$$\langle T,\varphi\rangle = \text{“}\int_{\mathbb{R}} T(x)\,\varphi(x)\,dx\text{”}$$

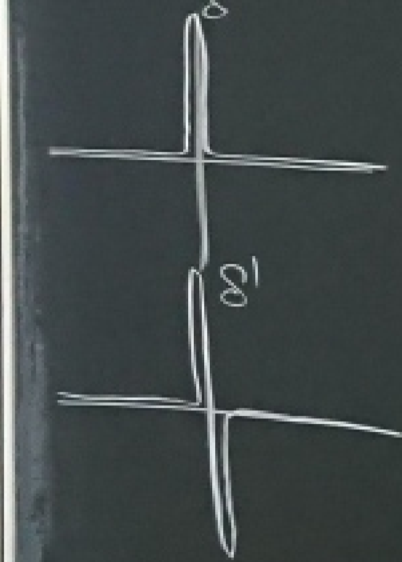
$$\langle T',\varphi\rangle = -\langle T,\varphi'\rangle$$

$$\langle \delta,\varphi\rangle = \varphi(0)$$

$$\langle fT,\varphi\rangle = \langle T, f\varphi\rangle$$

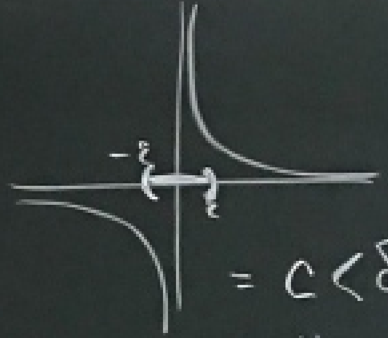
$$f(x)\,\delta(x) = f(0)\,\delta(x)$$

$$\begin{aligned} f(x)\,\delta'(x) &= (f(x)\,\delta(x))' - f'(x)\,\delta(x) \\ &= (f(0)\,\delta(x))' - f'(0)\,\delta(x) \\ &= f(0)\,\delta'(x) - f'(0)\,\delta(x) \end{aligned}$$



$$\begin{aligned} \langle f\delta',\varphi\rangle &= \langle \delta', f\varphi\rangle = \\ &= -\langle \delta,(f\varphi)'\rangle = -(f\varphi)'(0) \\ &= -f'(0)\varphi(0) - f(0)\varphi'(0) \\ &= f(0)\langle \delta',\varphi\rangle - f'(0)\langle \delta,\varphi\rangle \\ &= \langle f(0)\delta' - f'(0)\delta,\ \varphi\rangle \end{aligned}$$

$$T = \text{“}\frac{1}{x}\text{”}$$



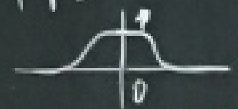
$$xT = 1$$

$$xT = 0 \iff \boxed{T = c\,\delta}$$

$$0 = \langle xT,\varphi\rangle = \langle T, x\varphi\rangle$$

$$\forall \psi \quad \psi(0) = 0 \quad \langle T,\psi\rangle = 0$$

$$\chi \quad \chi(0) = 1$$



$$\Rightarrow \varphi = \varphi(0)\,\chi + \psi \qquad \psi(0) = 0$$

$$\langle T,\varphi\rangle = \langle T,\varphi(0)\chi\rangle + \overset{0}{\langle T,\psi\rangle} = \overbrace{\langle T,\chi\rangle}^{c}\varphi(0)$$

$$= c\,\langle \delta,\varphi\rangle$$

$$\Downarrow$$

$$\boxed{T = c\,\delta}$$

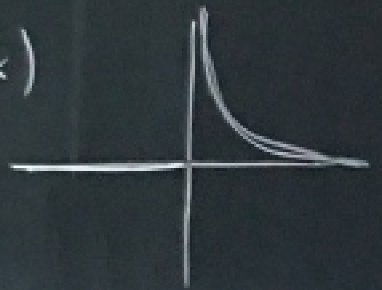
$$x\,P\frac{1}{x} = 1$$

$$T = P\frac{1}{x} + c\,\delta$$

$$xT = 1$$

$$\left\langle P\frac{1}{x},\varphi\right\rangle = \lim_{\varepsilon\to 0}\int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{x}\,\varphi(x)\,dx$$

$$\frac{1}{x}\,\theta(x)$$



$$\langle T, \varphi \rangle = \text{"} \int_{\mathbb{R}} T(x)\,\varphi(x)\,dx \text{"}$$

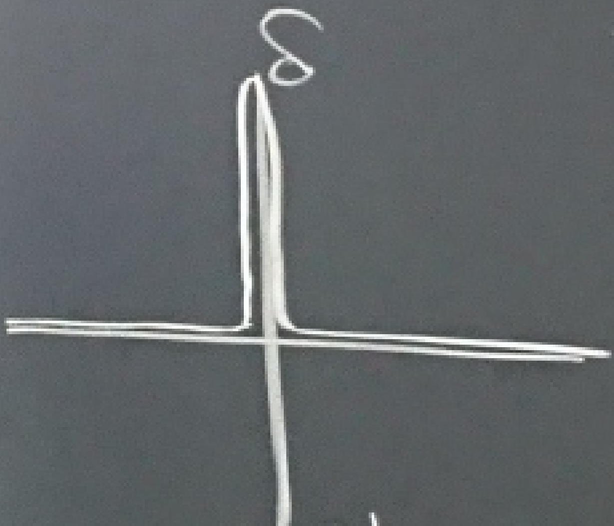
$$\langle T', \varphi \rangle = -\langle T, \varphi' \rangle$$

