langle T', f\varphi \rangle + \langle f'T, \varphi \rangle$$

$$= \underline{\langle fT' + f'T, \varphi \rangle}$$



$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

$$\langle \Theta, \varphi \rangle = \int_{\mathbb{R}} \Theta(x)\varphi(x)\,dx =$$

$$= \int_{\mathbb{R}^+} \varphi(x)\,dx$$

$$\langle \Theta', \varphi \rangle = -\langle \Theta, \varphi' \rangle$$

$$= -\int_0^\infty \varphi'(x)\,dx =$$

$$= -[\varphi]_0^\infty = \varphi(0)$$

$$= \langle \delta, \varphi \rangle$$

$$\Theta' = \delta$$

$\mathcal{D}$ prostor test. fcí

– hladké fce s komp. nosičem

$\varphi_n \xrightarrow{\mathcal{D}} \varphi$

$\exists$ komp. K $\forall n$ $\operatorname{supp} \varphi_n \subset K$

$\varphi_n^{(k)} \overset{K}{\rightrightarrows} \varphi^{(k)}$

$\mathcal{D}'$ prostor distribucí

$\mathcal{D} \subset \mathcal{D}'$

Distribuce a operace na nich

$\mathcal{D}'$ topol. duál z $\mathcal{D}$

$$\langle T, \varphi \rangle = \text{"}\int_{\mathbb{R}} T(x)\,\varphi(x)\,dx\text{"}$$

$T_1 + T_2 \qquad \langle T_1 + T_2, \varphi \rangle = \langle T_1, \varphi \rangle + \langle T_2, \varphi \rangle$

$\lambda T \qquad \langle \lambda T, \varphi \rangle = \lambda \langle T, \varphi \rangle$

$fT \qquad \langle fT, \varphi \rangle = \langle T, f\varphi \rangle$

hladká fce $\qquad \int f T \varphi \, dx = \int T (f\varphi)\, dx$

$$\int_{\mathbb{R}} T' \varphi \, dx = \cancel{\int_{\mathbb{R}} (T\varphi)' \, dx} - \int_{\mathbb{R}} T \varphi' \, dx$$

$$0 \qquad [T\varphi]_{-\infty}^{+\infty}$$

$$\langle T', \varphi \rangle \stackrel{\text{def}}{=} -\langle T, \varphi' \rangle$$

$$\langle \underline{\underline{(fT)'}}, \varphi \rangle = -\langle fT, \varphi' \rangle$$

$$= -\langle T, f\varphi' \rangle = -\langle T, (f\varphi)' \rangle + \langle T, f'\varphi \rangle$$

$$= \langle T', f\varphi \rangle + \langle f'T, \varphi \rangle$$

$$= \langle \underline{\underline{fT' + f'T}}, \varphi \rangle$$

$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

$$\langle \Theta, \varphi \rangle = \int_{\mathbb{R}} \Theta(x)\varphi(x)\,dx =$$

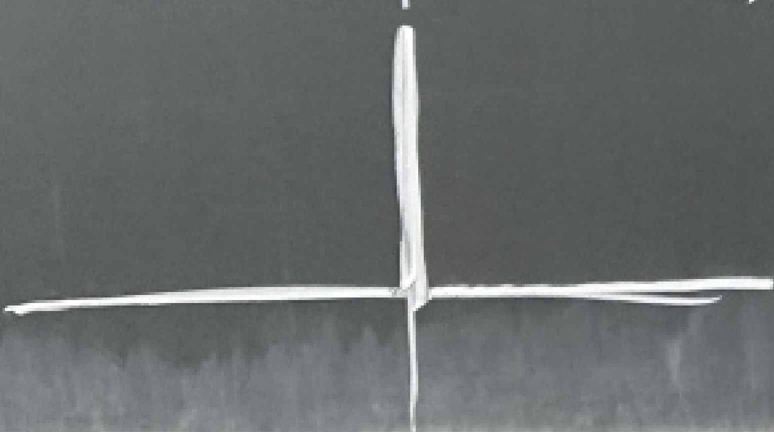
$$= \int_{\mathbb{R}^+} \varphi(x)\,dx$$

$$\langle \Theta', \varphi \rangle = -\langle \Theta, \varphi' \rangle$$

$$= -\int_0^\infty \varphi'(x)\,dx =$$

$$= -[\varphi]_0^\infty = \varphi(0)$$

$$= \langle \delta, \varphi \rangle$$



$$\Theta' = \delta$$

$$\langle T, \varphi \rangle = \int_{\mathbb{R}} T(x)\,\varphi(x)\,dx$$

$\mathcal{D}$ hladké fce s komp. nosičem $\rightarrow$ $\mathcal{D}'$ neomezené distribuce