$$\langle \delta, \varphi \rangle = \varphi(0)$$

$$\langle fT, \varphi \rangle = \langle T, f\varphi \rangle$$

$$f(x)\,\delta(x) = f(0)\,\delta(x)$$

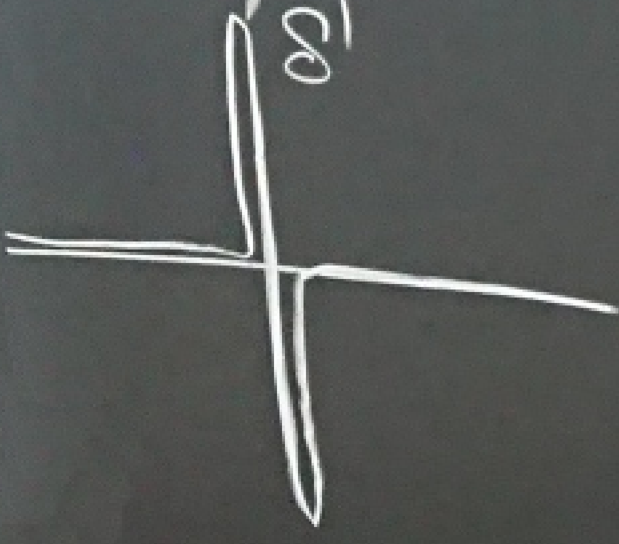
$$f(x)\,\delta'(x) = \left(f(x)\,\delta(x)\right)' - f'(x)\,\delta(x)$$



$$= \left(f(0)\,\delta(x)\right)' - f'(0)\,\delta(x)$$

$$= f(0)\,\delta'(x) - f'(0)\,\delta(x)$$

$$\langle f\delta', \varphi\rangle = \langle \delta', f\varphi\rangle =$$



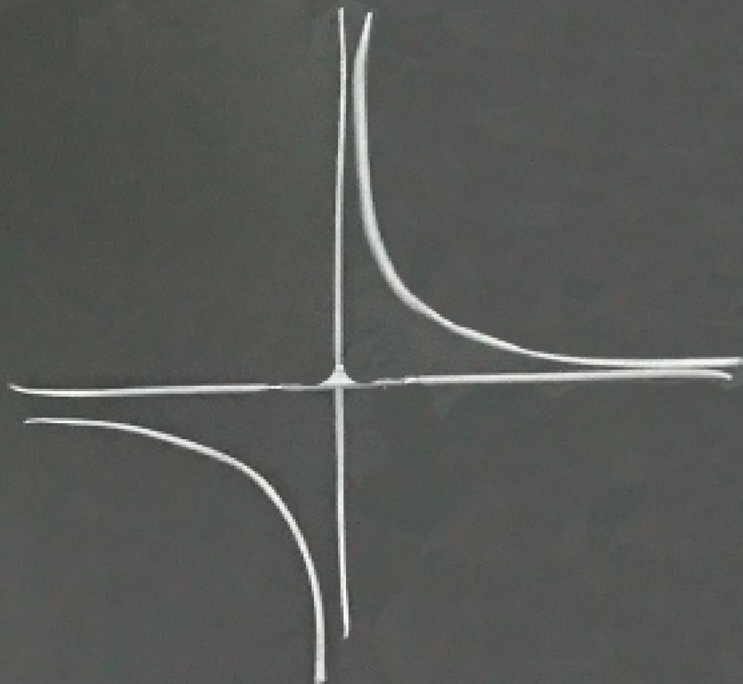
$$= -\langle \delta, (f\varphi)'\rangle = -(f\varphi)'(0)$$

$$= -f'(0)\,\varphi(0) - f(0)\,\varphi'(0)$$

$$= f(0)\langle \delta', \varphi\rangle - f'(0)\langle \delta, \varphi\rangle$$

$$= \langle f(0)\delta' - f'(0)\delta, \varphi\rangle$$

$$T = \text{"}\frac{1}{x}\text{"}$$



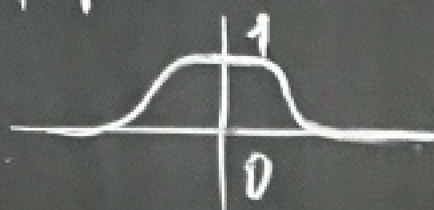
$$xT = 1$$

$$xT = 0 \iff T = c\delta$$

$$0 = \langle xT, \varphi \rangle = \langle T, x\varphi \rangle$$

$$\forall \psi \quad \psi(0) = 0 \quad \langle T, \psi \rangle = 0$$

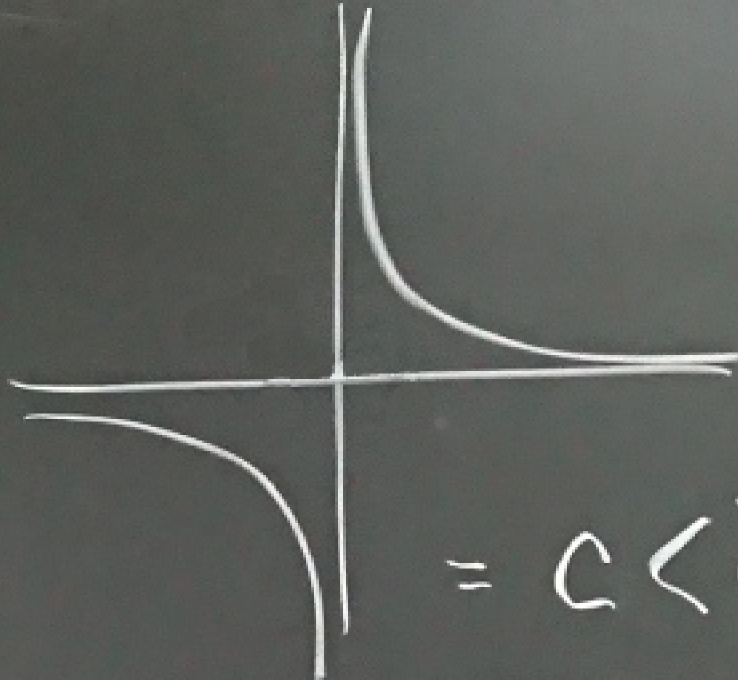
$$\chi \quad \chi(0) = 1$$



$$\Rightarrow \varphi = \varphi(0)\chi + \psi \qquad \psi(0) = 0$$

$$\langle T, \varphi \rangle = \langle T, \varphi(0)\chi \rangle + \cancelto{0}{\langle T, \psi \rangle} = \overbrace{\langle T, \chi \rangle}^{c} \varphi(0)$$