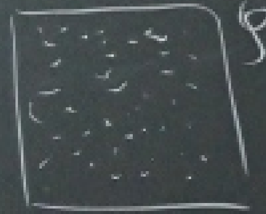
$\mathcal{E}$ hladké fce $\rightarrow$ $\mathcal{E}'$ distrib s komp. nosičem

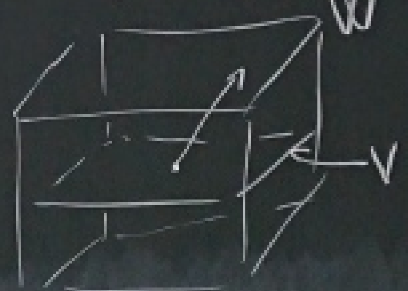
$\mathcal{S}$ rychle kles. fce $\rightarrow$ $\mathcal{S}'$ temperované distr

$\mathcal{L}^2$ kvadr. integr. fce $\rightarrow$ $\mathcal{L}^{2\prime} = \mathcal{L}^2$

hustá

$\mathcal{D} \subset \mathcal{S}$ $\qquad$ $\mathcal{S}' \subset \mathcal{D}'$ $\qquad$ $V \subset W$ $\qquad$ $V^* \subset W^*$





$\mathcal{D}$ prostor test. fcí

- hladké fce s komp. nosičem

$\varphi_n \xrightarrow{\mathcal{D}} \varphi$

$\exists$ komp. K $\forall n$ $\operatorname{supp} \varphi_n \subset K$

$\varphi_n^{[k]} \overset{K}{\rightrightarrows} \varphi^{[k]}$

$\mathcal{D}'$ prostor distribucí

$\mathcal{D} \subset \mathcal{D}'$

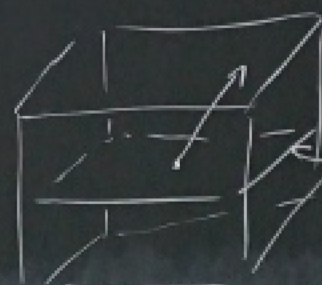
$\mathcal{D}$ hladké fce s komp. nosič. $\to$ $\mathcal{D}'$ neomezené distribuce

$\mathcal{E}$ hladké fce $\to$ $\mathcal{E}'$ distrib s komp. nosičem

$\mathcal{S}$ rychle kles. fce $\to$ $\mathcal{S}'$ temperované distr.

$\mathcal{L}^2$ kvadr. integr. fce $\to$ $\mathcal{L}^{2\prime} = \mathcal{L}^2$

$$\mathcal{D} \overset{\text{hustá}}{\subset} \mathcal{S} \qquad \mathcal{S}' \subset \mathcal{D}' \qquad V \subset W \qquad V^* \subset W^*$$



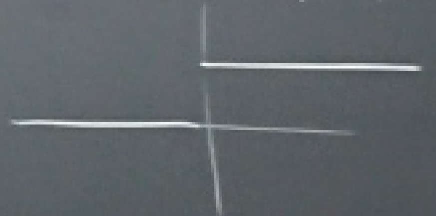
Derivace distribucí

$$\langle fT, \varphi\rangle = \langle T, f\varphi\rangle$$

$$\langle T', \varphi\rangle = -\langle T, \varphi'\rangle$$

$$\langle T^{[k]}, \varphi\rangle = (-1)^k\langle T, \varphi^{[k]}\rangle$$

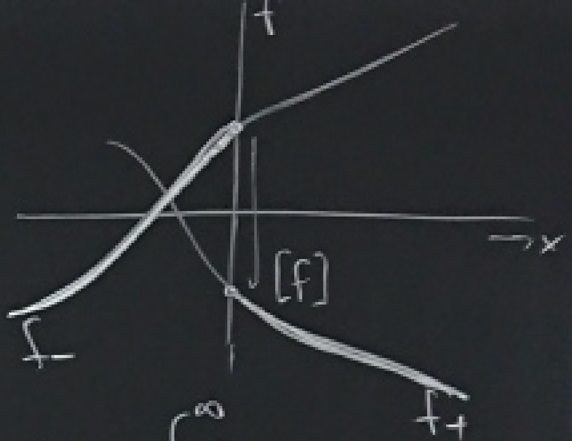
$$\Theta' = \delta$$



$$\langle T, \varphi\rangle = \text{"}\int_{\mathbb{R}} T(x)\,\varphi(x)\,dx\text{"}$$

$$f = \begin{cases} f_- & x<0 \\ f_+ & x>0 \end{cases} \qquad [f] = f_+(0) - f_-(0)$$

$$f^{(1)} = \begin{cases} f_-' & x<0 \\ f_+' & x>0 \end{cases} \qquad \boxed{f' = f^{(1)} + [f]\delta}$$



$$\langle f', \varphi\rangle = -\langle f, \varphi'\rangle = -\int_{\mathbb{R}} f(x)\varphi'(x)\,dx = -\int_{-\infty}^{0} f_-(x)\varphi'(x)\,dx - \int_0^{\infty} f_+(x)\varphi'(x)\,dx$$

$$= -\int_{-\infty}^{0}(f_-\varphi)'\,dx + \int_{-\infty}^{0} f_-'\varphi\,dx - \int_0^{\infty}(f_+\varphi)'\,dx + \int_0^{\infty} f_+'\varphi\,dx$$

$$= \langle f^{(1)}, \varphi\rangle - [f_-\varphi]_{-\infty}^{0} - [f_+\varphi]_0^{\infty} = \langle f^{(1)}, \varphi\rangle + \left(-f_-(0) + f_+(0)\right)\varphi(0)$$

$$= \langle f^{(1)}, \varphi\rangle + [f]\langle \delta, \varphi\rangle = \langle f^{(1)} + [f]\delta, \varphi\rangle$$

$$f' = f^{(1)} + [f]\delta$$

$$f'' = f^{(11)} + [f^{(1)}]\delta + [f]\delta'$$

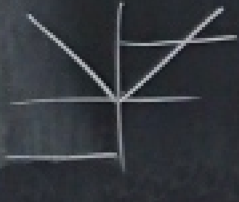
$$\vdots$$

$$f^{[k]} = f^{(k)} + [f^{(k-1)}]\delta + [f^{(k-2)}]\delta' + \dots + [f]\delta^{[k-1]}$$

$$\langle \delta', \varphi\rangle = -\langle \delta, \varphi'\rangle = -\varphi'(0)$$

$$\langle \delta'', \varphi\rangle = -\langle \delta', \varphi'\rangle = \langle \delta, \varphi''\rangle = \varphi''(0)$$