$T = "\frac{1}{x}"$



$= c\langle \delta, \varphi \rangle$

$\Downarrow$

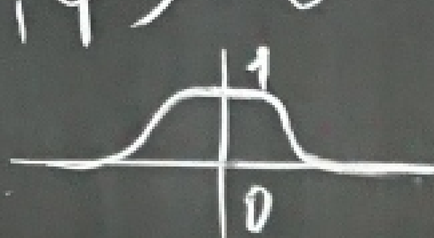
$xT = 1$

$xT = 0 \iff \boxed{T = c\delta}$

$0 = \langle xT, \varphi \rangle = \langle T, x\varphi \rangle$

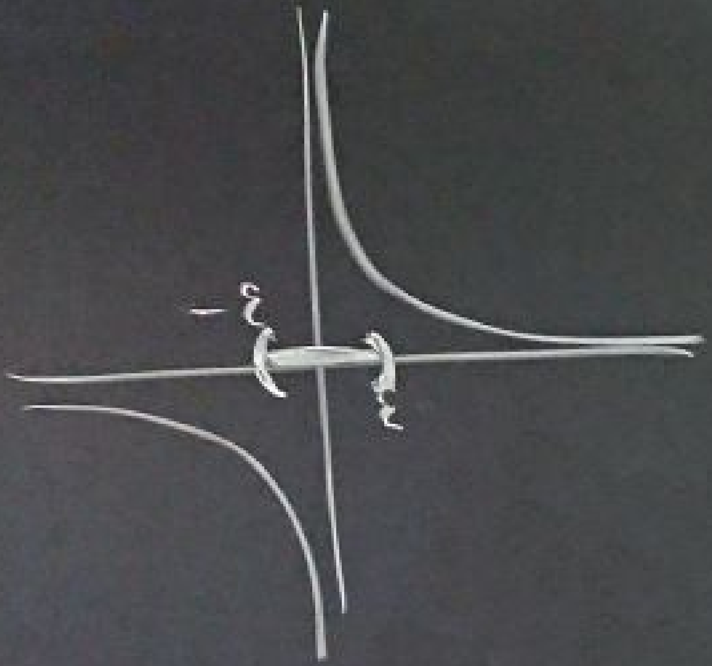
$\forall \psi \quad \psi(0) = 0 \quad \langle T, \psi \rangle = 0$

$\chi \quad \chi(0) = 1$



$\Rightarrow \varphi = \varphi(0)\chi + \psi \qquad \psi(0) = 0$

$\langle T, \varphi \rangle = \langle T, \varphi(0)\chi \rangle + \cancelto{0}{\langle T, \psi \rangle} = \overbrace{\langle T, \chi \rangle}^{c} \varphi(0)$



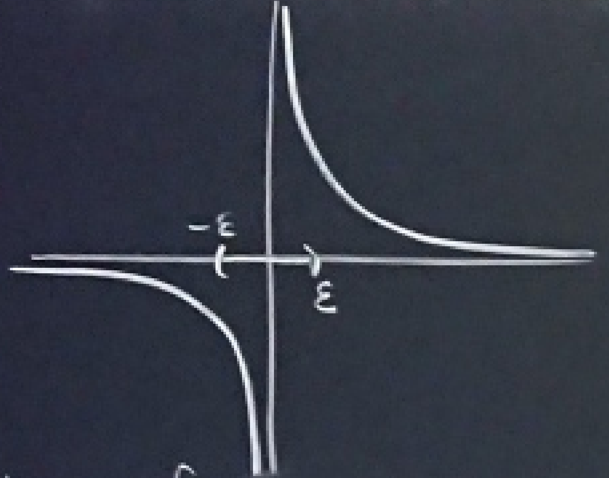
$$x\,P\frac{1}{x} = 1$$

$$T = P\frac{1}{x} + c\delta$$

$$xT = 1$$

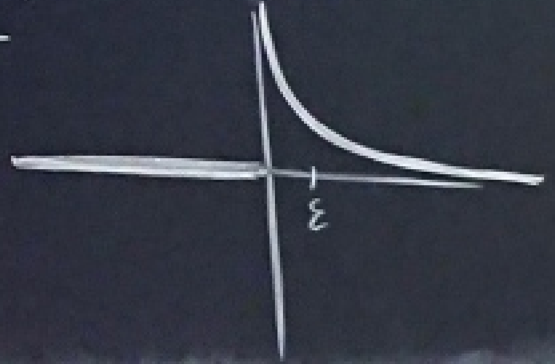
$$\left\langle P\frac{1}{x}, \varphi \right\rangle = \lim_{\varepsilon \to 0} \int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{x}\varphi(x)\,dx$$

$$\mathcal{P}\frac{1}{x}$$

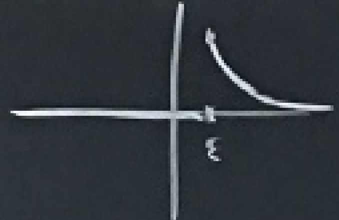


$$\left\langle \mathcal{P}\frac{1}{x}, \varphi \right\rangle = \lim_{\varepsilon \to 0} \int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{x}\varphi(x)\,dx$$

$$\text{"}\frac{\Theta}{x}\text{"}$$



$$\mathcal{R}_\varepsilon(x) = \frac{\Theta(x-\varepsilon)}{x}$$



$$\langle \mathcal{R}_\varepsilon, \varphi \rangle = \int_\varepsilon^\infty \frac{1}{x}\varphi(x)\,dx$$

$$\varphi(0) + x\varphi'(0) + \tfrac{1}{2}x^2\varphi''(0) + \dots$$

$$\langle \mathcal{R}_{\varepsilon_1} - \mathcal{R}_{\varepsilon_2}, \varphi \rangle = \int_{\varepsilon_1}^{\varepsilon_2} \frac{1}{x}\varphi(x)\,dx = \int_{\varepsilon_1}^{\varepsilon_2} \left(\frac{1}{x}\varphi(0) + \varphi'(0) + x\varphi''(0) + \dots \right) dx$$