$$|x|' = \operatorname{sign} x \qquad |x|'' = 0 + 2\delta$$



$$[|x|] = 0$$

$$[|x|^{(1)}] = 2$$

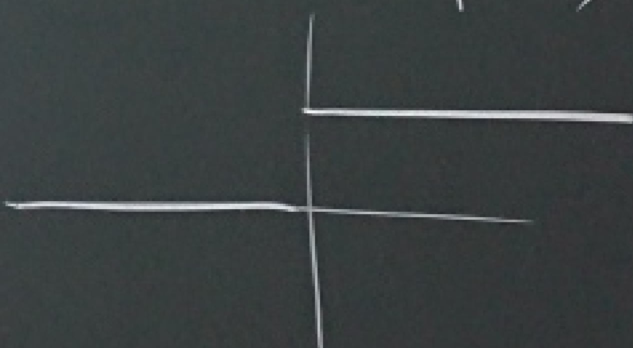
Derivace distribucí

$$\langle fT, \varphi \rangle = \langle T, f\varphi \rangle$$

$$\langle T', \varphi \rangle = -\langle T, \varphi' \rangle$$

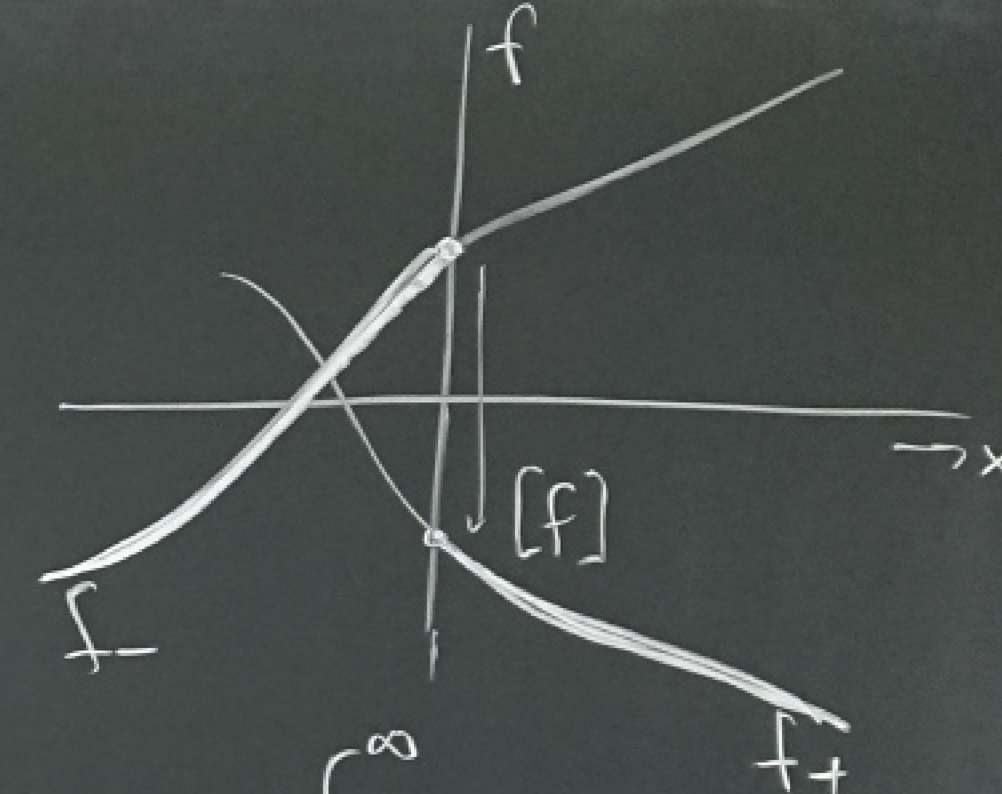
$$\langle T^{[k]}, \varphi \rangle = (-1)^k \langle T, \varphi^{[k]} \rangle$$

$$\Theta' = \delta$$



$$\langle T, \varphi \rangle = \text{"}\int_{\mathbb{R}} T(x)\,\varphi(x)\,dx\text{"}$$

$$f = \begin{cases} f_- & x<0 \\ f_+ & x>0 \end{cases} \qquad [f] = f_+(0) - f_-(0)$$



$$f^{(1)} = \begin{cases} f_-' & x<0 \\ f_+' & x>0 \end{cases} \qquad \boxed{f' = f^{(1)} + [f]\,\delta}$$

$$\langle f',\varphi\rangle = -\langle f,\varphi'\rangle = -\int_{\mathbb{R}} f(x)\,\varphi'(x)\,dx = -\int_{-\infty}^{0} f_-(x)\,\varphi'(x)\,dx - \int_{0}^{\infty} f_+(x)\,\varphi'(x)\,dx$$

$$= -\int_{-\infty}^{0} (f_-\varphi)'\,dx + \int_{-\infty}^{0} f_-'\varphi\,dx - \int_{0}^{\infty} (f_+\varphi)'\,dx + \int_{0}^{\infty} f_+'\varphi\,dx$$

$$= \langle f^{(1)},\varphi\rangle - [f_-\varphi]_{-\infty}^{0} - [f_+\varphi]_{0}^{\infty} = \langle f^{(1)},\varphi\rangle + \big(-f_-(0) + f_+(0)\big)\varphi(0)$$

$$= \langle f^{(1)},\varphi\rangle + [f]\langle\delta,\varphi\rangle = \langle f^{(1)} + [f]\delta,\varphi\rangle$$

$$f' = f^{(1)} + [f]\,\delta$$

$$f'' = f^{(11)} + [f^{(1)}]\,\delta + [f]\,\delta'$$

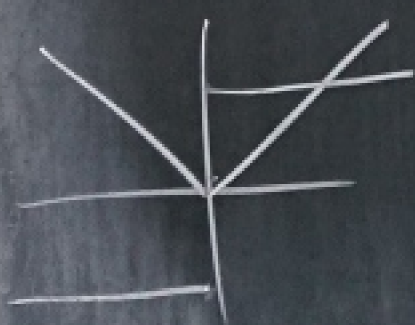
$$\vdots$$

$$f^{[k]} = f^{(k)} + [f^{(k-1)}]\,\delta + [f^{(k-2)}]\,\delta' + \dots + [f]\,\delta^{[k-1]}$$

$$\langle \delta', \varphi \rangle = -\langle \delta, \varphi' \rangle = -\varphi'(0)$$

$$\langle \delta'', \varphi \rangle = -\langle \delta', \varphi' \rangle = \langle \delta, \varphi'' \rangle = \varphi''(0)$$

$|x|' = \operatorname{sign} x \qquad |x|'' = 0 + 2\delta$



$[|x|] = 0$

$[|x|^{(1)}] = 2$

Substituce v distribuci

$$y = y(x)$$
$$x = x(y)$$
$$x = x(y(x))$$

$f(x) \qquad f(y(x)) \qquad f \circ y$

$T \qquad T \circ y$

$$\delta_{x_0}(x) = \delta(x - x_0)$$

$$\langle T, \varphi \rangle = \int T(x)\,\varphi(x)\,dx$$

$$\langle T \circ y, \varphi \rangle = \int_{\mathbb{R}} T(y(x))\,\varphi(x)\,dx$$

$y = y(x)$

$x = x(y) \qquad dx = \frac{dx}{dy}\,dy$

$$= \int_{\mathbb{R}} T(y)\,\varphi(x(y))\left|\frac{dx}{dy}(y)\right| dy$$

$$= \left\langle T, \varphi \circ x \frac{dx}{dy} \right\rangle = \left\langle T, \frac{\varphi}{\left|\frac{dy}{dx}\right|} \circ x \right\rangle$$

$$\langle T \circ y, \varphi \rangle = \left\langle T, \frac{\varphi}{\left|\frac{dy}{dx}\right|} \circ x \right\rangle$$

$$\delta(x) \longrightarrow (\delta \circ y)(x)$$

$$\langle \delta \circ y, \varphi \rangle = \left\langle \delta, \frac{\varphi}{\left|\frac{dy}{dx}\right|} \circ x \right\rangle$$

$$= \frac{\varphi}{\left|\frac{dy}{dx}\right|}(x(0))$$

$$= \frac{\varphi(x_0)}{\left|\frac{dy}{dx}(x_0)\right|}$$

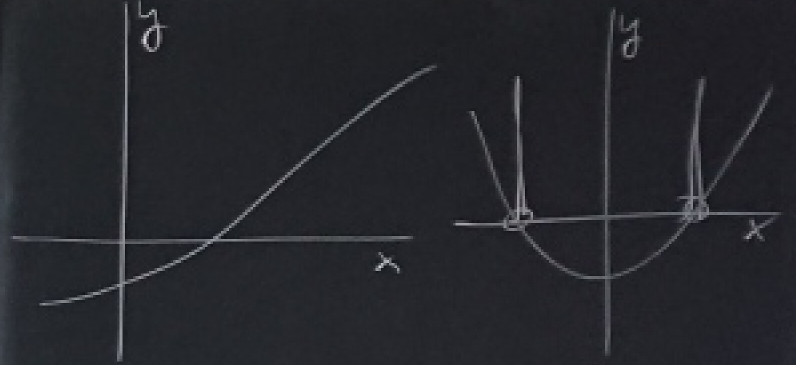
$$= \left\langle \frac{1}{\left|\frac{dy}{dx}(x_0)\right|}\delta_{x_0}, \varphi \right\rangle$$

$$\delta(\underbrace{y(x)}_{0})$$

$x_0 = x(0)$

$y(x_0) = 0$

$$\delta \circ y = \frac{1}{\left|\frac{dy}{dx}(x_0)\right|}\delta_{x_0} \qquad \delta(y(x)) = \frac{1}{\left|\frac{dy}{dx}(x_0)\right|}\delta_{x_0}(x)$$