$$= \left[\log x\right]_{\varepsilon_1}^{\varepsilon_2} \varphi(0) + (\varepsilon_2 - \varepsilon_1)\varphi'(0) + O(\varepsilon^2)$$

$$= \log\frac{\varepsilon_2}{\varepsilon_1}\,\varphi(0) + O(\varepsilon)$$

$$\mathcal{R}_{\varepsilon_1} - \mathcal{R}_{\varepsilon_2} = \log\frac{\varepsilon_2}{\varepsilon_1}\,\delta + O(\varepsilon)$$

$$(\log\varepsilon_2 - \log\varepsilon_1)\,\delta$$

$$\mathcal{R}\frac{\Theta(x)}{x} = \lim_{\varepsilon \to 0}\left(\mathcal{R}_\varepsilon(x) + \log\varepsilon\,\delta\right)$$

$$\mathcal{R}_{\varepsilon_1} + \log_{\varepsilon_1}\delta = \mathcal{R}_{\varepsilon_2} + \log_{\varepsilon_2}\delta + O(\varepsilon)$$

$$\left\langle \mathcal{R}\frac{\Theta}{x}, \varphi \right\rangle = \lim_{\varepsilon \to 0} \left[\int_\varepsilon^\infty \frac{1}{x}\varphi(x)\,dx + \varphi(0)\log\varepsilon \right]$$

$$\mathcal{R}\frac{\Theta(-x)}{x}$$

$$\mathcal{R}\frac{1}{|x|} = \mathcal{R}\frac{\Theta(x)}{x} - \mathcal{R}\frac{\Theta(-x)}{x}$$

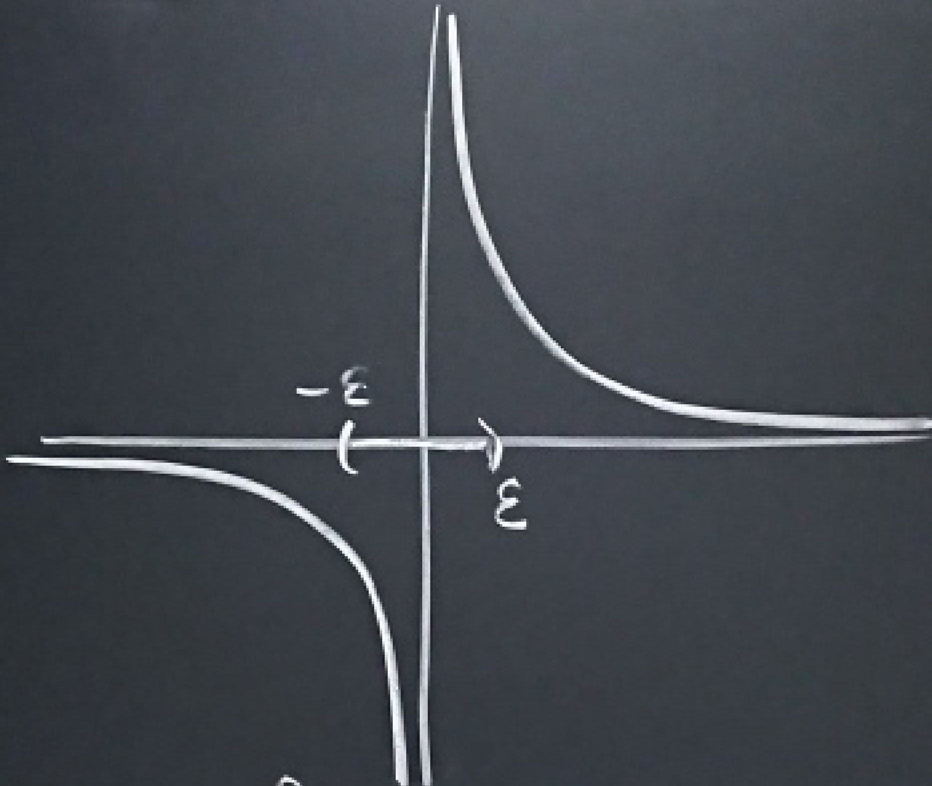
$$\left\langle \mathcal{R}\frac{1}{|x|}, \varphi \right\rangle = \lim_{\varepsilon \to 0} \left[\int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{|x|}\varphi(x)\,dx + 2\varphi(0)\log\varepsilon \right]$$

$$\mathcal{P}\frac{1}{x} = \mathcal{R}\frac{\Theta(x)}{x} + \mathcal{R}\frac{\Theta(-x)}{x}$$

$$\mathcal{P}\frac{1}{ax} = \frac{1}{a}\mathcal{P}\frac{1}{x}$$

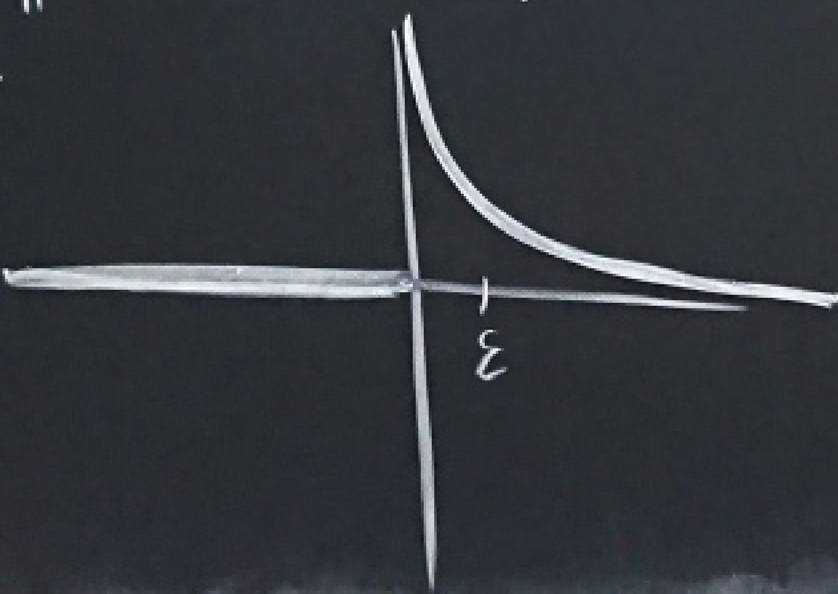
$$\mathcal{R}\frac{\Theta(\pm x)}{ax} = \frac{1}{a}\mathcal{R}\frac{\Theta(\pm x)}{x} \pm \frac{\log a}{a}\delta(x)$$