$\delta(y)$

$$\delta(y(x)) = \sum_{x_0} \frac{1}{\left|\frac{dy}{dx}(x_0)\right|}\delta_{x_0}(x)$$

x_0 řešení $y(x_0) = 0$ $\qquad a > 0$

$y(x) = x^2 - a^2 \qquad x_0 = a \quad x_0 = -a$

$$\delta(x^2 - a^2) = \frac{1}{2a}\delta(x-a) + \frac{1}{2a}\delta(x+a)$$

Substituce v distribuci

$$y = y(x)$$
$$x = x(y)$$

$$x = x(y(x))$$

$f(x)$ $\qquad$ $f(y(x))$ $\qquad$ $f \circ y$

T $\qquad$ $T \circ y$

$$\langle T, \varphi \rangle = \int T(x)\,\varphi(x)\,dx$$

$$\langle T \circ y, \varphi \rangle = \int_{\mathbb{R}} T(y(x))\,\varphi(x)\,dx$$

$$y = y(x)$$

$$x = x(y) \qquad dx = \frac{dx}{dy}\,dy$$

$$= \int_{\mathbb{R}} T(y)\,\varphi(x(y))\left|\frac{dx}{dy}(y)\right| dy$$

$$= \left\langle T, \varphi \circ x \frac{dx}{dy} \right\rangle = \left\langle T, \frac{\varphi}{\left|\frac{dy}{dx}\right|} \circ x \right\rangle$$

$$\langle T \circ y, \varphi \rangle = \left\langle T, \frac{\varphi}{\left|\frac{dy}{dx}\right|} \circ x \right\rangle$$

$$\delta(x) \longrightarrow (\delta \circ y)(x)$$

$$\langle \delta \circ y, \varphi \rangle = \left\langle \delta, \frac{\varphi}{\left|\frac{dy}{dx}\right|} \circ x \right\rangle$$

$$= \frac{\varphi}{\left|\frac{dy}{dx}\right|}(x(0))$$

$$= \frac{\varphi(x_0)}{\left|\frac{dy}{dx}(x_0)\right|}$$

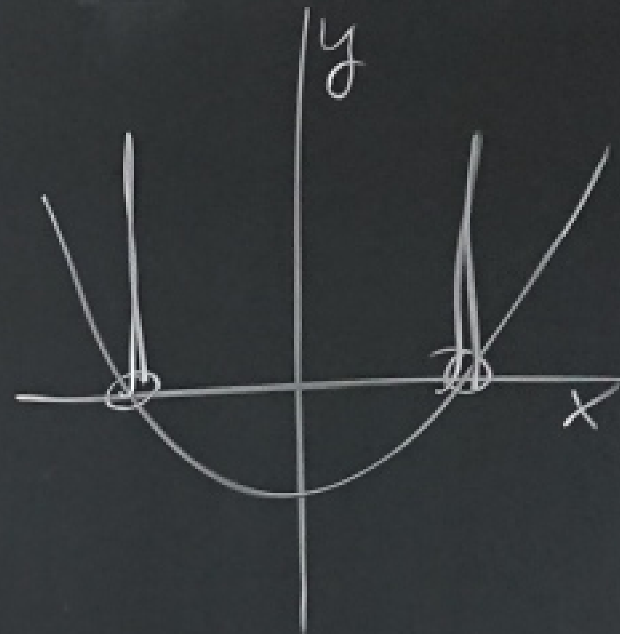
$$= \left\langle \frac{1}{\left|\frac{dy}{dx}(x_0)\right|} \delta_{x_0}, \varphi \right\rangle$$

$$\delta(\underbrace{y(x)}_{0})$$

$$x_0 = x(0)$$

$$y(x_0) = 0$$

$$\delta \circ y = \frac{1}{\left|\frac{dy}{dx}(x_0)\right|} \delta_{x_0} \qquad \delta(y(x)) = \frac{1}{\left|\frac{dy}{dx}(x_0)\right|} \delta_{x_0}(x)$$



$\delta(y)$

$$\delta(y(x)) = \sum_{x_0} \frac{1}{\left|\frac{dy}{dx}(x_0)\right|} \delta_{x_0}(x)$$

x_0 ře ōčle

$y(x_0) = 0$

$a > 0$

$y(x) = x^2 - a^2 \qquad x_0 = a \qquad x_0 = -a$

$$\delta(x^2 - a^2) = \frac{1}{2a}\delta(x-a) + \frac{1}{2a}\delta(x+a)$$

$$\delta_{x_0}(x) = \delta(x - x_0)$$