$\mathcal{P}\frac{1}{x}$

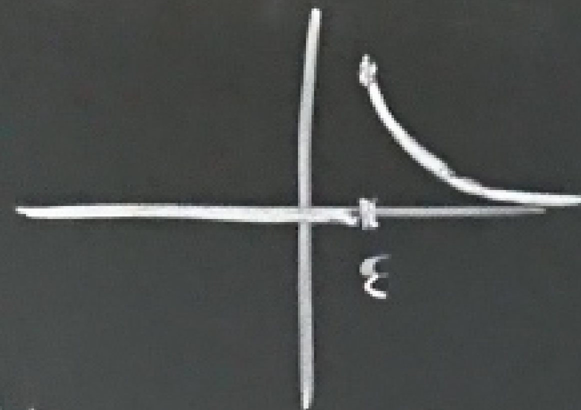


$$\langle \mathcal{P}\tfrac{1}{x}, \varphi \rangle = \lim_{\varepsilon \to 0} \int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{x}\,\varphi(x)\,dx$$

" $\frac{\Theta}{x}$ "



$$\mathcal{R}_\varepsilon(x) = \frac{\Theta(x-\varepsilon)}{x}$$



$$\langle \mathcal{R}_\varepsilon , \varphi \rangle = \int_\varepsilon^\infty \frac{1}{x}\varphi(x)\,dx$$

$$\varphi(0) + x\varphi'(0) + \frac{1}{2}x^2\varphi''(0) + \dots$$

$$\langle \mathcal{R}_{\varepsilon_1} - \mathcal{R}_{\varepsilon_2} , \varphi \rangle = \int_{\varepsilon_1}^{\varepsilon_2} \frac{1}{x}\varphi(x)\,dx = \int_{\varepsilon_1}^{\varepsilon_2}\left(\frac{1}{x}\varphi(0) + \varphi'(0) + x\varphi''(0) + \dots\right)dx$$

$$= \left[\log x\right]_{\varepsilon_1}^{\varepsilon_2}\varphi(0) + (\varepsilon_2 - \varepsilon_1)\varphi'(0) + O(\varepsilon^2)$$

$$= \log\frac{\varepsilon_2}{\varepsilon_1}\varphi(0) + O(\varepsilon)$$

$$\mathcal{R}_{\varepsilon_1} - \mathcal{R}_{\varepsilon_2} = \log\frac{\varepsilon_2}{\varepsilon_1}\delta + O(\varepsilon)$$

$$(\log\varepsilon_2 - \log\varepsilon_1)\delta$$

$$R\frac{\Theta(x)}{x} = \lim_{\varepsilon\to 0}\left(\mathcal{R}_\varepsilon(x) + \log\varepsilon\,\delta\right)$$

$$\mathcal{R}_{\varepsilon_1} + \log\varepsilon_1\,\delta = \mathcal{R}_{\varepsilon_2} + \log\varepsilon_2\,\delta + O(\varepsilon)$$

$$\langle R\frac{\Theta}{x} , \varphi \rangle = \lim_{\varepsilon\to 0}\left[\int_\varepsilon^\infty \frac{1}{x}\varphi(x)\,dx + \varphi(0)\log\varepsilon\right]$$

$$\mathcal{R}\frac{\Theta(-x)}{x}$$

$$\mathcal{R}\frac{1}{|x|} = \mathcal{R}\frac{\Theta(x)}{x} - \mathcal{R}\frac{\Theta(-x)}{x}$$

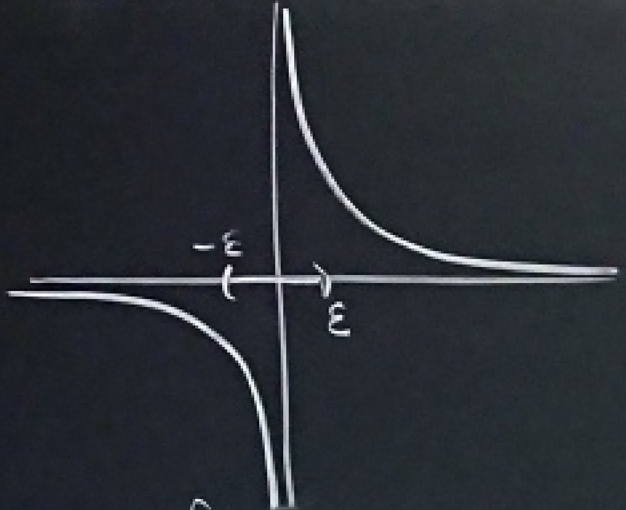
$$\left\langle \mathcal{R}\frac{1}{|x|}, \varphi \right\rangle = \lim_{\varepsilon \to 0}\left[\int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{|x|}\varphi(x)\,dx + 2\varphi(0)\log\varepsilon\right]$$

$$\mathcal{P}\frac{1}{x} = \mathcal{R}\frac{\Theta(x)}{x} + \mathcal{R}\frac{\Theta(-x)}{x}$$

$$\mathcal{P}\frac{1}{ax} = \frac{1}{a}\mathcal{P}\frac{1}{x}$$

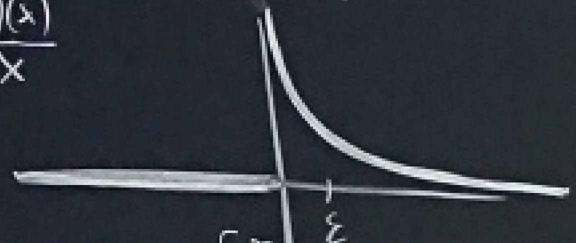
$$\mathcal{R}\frac{\Theta(\pm ax)}{ax} = \frac{1}{a}\mathcal{R}\frac{\Theta(\pm x)}{x} \pm \frac{\log a}{a}\delta(x)$$

$P\frac{1}{x}$



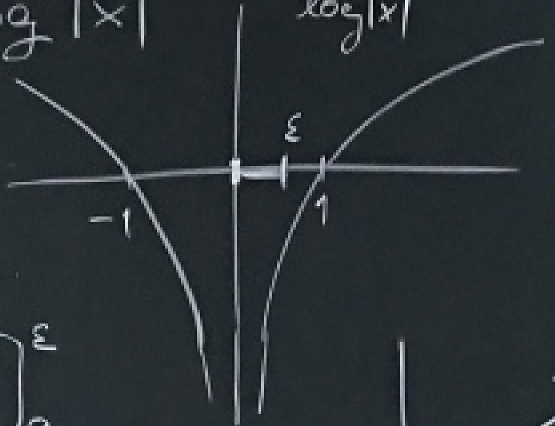
$$\langle P\tfrac{1}{x}, \varphi\rangle = \lim_{\varepsilon\to 0}\int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{x}\varphi(x)\,dx$$

$R\frac{\theta(x)}{x}$

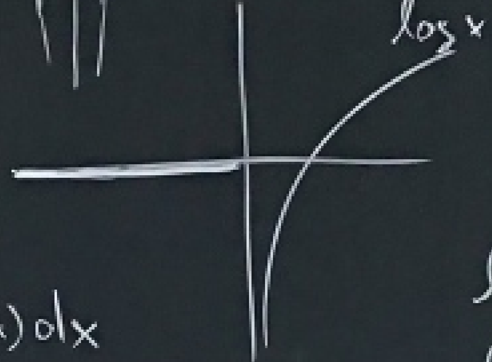


$$\langle R\tfrac{\theta(x)}{x}, \varphi\rangle = \lim_{\varepsilon\to 0}\left[\int_\varepsilon^\infty \frac{1}{x}\varphi(x)\,dx + \log\varepsilon\,\varphi(0)\right]$$

logarithms $\log|x|$



$$\int_0^\varepsilon \log x\,\varphi(x)\,dx =$$
$$\varphi(0) + x\varphi'(0)$$
$$= \varphi(0)\left[-x(1-\log x)\right]_0^\varepsilon$$
$$\sim O(\varepsilon\log\varepsilon)$$



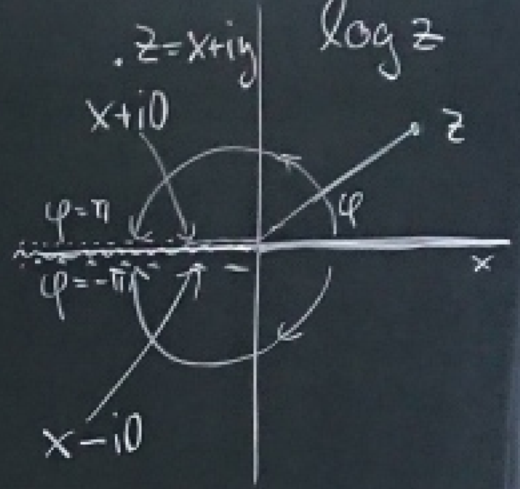
$$\langle \log|x|, \varphi\rangle = \int_{\mathbb{R}} \log|x|\,\varphi(x)\,dx$$

$$(\log|x|)' = P\frac{1}{x}$$

$$(\theta(x)\log x)' = R\frac{\theta(x)}{x} \qquad (\theta(-x)\log|x|)' = R\frac{\theta(-x)}{x}$$

$$R\frac{1}{|x|} = (\operatorname{sign}x\,\log|x|)'$$

$\log z$

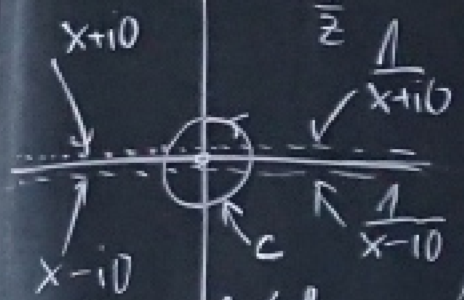


$$z = |z|\exp(i\varphi)$$
$$\log z = \log|z| + i\varphi$$
$$\log(x+i0) = \log|x| + i\pi$$
$$\log(x-i0) = \log|x| - i\pi$$
$x<0$

$$\langle \delta, \varphi\rangle = \varphi(0) = \frac{1}{2\pi i}\int_C \frac{\varphi(z)}{z}\,dz$$

$$\log(x\pm i0)'$$

$$\frac{1}{x\pm i0} = P\frac{1}{x} \mp i\pi\,\delta(x)$$
C

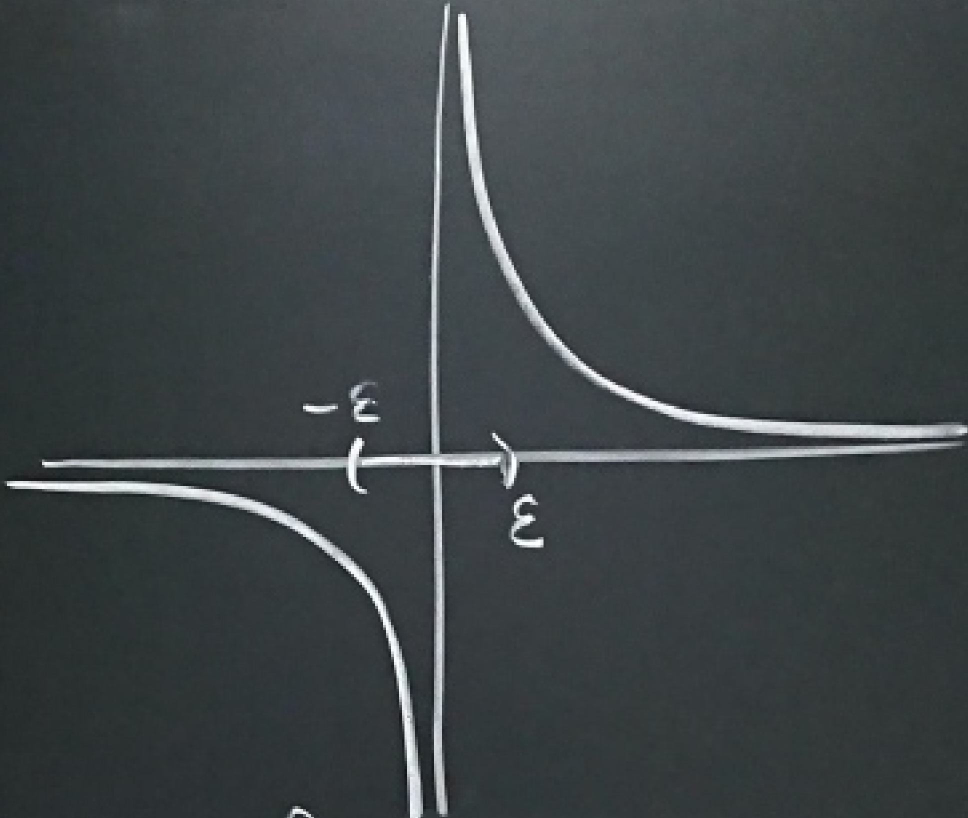


$$x\,\frac{1}{x\pm i0} = 1$$

$$P\frac{1}{x} = \frac{1}{2}\left(\frac{1}{x+i0} + \frac{1}{x-i0}\right)$$

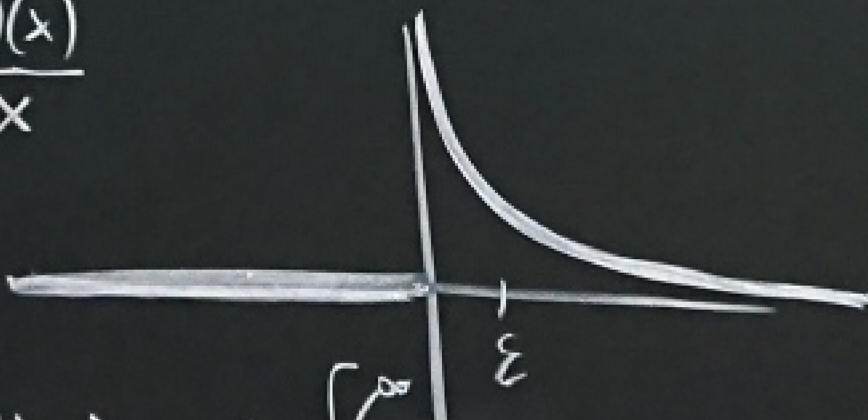
$$\delta(x) = \frac{1}{2\pi i}\left(\frac{1}{x-i0} - \frac{1}{x+i0}\right)$$

$\mathcal{P}\frac{1}{x}$



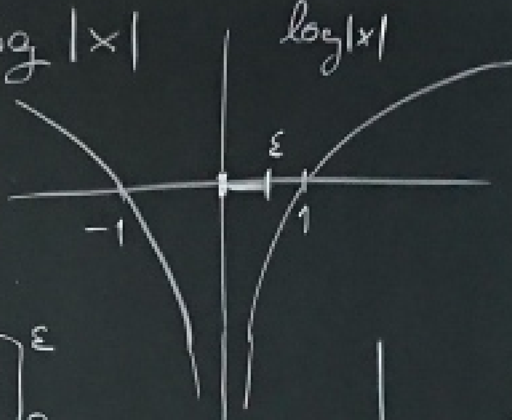
$$\left\langle \mathcal{P}\frac{1}{x}, \varphi \right\rangle = \lim_{\varepsilon \to 0} \int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{x}\varphi(x)\,dx$$

$\mathcal{R}\frac{\theta(x)}{x}$



$$\left\langle \mathcal{R}\frac{\theta(x)}{x}, \varphi \right\rangle = \lim_{\varepsilon \to 0} \left[\int_{\varepsilon}^{\infty} \frac{1}{x}\varphi(x)\,dx + \log\varepsilon\, \varphi(0) \right]$$

logarithms $\log|x|$

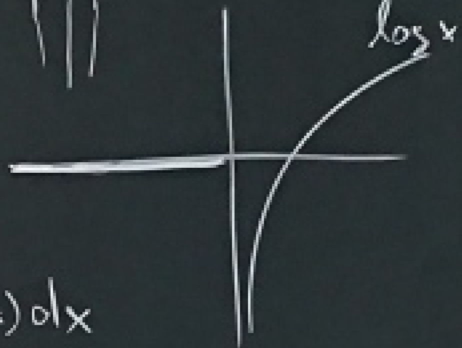


$$\int_0^{\varepsilon} \log x \, \varphi(x)\, dx =$$

$$\varphi(0) + x\varphi'(0)$$

$$= \varphi(0)\left[-x(1-\log x)\right]_0^{\varepsilon}$$

$$\sim O(\varepsilon \log \varepsilon)$$



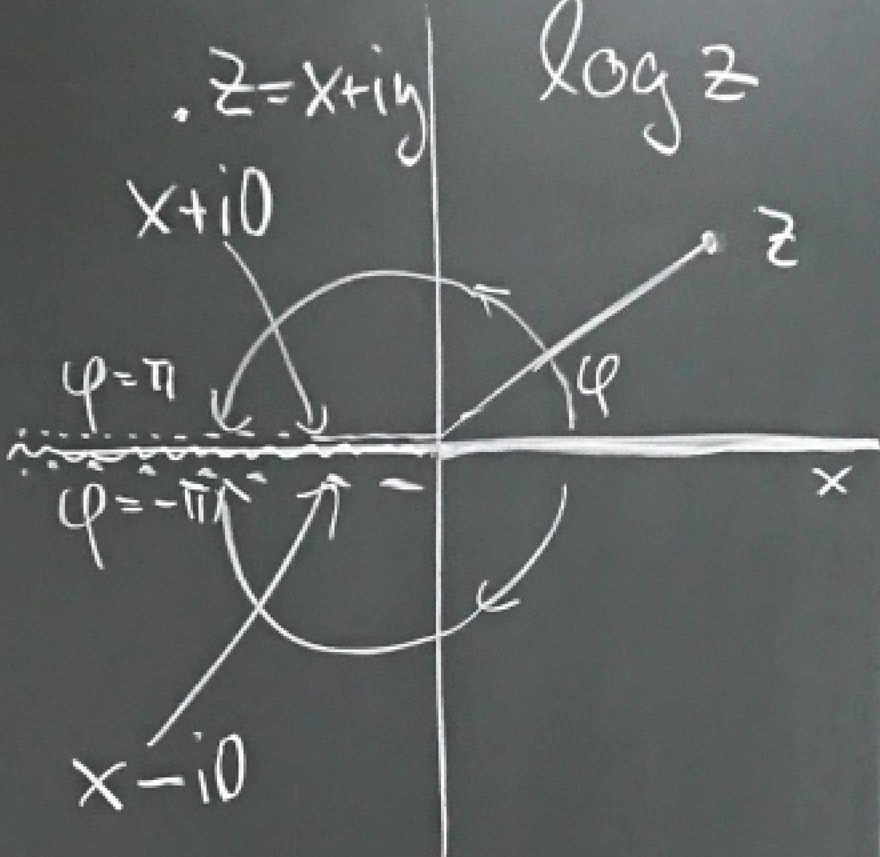
$$\langle \log|x|, \varphi \rangle = \int_{\mathbb{R}} \log|x|\, \varphi(x)\, dx$$

$$(\log|x|)' = P\frac{1}{x}$$

$$(\theta(x)\log x)' = R\frac{\theta(x)}{x} \qquad (\theta(-x)\log|x|)' = R\frac{\theta(-x)}{x}$$

$$R\frac{1}{|x|} = (\operatorname{sign} x \, \log|x|)'$$

$\log z$



$$z = |z| \exp(i\varphi)$$

$$\log z = \log|z| + i\varphi$$

$$\log(x+i0) = \log|x| + i\pi$$

$$\log(x-i0) = \log|x| - i\pi$$

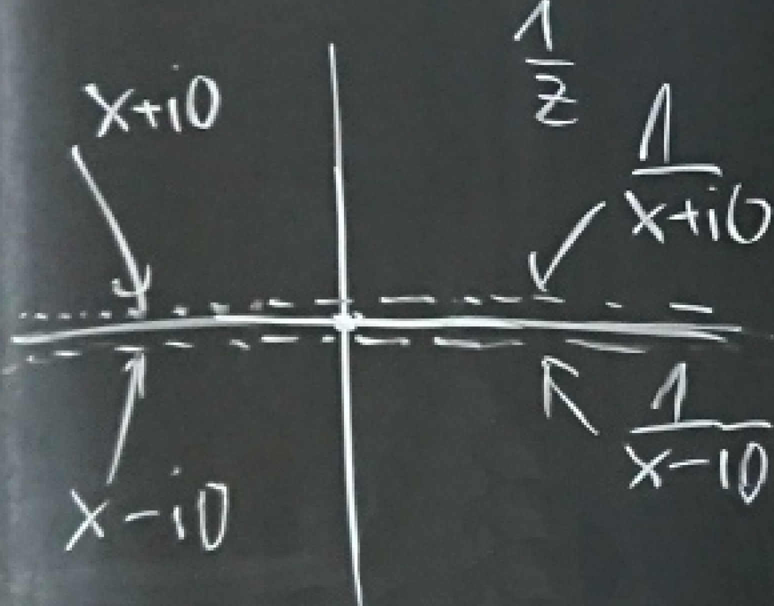
$$x<0$$

$$\log(x\pm i0) = \log|x| \pm i\pi\,\Theta(-x)$$

$$\log(x\pm i0)'$$

$$\frac{1}{x\pm i0} = P\frac{1}{x} \mp i\pi\,\delta(x)$$

c

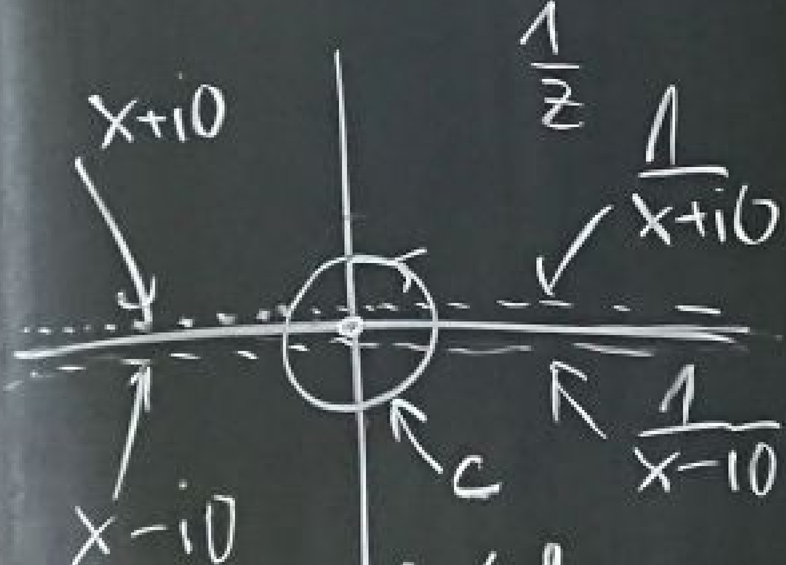


$$x\,\frac{1}{x\pm i0} = 1$$

$$\log(x\pm i0)'$$

$$\frac{1}{x\pm i0} = P\frac{1}{x} \mp i\pi\,\delta(x)$$

C



$$x\frac{1}{x\pm i0} = 1$$

$$P\frac{1}{x} = \frac{1}{2}\left(\frac{1}{x+i0} + \frac{1}{x-i0}\right)$$

$$\delta(x) = \frac{1}{2\pi i}\left(\frac{1}{x-i0} - \frac{1}{x+i0}\right)$$

$$\langle\delta,\varphi\rangle = \varphi(0) = \frac{1}{2\pi i}\int_C \frac{\varphi(z)}{z}\,dz$$